

# Geometric Aspects of Evolution and Control

## Book of abstracts

FernUniversität in Hagen, April 17–21, 2023

### Speakers

1	Andrea Adriani	2
2	Sergei Avdonin	2
3	İlknur Kuçbeyzi Aybar	3
4	Krzysztof Bogdan	3
5	Leonie Brinker	4
6	Michela Egidi	4
7	Pavel Exner	5
8	Florian Fischer	5
9	Tuncay Gençoğlu	6
10	Alexander Grigor'yan	6
11	Amru Hussein	7
12	Marc Hütt	7
13	James Kennedy	8
14	Joachim Kerner	9
15	Martin Lukarevski	9
16	José Mazón	9
17	Michael McGettrick	10
18	Ivan Moyano	10
19	Serge Nicaise	11
20	Mats-Erik Pistol	11
21	Vyacheslav Pivovarchik	12
22	Sergio Polidoro	12
23	Olaf Post	12
24	Aleksandra Puchalska	13

25	Christian Rose	13
26	Stefan Rotter	14
27	Benjamin Schäfer	15
28	Arick Shao	15
29	Tom ter Elst	15
30	Ivan Veselić	16
31	Enzo Vitillaro	16
32	John Wainwright	17
33	Enrique Zuazua	17

## 1 Andrea Adriani

(Università degli Studi dell'Insubria)

Tuesday, 16:00 – 16:30

*The  $L^1$ -Liouville property on graphs*

In this talk we investigate the  $L^1$ -Liouville property on infinite graphs, underlining its connection with stochastic completeness and other structural features of infinite (weighted) graphs. We give a characterization of the  $L^1$ -Liouville property in terms of the Green function of a graph and use it to prove its equivalence with stochastic completeness in the case of weakly spherically symmetric graphs. Moreover, we show that there exist stochastically incomplete graphs which satisfy the  $L^1$ -Liouville property and state some comparison theorems for general graphs based on inner-outer curvatures. We then introduce the Dirichlet  $L^1$ -Liouville property of subgraphs and state that if a graph has a Dirichlet  $L^1$ -Liouville subgraph, then it is  $L^1$ -Liouville itself. As a consequence, we obtain that the  $L^1$ -Liouville property is not affected by a finite perturbation of the graph and, just as in the continuous setting, a graph is  $L^1$ -Liouville provided that at least one of its ends is Dirichlet  $L^1$ -Liouville.

## 2 Sergei Avdonin

(University of Alaska Fairbanks, United States)

Wednesday, 11:45 – 12:15

*Exact controllability for wave equation on general metric graphs with non-smooth controls*

We study the exact controllability problem for the wave equation on a finite metric graph with the Kirchhoff-Neumann matching conditions. Among all

vertices and edges we choose certain active vertices and edges, and give a constructive proof that the wave equation on the graph is exactly controllable if  $(H^1(0, T))'$  Neumann controllers are placed at the active vertices and  $L^2(0, T)$  Dirichlet controllers are placed at the active edges. The proofs for the shape and velocity controllability are purely dynamical, while the proof for the exact controllability utilizes both dynamical and moment method approaches. The control time for this construction is determined by the chosen orientation and path decomposition of the graph. The talk is based on joint work with Julian Edward (FIU).

### 3 İlknur Kuçbeyzi Aybar

(Yeditepe University, Istanbul, Turkey)

Tuesday, 14:45 – 15:15

*The Geometry of Neuronal Excitability and Limit Cycle Bifurcations*

Regular spiking neurons can be described as oscillators, raising the investigation of the existence and stability of the steady-states and the limit cycle bifurcations in neuronal excitability. When sorting electrophysiological data, the elements that can switch the neuronal responses from undergoing excitatory to inhibitory (or v.v.), such as the frequency of neuronal synapses, amplitude, and the area under the spike, can be qualitatively indicated by obtaining the conditions for the existence and stability of the steady-states and the limit cycle bifurcations. By considering the geometry of the phase portraits in these equilibria and bifurcations, one can understand many electrophysiological characteristics of neurons, such as excitability threshold, spike frequency, spike amplitude, spike states, and source of spike delays. Then, one can understand how these characteristics, which can switch a neuron to an excitatory state, are related dynamically.

