On Overdissipation of Rents in Contests with Endogenous Intrinsic Motivation

Volker Schlepütz

Diskussionsbeitrag Nr. 421
Februar 2008

Diskussionsbeiträge der Fakultät für Wirtschaftswissenschaft
der FernUniversität in Hagen

Herausgegeben vom Dekan der Fakultät

On Overdissipation of Rents in Contests with Endogenous Intrinsic Motivation

Volker Schlepütz*
FernUniversität in Hagen, Department of Economics,
Profilstrasse 8, D-58084 Hagen, Germany

February 21, 2008

Abstract

In this paper, we show how endogenously determined intrinsic motivation affects the rents dissipated in a contest. If players just add an intrinsic benefit to their taking part in the contest, then in the unique Nash equilibrium all players activate their intrinsic motivation. Most important, if the intrinsic value players attach to their taking part in the contest is sufficiently high, overdissipation of rents occurs. If, however, players maximize a weighted sum of extrinsic and intrinsic payoffs, they deactive their intrinsic motivation and overdissipation of rents does not occur.

Keywords: Contest; Intrinsic motivation; Rent dissipation
JEL *classification:* D72, D74, C72

*corresponding address: volker.schlepuetz@fernuni-hagen.de; phone: +49(0)2331 987 2591, fax: +49(0)2331 987 347.

1 Introduction

In recent years, increased attention has been devoted to how insights from behavioral economics change standard results in economic decision making. An important contribution to behavioral economics emerged from the recognition of intrinsic motivation, a psychological construct introduced by Deci (1975). Intuitively, intrinsic motivation can be regarded as a mental resource. Therefore, increased effort is expected if these resource is activated. Economists have adopted this concept in the past mainly to show how agents' intrinsic motivation crowds in or out under different kinds of incentives in principal-agent relations.[1] The main insight from this literature is that external incentives should be carefully designed as to secure intrinsic motivation since in principal-agent relations increased effort is usually desired by the principal.

However, increased effort is not always socially valuable. An important example is a contest where players compete for a prize of a given size. In this case, efforts made by players to win the prize are completely wasteful because nothing socially valuable is produced.[2] The aim of this paper is to demonstrate that the players' possibility of activating intrinsic motivation is indeed used in a contest, and that this psychic engagement may lead to overdissipation of rents in the standard rent seeking contest of Tullock (1980).

Our contribution adds to the emerging field of behavioral contest design. Konrad (2004) shows how preferences for altruism and envy influences the equilibrium efforts in a contest, and how such preferences shape the population mixture of altruistic and envious players. In our paper, we endow players with a possible preference for intrinsic motivation and analyze how these preferences develop in equilibrium of a contest.

Riechmann (2007) demonstrates that the dissipation of rents is increased if contestants are assumed to maximize relative payoff rather than absolute payoff. Our paper is closest to Riechmann (2007) since exogenously given intrinsic motivation can be regarded as a player's mental ability. Such abilities decrease the net costs of efforts. Contrary to Riechmann, however, our work treats abilities as an additional strategy of the players. Mental abilities are thus *endogenised* in our contest.

Our approach is also similar to James (2005) who analyses intrinsic moti-

[1] For an overview, see Frey and Jegen (2001).

[2] See Corchon (2007) for on overview on the design of optimal contest from a social welfare point of view.

vation in the framework of a principal-agent relation. While his paper focuses on the crowding out of intrinsic motivation induced by incentive payments, we are interested in analyzing the strategic interaction of motivational choices among players in a contest competing for a given prize.

The analysis reveals that the activation of intrinsic motivation depends on the precise definition of the utility potentially motivated players derive in the contest. If intrinsic benefits are just added to the expected prize of the contest as in the model of James, a unique Nash equilibrium is characterized by all players taking part in the contest with activated intrinsic motivation. Most important, for sufficiently high intrinsic values, overdissipation of rents occurs. As the number of players approach infinity, overdissipation of rents occurs even with a slight spark of intrinsic motivation. If, however, players maximize a weighted sum of extrinsic and intrinsic incentives, an approach followed by Janssen and Mendys-Kamphorst (2004), then the unique Nash equilibrium corresponds to the Tullock equilibrium with deactivated intrinsic motivation.

The paper is organized as follows. Section 2 analysis the contest with an add-on utility derived in the contest. Section 3 derives the contest equilibrium under the weighted sum approach. Section 4 concludes.

