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The Kuznets-Kaldor-Puzzle and Neutral Cross-Capital-Intensity Structural Change*†

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Abstract The Kuznets-Kaldor stylized facts are one of the most striking empirical observations about the development process in the industrialized countries: While massive factor reallocation across technologically distinct sectors takes place, the aggregate ratios of the economy are quite stable. This implies that cross-technology factor reallocation has a relatively weak impact on the aggregates, which is a puzzle from a theoretical point of view. We provide a model that can explain this puzzle. Furthermore, we show by empirical evidence that this model is in line with 55% of structural change.

Keywords *neoclassical growth models, balanced growth, structural change, Kaldor facts, labor reallocation across sectors, sector technology.*

EL O14, O41

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1. Introduction

As shown by Kongsamut et al. (1997, 2001), the development process of the industrialized countries during the last century satisfies two types of stylized facts: “Kuznets facts” and “Kaldor facts”. Generally speaking, Kuznets facts state that massive structural change takes place during the development process.[1] Especially, in the early stages of economic development factors are primarily reallocated from the agricultural sector to the industrial sector and in later stages of development factors are primarily reallocated from the manufacturing sector to the services sector. (It has also been shown, that structural change takes place at more disaggregated level.) On the other hand, the Kaldor facts state that some key aggregate measures of the economy are quite stable during the development process; especially, the aggregate capital-to-output ratio and the aggregate income shares of capital and labor are quite stable whereas the aggregate capital-to-labor ratio increases (at a fairly constant rate) in the industrialized countries.[2] That is, the growth process seems to be “balanced” at the aggregate level.

It is generally accepted that sectors of an economy differ by several technological properties, especially there are cross-sector differences in total factor productivity (TFP) and in capital-intensities. (Empirical evidence regarding this fact is reviewed in the following section.) Therefore, from a theoretical perspective, the coexistence of Kuznets and Kaldor facts is a puzzle (henceforth we name it “Kuznets-Kaldor-puzzle”): If during the development process factors are reallocated across technologically distinct sectors, the average technology structure

[1] Papers that provide empirical evidence for the massive labor reallocation across sectors during the growth process are e.g. Kuznets (1976), Maddison (1980), Kongsamut et al. (1997, 2001) and Ngai and Pissarides (2004). Kongsamut et al. (1997, 2001) formulate the following stylized facts of structural change for the last hundred years: 1.) the employment share of agriculture decreases during the growth process; 2.) the employment share of services increases during the growth process; 3.) the employment share of manufacturing is constant. Ngai and Pissarides (2007) note that the development of the manufacturing employment share can be regarded as “hump-shaped” in the longer run.

[2] In detail, Kaldor’s stylized facts state that the growth rate of output per capita, the real rate of return on capital, the capital-to-output ratio and the income distribution (between labor and capital) are nearly constant in the long

of the economy changes and aggregates of the economy are not stable (i.e. Kaldor facts are not satisfied). Implicitly, this fact is discussed by Baumol (1967) under the headline "unbalanced growth": In fact, Baumol's model implies that the aggregate growth rate is not constant (hence Kaldor facts are not satisfied), provided that there are at least two sectors with different technologies. However, the Kuznets-Kaldor-puzzle is explicitly mentioned for the first time by Kongsamut et al. (1997, 2001). Our paper deals with the explanation of the Kuznets-Kaldor-puzzle.

In general, understanding the Kuznets-Kaldor-puzzle is essential for understanding the relationship between structural change and aggregate growth and for assessing the future growth impact of structural change. Therefore, it is not surprising that some literature has already dealt with the Kuznets-Kaldor-puzzle. The literature can be divided into two groups depending on whether structural change is caused *by non-homothetic preferences* or by technological cross-sector-disparities, where the latter include *cross-sector differences in TFP-growth* and *cross-sector differences in capital-intensities* (in the next section we will discuss the empirical evidence on these "structural change determinants"): Kongsamut et al. (1997, 2001), Meckl (2002) and Foellmi and Zweimueller (2008) study the role of non-homothetic preferences for structural change; Ngai and Pissarides (2007) focus on cross-sector-disparities in TFP-growth and Acemoglu and Guerrieri (2008) focus on cross-sector-disparities in capital-intensities.