#### References:

- [1] Izhikevich, E.M. (2010) *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*. The MIT Press.
- [2] Kusbeyzi Aybar, I. (2021) Memristor-based oscillatory behavior in the Fitz-Hugh-Nagumo and Hindmarsh–Rose models. *J. Nonlinear Dyn.* 103:2917-2929.
- [3] Romanovski, V.G., Shafer, D.S. (2009) *The Center and Cyclicity Problems*. Birkhauser, Boston.

### 4 Krzysztof Bogdan

(Wrocław University of Science and Technology, Poland)

Tuesday, 09:00–09:30

*Concatenation of Markov processes and nonlocal boundary value problems*

Local and nonlocal Schrödinger perturbations of integral kernels are now well understood. We will discuss how to use them to pose and resolve nonlocal boundary value problems, using the setting of Markovian semigroups and processes. Our discussion will be based on papers:

**References:**

- [1] Time-dependent Schrödinger perturbations of transition densities, K. Bogdan, W. Hansen, T. Jakubowski, *Studia Math.* 189 (2008),
- [2] On nonlocal perturbations of integral kernels, K. Bogdan, S. Sydor, in *Semigroups of operators—theory and applications*, Springer Proc. Math. Stat., 113 (2015).
- [3] The fractional Laplacian with reflections, K. Bogdan, M. Kunze, arXiv:2211.05511 (2022).

## 5 Leonie Brinker

(FernUniversität in Hagen, Germany)

Tuesday, 11:00 – 11:30

*Optimal stochastic control of the weighted occupation time in an unfavourable set (with reflection)*

Stochastic control can be used as a basis for decision making for various real-world problems, for example from economics and physics. A popular solution approach is to consider Hamilton-Jacobi-Bellman differential equations, which can be understood as infinitesimal 'dynamic programming' for stochastic control problems. For different control mechanisms, we show how the HJB approach can be adapted to minimise the expected time during which a stochastic process is bounded away by at least  $d > 0$  from its own running maximum. The results can be interpreted in the context of financial mathematics, where the drawdown (i.e. the relative loss in comparison to the last historical record high) serves as a performance-adjusted measure of business risk.

## 6 Michela Egidi

(University of Rostock, Germany)

Thursday, 14:45 – 15:15

*From harmonic analysis to control theory: Logvinenko-Sereda-type Theorems*

The uncertainty principle known as the Logvinenko-Sereda Theorem has become popular in control theory as a tool to show null-controllability of the heat equation on  $\mathbb{R}^d$ . In this talk I will survey over its manipulation tailored to applications in control theory and present a general framework to obtain it.

## **7 Pavel Exner**

(Doppler Institute for Mathematical Physics and Applied Mathematics, Prague, Czech Republic)

Monday, 09:00 – 09:30

*Geometrically induced magnetic transport*

A homogeneous magnetic field localizes motion of a charged particle in the perpendicular direction, both classically and quantum mechanically, and in order to achieve a transport in such a system, not parallel with the field, an infinitely long ‘obstacle’ is needed. We recall several types of such perturbations which make the spectrum of the corresponding Hamiltonian absolutely continuous, devoting the main attention to geometric perturbations of layers giving rise to a magnetic transport; we note that this is an effect without a classical analogue. We also mention open problems in this area.