2 Contest analysis with add-on intrinsic benefits

Consider a contest with n risk-neutral players. Players simultaneously choose efforts e_i to win a prize of value $V > 0$. We assume the standard Tullock contest success function

$$p_i(e_1, ..., e_n) = \frac{e_i}{\sum_{j=1}^{n} e_j}. \tag{1}$$

We allow players not only to choose their effort levels but also to be able to decide whether they want to be intrinsically motivated in the contest or not. To incorporate such a choice, we treat I_i as an additional strategy of the players. We follow James (2005) by assuming that player are restricted to an activation/deactivation choice. We also assume that both strategies are chosen simultaneously. Thus, the strategy space of each player is given by $e_i \geq 0$ and $I_i \epsilon \{0, 1\}$. This contest can be defined as a game

$$G = \{I_1 \epsilon \{0, 1\} \times e_1, ..., I_n \epsilon \{0, 1\} \times e_n, U_1, ..., U_n\}, \tag{2}$$

where

$$U_i = \left[V \frac{e_i}{\sum_{j=1}^{n} e_j} - e_i \right] + I_i \delta (e_i - \overline{e_i}) \text{ for } i = 1, ..., n, \tag{3}$$

and $\delta \epsilon (0, 1)$ is assumed.[3] The term in brackets in (3) is the material payoff to the players as it is usually assumed in the analysis of contests. The second term represents the motivational component of utility derived from intrinsic motivation. This extra benefit can be regarded as the pure joy of taking part in the contest which is an increasing function of the effort put into the contest. In the context of the principal-agent framework, James interprets the second term in (3) as a social norm, where $\overline{e}_i$ is a given reference level of effort. To avoid psychic costs the agent should provide at least some minimal level of effort when employed. In the context of a contest, $\overline{e}_i$ may be more appropriately interpreted as an individual aspiration level. The effect of such an aspiration level is that the contestants suffer from psychic costs unless their effort is as least as high as their own aspiration level. Since $\delta \overline{e}_i > 0$ are fixed psychic costs, the equilibrium effort level might yield an expected gross payoff that falls short of such fixed costs. However, we assume that agents are able to adjust their aspiration levels downwards in a way to participate in the contest. Such adjustments seem to be realistic in the view of the utility lost by not adjusting. Therefore, in the following analysis we assume $\overline{e}_i = 0$.

The equilibrium of this game is determined by the set of n first order conditions

$$V \left[\frac{\sum_{j \neq i}^{n} e_j(I_j) - e_i(I_i)}{\left(\sum_{j \neq 1}^{n} e_j(I_j) + e_i(I_i) \right)^2} \right] - (1 - I_i \delta) = 0 \text{ for } i = 1, ..., n \tag{4}$$

where $e_j(I_j)$ are the effort levels of the other players, given their motivational choice, and the optimal motivational choice strategies I_i^* for $i = 1, ..., n$.

[3] In principle, there is no plausbile reason to restrict intrinsic value to an upper bound. However, given a standard Tullock rent seeking contest with linear effort costs, such unrestricted intrinsic values would not be interesting to study. This is because in this case, the maximum effort is always the optimal effort. However, we are interested in analyzing the extend of rent dissipation with small intrinsic values, since even low intrinsic values may result in overdissipation of rents. That such overdissipation of rents may be derived with unrestricted intrinsic values should be rather clear without further analysis.

Consider first a strategy profile $I_i = 0$ for all $i = 1, ..., n$ players. Then, each player chooses the same effort level derived from the corresponding optimally condition

$$V(n-1)e_i = n^2 (e_i)^2 . \qquad (5)$$

We have to show whether all players prefer to deactivate intrinsic motivation. The result of this investigation is summarized in our first proposition.

> **Proposition 1:** Given $0 < \delta < 1$, a strategy profile with all players deactivating their intrinsic motivation cannot be a contest equilibrium.

Proof: Consider a strategy profile $I_i = 0$ for $i = 1, ..., n$ in combination with corresponding optimal effort levels $e_i^*(0) = e(0) = V(n-1)/n^2$. Given this strategy profile, the utility of the players amounts to $U_i = V/n^2$. To see that this is not an equilibrium, it suffices to show that a player (say player k) has an incentive to deviate by activating his intrinsic motivation, given the other players' effort levels $e_{i\neq k}^*(0)$. The utility of player k is given by

$$U_k = V \frac{e_k}{e_k + (n-1)e(0)} - (1-\delta)e_k. \qquad (6)$$

Define $z := \sqrt{1-\delta}$, then from the first order condition of (6), the optimal effort level can be obtained as

$$\widehat{e}_k(1) = V(n-1) \left[\frac{n - z(n-1)}{zn^2} \right]. \qquad (7)$$

Inserting (7) into (6) yields

$$\widehat{U}_k = \frac{V}{n^2} \left[n - z(n-1) \right]^2 . \qquad (8)$$

Using $z := \sqrt{1-\delta}$, we have $\widehat{U}_k > U_i$ since $(n-1)(1-z) > 0$ holds for $\delta < 1$. Thus, activating intrinsic motivation and adjusting effort optimally yields a higher utility. Hence, all players deactivating intrinsic motivation cannot be a contest equilibrium. □