We learn from this literature that the relationship between *cross-sector-differences in capital-intensities* and the Kaldor facts is the key for solving the Kuznets-Kaldor-puzzle: In fact Acemoglu and Guerrieri (2008) show that the equilibrium growth path of an economy is completely unbalanced, if sectors differ by capital intensities; hence, Kaldor facts are not satisfied in general. (However, they show that for US-parameters their unbalanced growth path satisfies Kaldor facts approximately.) In contrast, the rest of the literature implies that the

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run; capital-to-labor ratio increases in the long run. It is widely accepted that these facts are an accurate other structural change determinants (i.e. non-homothetic preferences and cross-sector differences in TFP-growth) are not such a big challenge for the solution of the Kuznets-Kaldor-puzzle: Ngai and Pissarides (2007) show that *cross-sector-differences in TFP-growth* are compatible with an "aggregate balanced growth path" (ABGP). According to their definition, an ABGP is an equilibrium growth path that features at the same time balanced growth of aggregates and unbalanced growth of disaggregated variables. That is, along an ABGP, aggregate output and aggregate capital grow at a constant rate and at the same time structural change takes place (e.g. sectoral output shares change); hence, an ABGP is consistent with the Kuznets and Kaldor facts. Ngai and Pissarides (2007) show that an ABGP can exist under (otherwise) fairly general assumptions, if sectors differ by TFP(-growth rates) only (and if preferences are homothetic). Regarding, *non-homothetic preferences* the story is quite similar: although the papers by Kongsamut et al. (1997, 2001) and Meckl (2002) imply that with non-homothetic preferences the existence of an ABGP requires cross-parameter knife-edge-conditions (which is regarded by some authors with severe reservation, see e.g. Ngai and Pisssarides (2007)), Foellmi and Zweimueller (2008) show that such knife-edge conditions are not necessary for the existence of an ABGP when taking a more disaggregated view of the economy.

Overall, the key challenge is to understand the Kuznets-Kaldor-puzzle in the light of cross-sector-differences in capital intensity, and that is in focus of our paper.

There are two possible explanations for Acemoglu and Guerrieri's (2008) finding that Kaldor and Kuznets facts can be satisfied approximately in the US despite the fact that sectors differ by capital intensity:

(1) in general cross-sector capital-intensity-disparity has a large impact on the aggregate economy; however, the cross-sector capital-intensity-disparity is relatively small; hence, Kaldor facts are satisfied approximately despite structural change

shorthand description of the long run growth process (at the aggregate level) in industrialized countries.

and/or

(2) cross-sector capital-intensity-disparity is relatively large; however, cross sector capital-intensity-disparity has relatively weak impact on the aggregate ratios in general; hence Kaldor-facts are satisfied approximately.

It is important to distinguish between these two explanations, since they require different approaches in further research regarding the Kuznets-Kaldor-puzzle: If explanation (1) is dominant, it is necessary to analyze, why the cross-sector-capital-intensity-disparity is so small; then, this explanations could be integrated into predominantly unbalanced growth models to explain the Kuznets-Kaldor-puzzle (similar to the simulation approach by Acemoglu and Guerrieri (2008)). Otherwise, if case (2) is dominant, it is necessary to analyze why cross-sector-capital-intensity-disparity has such a small impact on aggregates; this explanation could be used to generate models where an ABGP exists, and the Kuznets-Kaldor-puzzle could be explained primarily by ABGP-models; this would be advantageous, since ABGP-models feature closed form solutions (hence, no simulations are necessary, which is more convenient and increases the generality of the results) and since it would support the mainstream-school of economic growth, the neoclassical growth theory.

Our theoretical and empirical findings are slightly in favor of explanation (2). However, both explanations seem to be true.

We develop a three-sector-model that depicts explanation (2). For example, the sectors could be interpreted as agriculture, manufacturing and services. The three sectors in our model have different capital intensities. We show that factors are reallocated across these three sectors, while at the same time the Kaldor-facts regarding the aggregate ratios are satisfied (exactly). Hence, for the first time in the literature our model postulates that it is possible to have factor reallocation across sectors that differ by capital intensity along an ABGP. (We name this type of factor reallocation “neutral cross-capital-intensity structural change”, or in short “NCCI structural change”.) This result contradicts Acemoglu and Guerrieri (2008). We were able to obtain our results, since, in contrast to Acemoglu and Guerrieri (2008), we assume a utility function that has non-unitary price elasticity of demand (i.e. each good has its own specific price elasticity) and since we assume that at least one of the three sectors uses two technologies. (As we will discuss in our paper, the latter assumption is consistent with empirical evidence, which postulates that e.g. the services sector is quite technologically heterogeneous.) Furthermore, in contrast to Acemoglu and Guerrieri (2008), we model sectors that feature non-constant output-elasticities of inputs.

Subsequently, we study the empirically observable patterns of cross-capital-intensity structural change. We develop an index of neutrality of cross-capital-intensity structural change and show with the data for the US between 1948 and 1987 that about 55% of structural change was NCCI structural change. Hence, neutrality of cross-capital-intensity structural change seems to be a relatively large explanatory variable regarding the Kuznets-Kaldor-puzzle.