## **8 Florian Fischer**

(Universität Potsdam, Germany)

Thursday, 16:00 – 16:30

*Optimal  $p$ -Hardy Weights on Locally Finite Graphs*

The Hardy inequality stands in the core of mathematical physics as having a Hardy inequality is strongly connected to hyperbolic manifolds and graphs, transient jump processes and subcritical energy functionals. Thus, it connects geometry, probability theory and analysis in a certain sense. Besides studying the validity of the inequality, the discussion of the optimality of the corresponding weights is of main interest. In this talk, we introduce the corresponding quasi-linear setting on locally finite graphs and show a method to obtain optimal Hardy weights. We illustrate this method on the natural numbers and on regular trees. If the time permits, we also discuss characterisations of having a Hardy inequality in the quasi-linear setting.

## 9 Tuncay Gençoğlu

(Firat University, Elazığ, Turkey)

Thursday, 17:30 – 18:00

*Quantum Transformation Based Blockchain*

Blockchain is one of the hottest topics today. The study shows how to strengthen this technology with quantum conversion and speed up the mining procedure using modified Grover's algorithm.

### References:

- [1] E.O. Kiktenko, N.O. Pozhar, M.N. Anufriev, A.S. Trushechkin, R.R. Yunusov, Y.V. Kurochkin, A.I. Lvovsky, and A.K. Fedorov. Quantum-secured blockchain. Technical Report arXiv:1705.09258, Cornell University Library, May 2017.
- [2] Jonathan Jogenfors. Quantum bitcoin: An anonymous and distributed currency secured by the no-cloning theorem of quantum mechanics. Technical Report arXiv:1604.01383, Cornell University Library, Apr 2016.
- [3] Kazuki Ikeda. qbitcoin: A peer-to-peer quantum cash system. Technical Report arXiv:1708.04955, Cornell University Library, Nov 2017.
- [4] Divesh Aggarwal, Gavin K. Brennen, Troy Lee, Miklos Santha, and Marco Tomamichel. Quantum attacks on bitcoin, and how to protect against them. Technical Report arXiv:1710.10377, Cornell University Library, Oct 2017.
- [5] Lov K. Grover. A fast quantum mechanical algorithm for database search. In Proceedings of the Twenty-eighth Annual ACM Symposium on Theory of Computing, STOC '96, pages 212-219, New York, NY, USA, 1996. ACM.

## 10 Alexander Grigor'yan

(Universität Bielefeld, Germany)

Tuesday, 16:45 – 17:15

*Propagation speed of non-linear parabolic equations on Riemannian manifolds*

We consider on arbitrary Riemannian manifold the Leibenson equation

$$\partial_t u = \Delta_p u^q.$$

This equation comes from hydrodynamics where it describes filtration of a turbulent compressible liquid in porous medium. Our main result is that if  $p > 1$  and  $q > 1/(p - 1)$  then bounded solutions of this equation have finite propagation speed. In  $R^n$  this result has been long known due to explicit Barenblatt

solutions. The proof on manifolds is based on a certain mean value inequality for subsolutions.

This work is joint with Philipp Sürig.

## 11 Amru Hussein

(Rheinland-Pfälzische Technische Universität Kaiserslautern-Landau, Germany)

Thursday, 11:00 – 11:30

*Non-self-adjoint boundary conditions on graphs: generators and invariances*

Classical boundary conditions for elliptic operators include for instance Dirichlet, Neumann and Robin boundary conditions. Going beyond this, I consider here the model case of a Laplacian on a finite metric graph subject to general non-self-adjoint matching conditions imposed at the graph's vertices. The generators of  $C_0$ -semigroups in  $L^2$ -spaces are characterized, and invariance properties such as reality, positivity and  $L^\infty$ -contractivity of the corresponding semigroup are investigated. It turns out that in each case the respective properties depend only on the boundary conditions, and moreover these can be read out of certain parametrizations of the boundary conditions.

The talk is based on joint works with David Krejcirik (Czech Technical University in Prague), Petr Siegl (TU Graz), Delio Mugnolo (FernUniversität Hagen), and Paul Beckermann (RPTU Kaiserslautern-Landau).