Intuitively, activating intrinsic motivation lowers the net costs of effort. Thus, an increase in effort results. From the properties of the contest success function, an increase in win probability follows. As can be shown, the cost effect is ambiguous.[4] However, as the proof of proposition 1 reveals, the overall effect of activating intrinsic motivation on utility is positive.

Our next step is to show that an equilibrium with all players activating their intrinsic motivation exists.

> **Proposition 2:** Given $0 < \delta < 1$, then
> (i) $I_i^* = 1$ and $e_i^* = V(n-1)/n^2(1-\delta)$ for all $i = 1, ..., n$ is a Nash equilibrium of the contest;
> (ii) for $\delta > 1/n$, overdissipation of rents occurs.

Proof: To prove (i), consider a strategy profile $I_i = 1$ for $i = 1, ..., n$ in combination with corresponding optimal effort levels derived from (4), $e_i^*(1) = V(n-1)/(1-\delta)n^2$. Note that, despite intrinsic motivation, in equilibrium payoffs are equivalent to an equilibrium without intrinsic motivation, and utility amounts equally to V/n^2. To see that this is indeed an equilibrium, let us denote $e_{i \neq k}^*(1)$ as the effort levels of the other motivated players. Setting $I_k = 0$, the optimal effort adjustment of player k is given by

$$\widehat{e}_k(0) = \max \left\{ 0, \frac{V(n-1)\left[1 + n(z-1)\right]}{z^2 n^2} \right\}. \tag{9}$$

Thus, player k does not enter the contest if $n > 1/(1-z)$ holds. Clearly, if he does not enter, he does not improve since $V/n^2 > 0$. Otherwise, he enters and gets a utility

$$\widehat{U}_k = \frac{V}{n^2} \left(\frac{nz + 1 - n}{z} \right)^2. \tag{10}$$

Since $\delta < 1$, the RHS of (10) is lower than V/n^2. Thus, activating intrinsic motivation by all players is a Nash equilibrium. To prove (ii), note that total rent-seeking costs sum up to $T = ne_i^* = V(n-1)/n(1-\delta) > V \Leftrightarrow \delta > 1/n$. □

Proposition 2 makes clear that intrinsic motivation by all players is indeed an equilibrium choice of the players. Intuitively, deactivating intrinsic

[4] To see this, the costs net of intrinsic benefit can be equivalently expressed as

$$e^*(0)(n - zn + z)z.$$

Since the middle term is larger than one while the third term is smaller than one, effort costs with intrinsic motivation can be lower or higher than the effort costs with deactivated intrinsic motivation, $e^*(0)$.

motivation lowers effort and thus lowers the probability of winning the prize. However, this negative effect on utility is weaker than the cost effect of deactivating intrinsic motivation, as the proof of proposition 2 clarifies. Thus, activating intrinsic motivation is beneficial, given the opponents strategies. Proposition 2 also shows that with a sufficiently high value players attach to taking part in the contest, the contest results in overdissipation of rents.

Our last step proves the uniqueness of the contest equilibrium stated in proposition 2(i).

> **Proposition 3:** Given $0 < \delta < 1$, the Nash equilibrium with n intrinsically motivated players according to proposition 2(i) is unique.

Proof: To prove uniqueness, consider an arbitrary allocation of motivational strategies with $1 \leq m_0 \leq n$ intrinsically motivated players, and the remaining $m_0 - n$ players with deactivated intrinsic motivation. Then, the optimal effort levels of motivated and non-motivated players follow from (4). Since motivated players choose the same effort levels within the group of motivated players and the non-motivated players choose the same effort level within the group of non-motivated players, equilibrium effort levels are determined by the solution of the two equations

$$\frac{(m_0-1)e(1)+(n-m_0)e(0)}{1-\delta} = [m_0e(1)+(n-m_0)e(0)]^2, \qquad (11)$$

$$(n-m_0-1)e(0)+m_0e(1) = [m_0e(1)+(n-m_0)e(0)]^2, \qquad (12)$$

where $e(0)$ denotes the effort level of a non-motivated and $e(1)$ denotes the effort level of a motivated player. The equilibrium effort levels of the non-motivated players are as follows