Last but not least, we argue that low (no) correlation between preference parameters and technology parameters can explain the prevalence (existence) of NCCI structural change in reality (our model).[3] We also argue that the assumption of uncorrelated preferences and technologies may be theoretically reasonable in long run growth models.

Overall, our empirical and theoretical results imply the following solution for the Kuznets-Kaldor-puzzle ("Why is there "stable" aggregate behavior despite massive structural change?"):

1) The biggest part of the factor-reallocation across sectors, which differ by capital intensity, is neutral regarding the aggregate ratios (NCCI structural change), hence being consistent with the Kaldor-Kuznets-facts.

[3] It should be noted here that previously it has been mentioned by Foellmi and Zweimueller (2008) that some type of independency between technology and preferences may be useful for generating aggregate balanced growth. However, this topic has not been studied further by them.

2) Therefore, the rest of the structural change ("non-neutral structural change") is quantitatively small; hence it has a relatively weak impact on aggregate growth.

In fact, this is the greatest part of the solution of the Kuznets-Kaldor-puzzle. Further research has to model some "micro-foundation" for these findings. We will argue that the point 1) can be explained by low correlation between preferences and technologies and that this low-correlation may be a reasonable assumption in long run growth models. In fact, point 2) has been shown by the simulation of Acemoglu and Guerrieri (2008).

In the next section (section 2) we provide some evidence on sectoral structures that are observed in reality, in order to provide an empirical basis for our discussion and model assumptions. Then, in section 3, we provide a model of structural change in order to show the existence of NCCI structural change. Section 4 is dedicated to the empirical analysis, where among others we develop an index of neutrality of structural change and analyze the cross-capital-intensity structural change patterns in detail. In section 5 we discuss the assumption of low-correlation between technology and preferences. Finally, in section 6 we provide some concluding remarks and hints for further research.

2. Stylized facts of sectoral structures

2.1 Stylized Facts regarding Cross-Sector-Heterogeneity in Production-Technology

Empirical evidence implies the following stylized facts of sectoral production functions:

1. TFP-growth differs across sectors. Empirical evidence implies that TFP-growth rates differ strongly across sectors. For example, Bernard and Jones (1996) (pp. 1221f.), who analyze sectoral TFP-growth in 14 OECD countries between 1970 and 1987, report that e.g. the average TFP-growth rate in agriculture (3%) was more than three times as high as in services

(0.8%). Similar results are obtained by Baumol et al. (1985), who report the TFP-growth-rates of US-sectors between 1947 and 1976.

2. Capital intensity differs across sectors. Empirical evidence implies that factor income shares differ strongly across sectors (hence, capital intensities differ strongly across sectors as well[4]). For example, Kongsamut, Rebelo and Xie (1997) provide evidence for the USA for the period 1959-1994. Their data implies that, for example, the labor income share was relatively high in manufacturing and construction (around 70%) in this period. At the same time, e.g. the labor income share in agriculture, finance, insurance and real estate was relatively low (around 20%). Similar results for the USA are obtained by Close and Shulenburger (1971) for the period 1948-1965 and by Acemoglu and Guerrieri (2008) for the period 1987-2004. Some new evidence for the USA (presented by Valentinyi and Herrendorf (2008)) supports these results as well. Gollin (2002) (p. 464) analyzes the data from 41 countries reported in the U.N. National Statistics. He confirms that factor income shares vary widely across sectors.

A model that analyzes structural change across *sectors* should be consistent with these "stylized" facts of *sectoral* production functions. This is especially important, since these stylized facts have an impact on structural change (and hence on aggregate balanced growth), as we will see now.

2.2 Structural change determinants

As proposed by Schettkat and Yocarini (2006), there are three main determinants of structural change in industrialized economies: shifts in final demand (non-homothetic preferences),

[4] If labor income shares (or: output elasticities of labor) differ across sectors, it follows that capital intensities differ across sectors as well, since in optimum capital intensity is determined by factor prices and by output elasticities of capital and labor. We will see later that this is true within our model.

shifts in intermediates production (outsourcing), and differences in productivity growth. Note that differences in productivity growth can arise due to differences in TFP-growth and due to differences in capital intensities across sectors. Empirical evidence on the impact of these determinants on structural change is reviewed, e.g., by Schettkat and Yocarini (2006). Overall, we can postulate the following key determinants of structural change in industrialized economies:

1. Non-homothetic preferences across sectors – relevance for structural change proved empirically and theoretically, e.g., by Kongsamut et al. (1997, 2001).

2. Differences in TFP-growth across sectors – empirical relevance for structural change shown, e.g., by Baumol (1985); theoretical relevance for structural change shown, e.g., by Ngai and Pissarides (2007).