## 12 Marc Hütt

(Constructor University, Bremen, Germany)

Thursday, 09:00 – 09:30

*A network embedded in a circular chromosome*

For a long time it has been hypothesized that bacterial gene regulation involves an intricate interplay of the transcriptional regulatory network (TRN) and the spatial organization of genes along the chromosome. In the first part of my talk, summarizing our previous work [1, 2, 3], I explore this hypothesis both on a structural and on a functional level, using a bacterial gene regulatory network as the main example.

On the structural level, we study the TRN as a spatially embedded network, where the embedding space is the circular chromosome. Our work is motivated by 'wiring economy' research in Computational Neuroscience. On the functional level, we analyze gene expression patterns from a network perspective ('digital

control'), as well as from the perspective of the spatial organization of the chromosome ('analog control'). This large-scale analysis supports the notion that two logically distinct types of genetic control are cooperating to regulate gene expression in a complementary manner.

In the second part of my talk I will provide a stylized perspective on this system by approximating the TRN as a random Boolean network and the impact of chromosomal organization as the local update rules of a cellular automaton running on the circular chromosome. By studying the complexity of the spatiotemporal dynamics as a function of network architecture and spatial computation, we can delineate, which systemic features enhance or reduce stability.

Summarizing, our statistical analyses and modeling efforts provide insight in this biological example of a control system shaped by evolution. Our results suggest that reliable gene expression patterns require the interplay of chromosomal organization and network regulation.

#### References:

- [1] Kosmidis, K., & Hütt, M. T. (2019). The E. coli transcriptional regulatory network and its spatial embedding. *European Physical Journal E*, 42, 1-9.
- [2] Kosmidis, K., Jablonski, K. P., Muskhelishvili, G., & Hütt, M. T. (2020). Chromosomal origin of replication coordinates logically distinct types of bacterial genetic regulation. *npj Systems Biology and Applications*, 6(1), 5.
- [3] Cakir, E., Lesne, A., & Hütt, M. T. (2021). The economy of chromosomal distances in bacterial gene regulation. *npj Systems Biology and Applications*, 7(1), 49.

## 13 James Kennedy

(University of Lisbon, Portugal)

Tuesday, 11:45 – 12:15

*Spectral geometry and surgery of quantum graphs: an overview*

We give a brief introduction to, and summary of, developments in the study of eigenvalue estimates for quantum graphs based on the geometry and topology of the graphs, both recent and over the last five to ten years. We will focus in particular on so-called surgery techniques, where the goal is to make generally small, localised changes to the graph which have a predictable effect on one or all eigenvalues.

This is based largely (but not entirely) on joint work with Gregory Berkolaiko, Pavel Kurasov and Delio Mugnolo.



## 14 Joachim Kerner

(FernUniversität in Hagen, Germany)

Thursday, 16:45 – 17:15

*Comparing the spectra of Schrödinger operators on metric and discrete graphs*

We start with comparing eigenvalues of Schrödinger operators on finite, compact metric graphs and derive asymptotic results for the mean value of spectral differences. To obtain those results, we derive a local Weyl law for Schrödinger operators on graphs which is valid also for general (local) boundary conditions. We then derive related results for discrete graphs and, based on the main result, discuss seemingly new notions of circumference (or surface area) for graphs. This discussion naturally leads to a certain class of large graphs which might prove interesting in the future when it comes to applications. This talk is based on joint work with Patrizio Bifulco (Hagen).

## 15 Martin Lukarevski

(University ‘Goce Delcev’, Stip, Macedonia)

Monday, 11:00 – 11:30

*Maximal regularity for evolution equations and application*

Maximal regularity is a useful tool for solving abstract parabolic evolution equations. We first give some definitions and properties of maximal regularity and then show how it can be applied to one particular free boundary problem – the Stefan problem. A variant of the one-phase quasistationary Stefan problem can be transformed to a fixed domain and then it can be reduced to a single evolution equation. Using  $H^\infty$ -calculus it can be shown that the operator in the evolution equation has maximal regularity. From an existence theorem for this type of evolution equation, we obtain the solvability of the Stefan problem.