$$e^*(0) = \left\{ \begin{array}{cc} 0 & for \quad \delta \geq 1/m_0 \\ \frac{V(n-1)(1-m_0\delta)}{(n-m_0\delta)^2} & for \quad \delta < 1/m_0 \end{array} \right\}, \qquad (13)$$

$$e^*(1) = \left\{ \begin{array}{cc} V\frac{(m_0-1)}{m^2(1-\delta)} & for \quad \delta \geq 1/m_0 \\ V\frac{(n-1)[\delta(n-m_0)+1-\delta]}{(n-m_0\delta)^2} & for \quad \delta < 1/m_0. \end{array} \right\}. \qquad (14)$$

Expression (13) defines a contest entry condition in the following sense: Given some value $\delta\epsilon\,(1/n,1)$, $n-m$ non-motivated players can be driven out of the contest by any number of players $m \epsilon [m_0, n]$. Thus, the following argument holds for any $m \epsilon [m_0, n]$:

Consider first an allocation of motivational strategies $[m, n-m]$ in case of $\delta < 1/m_0$. To see that this constellation cannot be an equilibrium, pick a non-motivated player, say player k. Given the optimal effort levels according to (13) and (14), his utility amounts to

$$U_k = V \frac{(1-m\delta)^2}{(n-m\delta)^2}. \tag{15}$$

By activating his intrinsic motivation, the number of non-motivated players is reduced by one such that total efforts of the remaining non-motivated players are given by $(n - m_0 - 1)e^*(0)$. The optimal effort level of player k is, therefore, derived from the utility

$$U_k = V \left[\frac{e_k}{e_k + me(1) + (n-m-1)e(0)} \right] - e_k. \tag{16}$$

From the first order condition of (16), the optimal adjusted effort level can be obtained as

$$\widehat{e}_k = \frac{V(n-1)}{(n-m\delta)^2} \left(\frac{n - m\delta - z(n-1)}{z} \right). \tag{17}$$

Note that $\widehat{e}_k > 0$, since $m \leq n$ and $\delta < 1$ implies $n > m\delta$. Thus, $e(0) \geq 0$ implies $\widehat{e}_k > 0$. Inserting (17) into (16), we have

$$\widehat{U}_k = \frac{V\left[n - m\delta - z(n-1)\right]^2}{(n-m\delta)^2}. \tag{18}$$

Comparing (18) with (15), it can be verified that for $\delta < 1$ a non-motivated player gets a higher expected payoff by activating his intrinsic motivation, independent of the actual number of motivated and non-motivated players. Thus, $[m, n-m]$ cannot be an equilibrium of the contest.

Now, consider $\delta \geq 1/m_0$. In this case, only a subgroup of $m \epsilon [m_0, n]$ motivated players compete for the price. By analyzing the behavioral change of a non-motivated player, let us consider his utility if he wants to become a motivated one

$$U_m = V \frac{\widehat{e}_m}{\widehat{e}_m + me(1)} - (1-\delta)\widehat{e}_m. \tag{19}$$

From the first order condition, we get his optimal adjusted effort level

$$\widehat{e}_m = \frac{V}{z}\left(\gamma - \gamma^2\right) \tag{20}$$

$$\text{where } \gamma = \sqrt{\frac{m-1}{m}}. \tag{21}$$

Insert (20) into (19), and note that

$$U_m(\widehat{e}_m) = V(1-\gamma) > 0 \tag{22}$$

since $\gamma < 1$. Thus, he has an incentive to take part in the contest as a motivated player. Hence, their cannot be a contest equilibrium with motivated and non-motivated players. Thus, the unique Nash equilibrium of the contest is the one where all players compete for the prize with activated intrinsic motivation. □

Intuitively, uniqueness stems from the fact that the contest is symmetric. Thus, an asymmetric equilibrium with some players not motivated should not be expected. The proof of proposition 3 confirms this intuition.