## 16 José Mazón

(Universitat de Valencia, Spain)

Monday, 09:45 – 10:15

*Torsional rigidity in random walk spaces*

This lecture deals with the (nonlocal) torsional rigidity in the ambient space of random walk spaces. We get the relation of the (nonlocal) torsional rigidity of a set  $\Omega$  with the spectral  $m$ -heat content of  $\Omega$ , what gives rise to a complete

description of the nonlocal torsional rigidity of  $\Omega$  by using uniquely probability terms involving the set  $\Omega$ ; and recover the first eigenvalue of the nonlocal Laplacian with homogeneous Dirichlet boundary conditions by a limit formula using these probability term. For the random walk in  $\mathbb{R}^N$  associated with a non singular kernel, we get a nonlocal version of the Saint-Venant inequality, and, under rescaling we recover the classical Saint-Venant inequality. We study the nonlocal  $p$ -torsional rigidity and its relation with the nonlocal Cheeger constants. We also get a nonlocal version of the Pólya-Makai-type inequalities. We relate the torsional rigidity given here for weighted graphs with the torsional rigidity on metric graphs.

## 17 Michael McGettrick

(University of Galway, Ireland)

Wednesday, 11:00 – 11:30

*Evolution of strategies for iterated Quantum Games on graphs*

We will present work in progress on the analysis of optimum strategies for iterated quantum games played on a graph. Quantum Games generalize classical game theory to allow operations (strategy choices) given by Unitary matrices. The (intrinsically quantum) property of entanglement has been shown in certain cases to give an advantage (to both/all players) not obtainable classically (for example in the celebrated CHSH game). The study of agents (players) interacting (playing games) on a network (graph) extends naturally to the study of quantum agents. One must consider issues such as (1) what are the strategy update rules, (2) how does one distribute/share any entanglement (which is a resource) on the graph, (3) convergence (or lack of such) of the strategy choices for each player. Iterated strategies can be viewed as a dynamical system. We present initial results for the simple example of the iterated quantum Prisoners Dilemma played on the Cycle graph  $C_n$ .

## 18 Ivan Moyano

(University of Nice, Sophia Antipolis, France)

Monday, 14:45 – 15:15

*Uncertainty principles in control theory of PDEs*

In this talk we review some classical and recent results relating the uncertainty principles for the Laplacian with the controllability and stabilisation of some linear PDEs. The uncertainty principles for the Fourier transforms state that a square integrable function cannot be both localised in frequency and space

without being zero, and this can be further quantified resulting in unique continuation inequalities in the phase spaces. Applying these ideas to the spectrum of the Laplacian on a compact Riemannian manifold, Lebeau and Robbaino obtained their celebrated result on the exact controllability of the heat equation in arbitrarily small time. The relevant quantitative uncertainty principles known as spectral inequalities in the literature can be adapted to a number of different operators, including the Laplace-Beltrami operator associated to  $C^1$  metrics or some Schrödinger operators with long-range potentials, as we have shown in recent results in collaboration with Gilles Lebeau (Nice) and Nicolas Burq (Orsay), with a significant relaxation on the localisation in space. As a consequence, we obtain a number of corollaries on the decay rate of damped waves with rough dampings, the simultaneous controllability of heat equations with different boundary conditions and the controllability of the heat equation with rough controls.

## 19 Serge Nicaise

(Université Polytechnique Hauts-de-France, France)

Friday, 09:00 – 09:30

*Stability properties of dissipative evolution equations with nonautonomous and nonlinear dampings*

In this talk, we will present some stability results of (abstract) dissipative evolution equations with a nonautonomous and nonlinear damping using the exponential stability of the retrograde problem with a linear and autonomous feedback and a comparison principle. We will further illustrate our abstract statements for different concrete examples, where new results are achieved.

## 20 Mats-Erik Pistol

(Lund University, Sweden)

Monday, 14:00 – 14:30

*Isospectral quantum graphs and their Titchmarsh-Weil  $m$ -functions*

We have found all 364 sets of equilateral but not isomorphic isospectral quantum graphs with at most nine vertices, using computer algebra.

This includes two isospectral partners to the loop. Furthermore we have found an algorithm to determine if two vertices have the same Titchmarsh-Weil  $m$ -function, which allows us to combinatorially generate arbitrarily large sets of isospectral quantum graphs. All sets of vertices with the same  $m$ -function have been found for all equilateral isospectral graphs with at most seven vertices.

We can also generate graphs with as many vertices with the same m-function as we want. We will give examples of isospectral trees and of isospectral graphs containing only loops to exemplify the diversity of isospectral graphs. The talk is based on: <https://arxiv.org/abs/2104.12885> but will include new results. If time allows we will demonstrate our software, which is open-source.

## 21 Vyacheslav Pivovarchik

(South Ukrainian National Pedagogical University, Odessa, Ukraine)

Tuesday, 14:00 – 14:30

*Recovering the shape of a quantum tree by two spectra*

We show how to find the shape of an equilateral tree using the spectra of the Neumann and the Dirichlet problems generated by the Sturm-Liouville equation on a metric tree.

In case of snowflake trees the spectra of the Neumann and Dirichlet problems uniquely determine the shape of the tree.

## 22 Sergio Polidoro

(University of Modena and Reggio Emilia, Italy)

Monday, 11:45 – 12:15

*Degenerate Kolmogorov Equations in Relativistic Kinetic Theory*

I first present a survey on the regularity theory of degenerate Kolmogorov equations and on their application in kinetic theory, then the most recent developments in the setting of special relativity will be discussed. This research has been developed in collaboration with Francesca Anceschi and Annalaura Rebusci.

## 23 Olaf Post

(University of Trier, Germany)

Wednesday, 09:00 – 09:30

*Resolvent convergence in varying spaces*

In this talk, I present some recent results on generalised norm resolvent convergence: Weidmann proposed such a concept by embedding everything in a common Hilbert space and consider convergence there. Another concept is to

use so-called identification operators close to unitary operators. I will also comment on applications to control theory. This is a joint work with Sebastian Zimmer (Uni Trier).

## 24 Aleksandra Puchalska

(University of Warsaw, Poland)

Thursday, 11:45 – 12:15

*Inviscid Burgers equation on a metric tree*

Consider the classical inviscid Burgers equation  $u_t + uu_x = 0$  in a monodimensional case. The basic interpretation of the system explains the motion of one wave, creation of shocks and rarefaction waves, but does not capture the interaction of waves, since big waves always overtake smaller ones. Hoping to receive passing through phenomenon between waves, think now about the extension of the monodimensional structure of the domain to a graph, giving the possibility for the solution to take different paths.

The above reasoning serves us as a motivation to develop the theory of the Burgers equation on the metric graph. Alike many network problems, the main difficulty here is to determine the behaviour in vertices to make the problem well-posed. In the talk we will show some interesting examples as well as key points of mathematical theory. In particular we concentrate on the motion that can change the direction of a flow in the vertex which allows for the backflow modelling.

### References:

- [1] Piotr B. Mucha, Aleksandra Puchalska, Burgers' Equation Revisited: Extension of Mono-Dimensional Case on a Network (2022) Journal of Mathematical Fluid Mechanics vol. 24, 112. DOI: 10.1007/s00021-022-00737-9.