3 A case against activating intrinsic motivation

In our first model, the utility of intrinsically motivated contestants is composed of an added intrinsic benefit to the usual material expected payoff in the standard Tullock contest. However, Janssen and Mendys-Kamphorst (2004) investigated how financial incentives in a public good setting may lead to the crowding out of intrinsic motivation, especially if the intrinsic motivation is directed to the social approval of their contribution to a public good. Of special interest for our purpose is that, in their model, some individuals (called altruists) maximize a weighted sum of both types of motivation, the utility from money they get in exchange for the public good contribution, and the intrinsic utility from contributing to the public good. To incorporate this idea in the contest, let us, as an alternative to (3), assume the following utility of contestants

$$U_i = (1 - I_i\delta)\frac{Ve_i}{\sum_{j=1}^{n} e_j} + I_i\delta e_i - e_i, \text{ for } i = 1, ..., n. \tag{23}$$

The utility defined in (23) now stresses the players relative importance of the expected prize and the intrinsic benefit, where $(1 - I_i\delta)$ is the weight put on the extrinsic incentives, i.e. the expected value of the prize, and $I_i\delta$ is the weight put on the intrinsic enjoyment of taking part in the contest. The higher the intrinsic value of taking part in the contest, the lower players value the prize of the contest. In this case, from a straightforward argument the following proposition can be stated:

> **Proposition 4:** Given the weighted sum of contestants' utility according to (23), the unique Nash equilibrium of the contest is characterized by the absence of intrinsic motivation.

Proof: Assume $I_i = 1$ by some individuals. Then, expected utility of a motivated player, say k, if given by

$$U_k = (1-\delta)\left[V\frac{e_k}{\sum_{j=1}^{n} e_j} - e_k\right]. \tag{24}$$

As can be seen from (24), optimal effort levels of motivated players are *unaffected* by the value δ of intrinsic motivation. Hence, motivated players choose the same effort as non-motivated players. The only difference between players is that motivated players derive less expected maximized utility. Thus, they would not prefer to activate intrinsic motivation. As a consequence, $I_i^* = 0$ for all players and the corresponding effort levels derived from (4) is the unique Nash equilibrium. □

The implication of proposition 4 is that under a weighted sum of different types of motivation in a contest, overdissipation of rents does not happen as it does not occur in the standard Tullock contest. This result even holds under more general assumptions. First, the result is independent of possible different motivational powers of the players. Moreover, all results derived in the contest literature with respect to the degree of rent seeking costs equally apply to our result since intrinsic motivation only devalues the maximized utility derived from any possible variations of the standard Tullock contest.

4 Conclusion

We have analyzed a contest with potentially motivated players. The analysis shows that the activation of intrinsic motivation crucially depends on the precise definition of the utility derived in a contest by potentially motivated players. If players act as if they just add the enjoyment of taking part in the contest to the material expected prize, intrinsic motivation is indeed activated. The implication of this behavioral choice is that overdissipation of rents may occur. This is especially expected in contests where intrinsic motivation is an important motivational drive of players, as can be assumed, for instance, in sports contests. In this case, the objective of the contest designer is important. If the contest designer dislikes increased efforts, our result can be regarded as a new challenge for contest design which behavioral economics brings about. Most important, a contest designer should care on how contests with intrinsically motivated players could be redesigned as to prevent overdissipation in the presence of intrinsic motivation. As the proofs of our propositions illuminate, the prize mechanism does not crowd out intrinsic motivation in our specific contest setting. As a consequence, in such contests, more sophisticated instruments are required to narrow the scope of rent seeking activities. However, if players devalue the prize of the contest to the extend they are motivated, i.e. they maximize a weighted sum of extrinsic and intrinsic incentives, then players do not activate their intrinsic motivation. The plausibility of defining the adequate utility function in contests with potentially motivated players remains a matter of empirical research.

References

Corchón, L.C. (2007), The theory of contests, *Review of Economic Design* 11, 69-100.

Deci, E. L. (1975), Intrinsic motivation. New York, NY: Plenum Press.

Frey, B. S. and Jegen, R. (2001), Motivation crowding theory, *Journal of Economic Surveys* 15(5), 589–611.

James, H.S. (2005), Why Did You Do That? An Economic Examination of the Effect of Extrinsic Compensation on Intrinsic Motivation and Performance, *Journal of Economic Psychology* 26(4), 549-566.

Janssen, M.C.W., Mendys-Kamphorst, E. (2004), The price of a price: on the crowding out and in of social norms, *Journal of Economic Behavior & Organization* 55, 377-395.

Konrad, K.A. (2004), Altruism and Envy in Contests: An Evolutionarily Stable Symbiosis, *Social Choice and Welfare* 22(3), 479-490.

Riechmann, T. (2007), An Analysis of Rent-Seeking Games with Relative-Payoff Maximizers, *Public Choice* 133(1-2), 147-155.
Tullock, G. (1980), Efficient rent-seeking. In: Buchanan, J.M., Tollison, R.D., Tullock, G. (Eds.), Toward a Theory of Rent-Seeking Society. Texas A&M University Press, College Station.