## 25 Christian Rose

(University of Potsdam, Germany)

Friday, 11:00 – 11:30

*Gaussian upper bounds for heat kernels on graphs with unbounded geometry*

The heat kernel as the minimal fundamental solution of the heat equation is one of the most important objects studied in geometric analysis and encodes the geometry of the underlying space in analytic terms. On graphs it is nowadays known that the short time behaviour of the heat kernel is very different from

the short time behaviour of heat kernels on manifolds due to the non-locality of the graph Laplacian. However, on large scales, i.e., large times or distances, heat kernels on graphs satisfying certain regularity properties are expected to behave like heat kernels on manifolds with corresponding properties. Our main result is an optimal Gaussian decay estimate for large times and distances for any graph satisfying volume doubling and Sobolev inequalities on large balls encoded via intrinsic metrics. I will illustrate our main result for the important cases of normalizing and counting measures. Our estimates particularly yield Gaussian bounds for heat kernels on anti-trees, for which such bounds had been unknown so far. This is joint work with Matthias Keller.

## 26 Stefan Rotter

(Bernstein Center Freiburg & Faculty of Biology, University of Freiburg, Germany)

Thursday, 09:45 – 10:15

*Associative remodeling and repair in self-organizing networks*

Structural plasticity and network remodeling are prominent and well-documented processes during brain development, maturation, learning, and aging. Stun-ningly high turnover rates have been observed under baseline conditions, and even more so upon stimulation or perturbation. However, these results raise many questions about the underlying biological mechanisms and biological function of this drastic form of brain plasticity. Since direct experiments are notoriously difficult to perform and analyze, mathematical models are crucial for predicting and understanding the emergent properties of graphs or networks with highly dynamic structure. To this end, we consider ensembles of directed multigraphs with constrained indegrees and outdegrees, the Directed Configuration Model, as this reflects well the limited material resources of brain networks. Similar to the law of mass action of chemistry, we outline a kinetic theory of random graph formation and structural self-organization. We study the equilibria of these networks and describe the transient effects of perturbation. We apply our theoretical findings to biological neural networks of the brain, where the vertices are neurons and the directed edges are chemical synapses between neurons. In this setting, stimulation leads to perturbation of the network structure, followed by dynamic reconfiguration and convergence to a new structural equilibrium that is different from the previous one. For specific perturbation strategies, we obtain models for brain maturation during adolescence and for engram formation during learning and memory. Potential applications in machine learning are also discussed.

## 27 Benjamin Schäfer

(Karlsruhe Institute of Technology, Germany)

Tuesday, 09:45 – 10:15

*Using data and machine learning to understand the power grid*

The transition towards renewable energy generation raises several questions about control, stability and operation and therefore requires a solid understanding of existing and future power systems. The power grid frequency is the central observable in power system control, as it measures the balance of electrical supply and demand. Here, we use a data-driven approach analysing the power grid frequency to work towards a quantitative understanding of the power grid. We present an open data base with time series from various synchronous areas such as Continental Europe, Great Britain, the Western and Texas Interconnection, as well as several European islands. We analyse the data and highlight significant deviations from Gaussianity in several regions, scaling laws and spatio-temporal dynamics. Furthermore, we utilize state-of-the-art machine learning approaches to forecast trajectories and identify risks and drivers for frequency stability. Overall, we offer a model-free and data-centered perspective on understanding power systems.

## 28 Arick Shao

(Queen Mary University of London, United Kingdom)

Friday, 09:45 – 10:15

*Control of parabolic equations with inverse square infinite potential wells*

We consider heat operators on a bounded convex domain, with a critically singular potential diverging as the inverse square of the distance to the boundary of the domain. We establish a general boundary controllability result for such operators in all spatial dimensions, in particular providing the first such result in more than one spatial dimension. The key step in the proof is a novel global Carleman estimate that captures both the relevant boundary asymptotics and the appropriate energy for this problem. The estimate is derived by combining two intermediate Carleman inequalities with distinct and carefully constructed weights involving non-smooth powers of the boundary distance.

## 29 Tom ter Elst

(University of Auckland, New Zealand)

Friday, 11:45 – 12:15

*Does diffusion determine the drum?*

The famous question of Kac is whether one can hear the shape of a drum. Or more precisely, whether all eigen frequencies of a drum determine the drum up to congruency. In general the answer to the latter question is negative if one allows polygon shaped drums. The eigen frequencies are equal if and only if there exists a unitary operator which maps the Laplacian on the first drum onto the Laplacian on the second drum. Equivalently, this unitary operator intertwines between the two heat semigroups on the drums. In this talk we discuss what happens if the unitary operator is replaced by an order isomorphism, i.e., if it maps positive functions to positive functions. Then that order isomorphism maps positive solution of the heat equation on the first drum onto positive solutions on the second drum. Phrased differently: the order isomorphism tells that the diffusion on the two drums are the same. Hence the question: if the diffusion on two drums is the same, are they then congruent?

This is joint work with W. Arendt and M. Biegert.

## **30 Ivan Veselić**

(Technische Universität Dortmund, Germany)

Monday, 16:45 – 17:15

*Spectral inequalities and observability for Schrödinger operators with unboundedly growing potentials*

For the heat equation on  $\mathbb{R}^d$  it is known that the heat equation is observable from a sensor set if and only if the set is thick. For (sufficiently regular) bounded domains observability of the heat equation holds already if the sensor set has positive Lebesgue measure. We consider a third class of models lying between the two just mentioned and motivated by kinetic theory. The semigroup generator is a Schroedinger operator with a quadratic or some other regularly growing potential. We identify classes of sensors sets leading to observability and null controllability. In particular, in some cases finite volume sensor sets are allowed, even though the configuration space is unbounded. This is joint work with Alexander Dicke and Albrecht Seelmann.

## **31 Enzo Vitillaro**

(Università di Perugia, Italy)

Wednesday, 09:45 – 10:15

*Three evolution problems modelling the interaction between acoustic waves and non-locally reacting surfaces*



We deal with three evolution problems arising in the physical modelling of acoustic phenomena of small amplitude in a homogeneous, ideal, nonviscous and compressible fluid, bounded by a surface of extended reaction. The problems are posed in a bounded and simply connected domain  $\Omega$  of  $\mathbb{R}^3$  with boundary  $\Gamma = \partial\Omega$  of class  $C^{r,1}$  with  $r = 2, 3, \dots, \infty$ , where  $\Gamma = \Gamma_0 \cup \Gamma_1$ ,  $\Gamma_1 \cap \Gamma_0 = \emptyset$ ,  $\Gamma_1$  being nonempty and connected. The first problem arises from modelling the phenomenon in an Eulerian framework, so the fields characterizing the problem are incremental pressure, velocity and boundary deformation. The second one arises from modelling the phenomenon in a Lagrangian framework, so the fields appearing are the displacement and the boundary deformation. The third one is the well-known wave equation with acoustic boundary conditions. In the talk we shall give well-posedness and regularity results for the Cauchy problems related to the first two models and give the exact relations between the three problems, which are not exactly equivalent.

## 32 John Wainwright

(Durham University)

Thursday, 14:00 – 14:30

*tba*

## 33 Enrique Zuazua

(University of Erlangen–Nuremberg, Germany)

Wednesday, 16:00 – 16:30

*Control and Machine Learning*

We present some recent results on the interplay between control and Machine Learning.

We adopt the perspective of the simultaneous or ensemble control of systems of Residual Neural Networks (ResNets) and present a genuinely nonlinear and constructive method, allowing to show that such an ambitious goal can be achieved, estimating the complexity of the control strategies.

This property is rarely fulfilled by the classical dynamical systems in Mechanics and the very nonlinear nature of the activation function governing the ResNet dynamics plays a determinant role.

The turnpike property is also analyzed in this context.

This lecture is inspired in joint work, among others, with Borjan Geshkovski

(MIT), Carlos Esteve (Cambridge), Domènec Ruiz-Balet (IC, London), Dario Pighin (Sherpa.ai) and Martin Hernández (FAU).