3. *Differences in capital intensities across sectors* – relevance for structural change proved empirically and theoretically, e.g., by Acemoglu and Guerrieri (2008).

4. *Shifts in intermediates production across sectors* – relevance for structural change proved empirically and theoretically, e.g., by Fixler and Siegel (1998).

(Note that the papers mentioned for each determinant are only examples. Further empirical evidence on the relevance of each determinant is discussed, e.g., by Schettkat and Yocarini (2006).)

So we can conclude that all these determinants influence the structural change patterns. Since the aggregate economy is the weighted average of its sectors, the aggregate behavior depends on the structural change patterns. That is, all four structural change determinants influence the behavior of the aggregate economy. Hence, only if we include all four structural change determinants into a model, we can adequately analyze whether balanced growth with respect to aggregates can coexist with structural change.

3. Model of Neutral Cross-Capital-Intensity Structural Change

3.1 Model Assumptions

Production

We assume an economy where two technologies exist (the model could be modified such that it includes more technologies; the key results would be the same). The technologies differ by capital intensity (i.e. output elasticities of inputs differ across technologies) and by total factor productivity (TFP) growth. TFP-growth rates are constant and exogenously given. Goods $i = 1,...n$ are produced in the economy. Goods $i = 1,...m$ are produced by using technology 1 and goods $i = m+1,...n$ are produced by using technology 2 ($n > m$). We assume that three inputs are used for production: capital (*K*), labor (*L*) and intermediates (*Z*). All capital, labor and intermediates are used in the production of goods $i = 1,...n$. The amount of available labor grows at constant rate (g_L). Since we want to model TFP-growth, we assume Hicks-neutral technological progress. It is well known that the existence of a balanced growth path in standard balanced growth frameworks requires the assumption of Cobb-Douglas production function(s) when technological progress is Hicks-neutral. (Later, we will see that the aggregate production function "inherits" the attributes of sectoral production functions along the ABGP, i.e. the aggregate production function is of type Cobb-Douglas.) These assumptions imply the following production functions:

$$Y_i = A(l_i L)^{\alpha} (k_i K)^{\beta} (z_i Z)^{\gamma}, \quad i = 1,...m \qquad (1)$$

where $\alpha + \beta + \gamma = 1; \quad \alpha, \beta, \gamma > 0; \quad \frac{\dot{A}}{A} = g_A = const.$

$$Y_i = B(l_i L)^{\chi} (k_i K)^{\nu} (z_i Z)^{\mu}, \quad i = m+1,...n \qquad (2)$$

where $\chi + \nu + \mu = 1; \quad \chi, \nu, \mu > 0; \quad \frac{\dot{B}}{B} = g_B = const.$

$$\sum_{i=1}^{n} l_i = 1; \qquad \sum_{i=1}^{n} k_i = 1; \qquad \sum_{i=1}^{n} z_i = 1 \qquad (3)$$

$$\frac{\dot{L}}{L} \equiv g_L = const. \qquad (4)$$

where Y_i denotes the output of good *i*; l_i, k_i and z_i denote respectively the fraction of labor, capital and intermediates devoted to production of good *i*. Note that we omit here the time index. Furthermore, note that the index i denotes *not* sectors but a good or a group of similar goods. We will define sectors later.

Of course, it is not "realistic" that there are only two technologies and that some goods are produced by identical production functions. However, every model simplifies to some extent and it is only important that the simplification does not affect the meaningfulness of the results. Our assumption is only a "technical assumption", which is necessary to make our argumentation as simple as possible. Our key arguments (namely the existence of NCCI structural change) could also be derived in a framework where each good is produced by a unique production function. (We show this fact in Proposition 4.) However, it would be much more difficult to formulate the independency assumptions (which are formulated in the next subsection). Instead of the simple restrictions, which we use in the next subsection, we would have to derive complex restrictions which would not be such transparent. Anyway, later our focus will be on the analysis of only three sectors (which are aggregates of the products i=1,…n); thus, two technologies are sufficient to generate technological heterogeneity between these three sectors. In this sense, we have introduced technological diversity into our framework in the simplest manner (by assuming that there are only two technologies).

It may easier to accommodate with our assumption of only two technologies by imagining that an economist divides the whole set of products of an economy into two groups (a technologically progressive and a technologically backward) and estimates the average production function for the two groups. Such approaches are prominent in the literature: e.g. Baumol et al. (1985) and Acemoglu and Guerrieri (2008) approach in similar way in the empirical parts of their argumentation. Furthermore, note that much of the new literature on the Kuznets-Kaldor-puzzle assume very similar sectoral production functions (e.g. Kongsamut et al. (2001) and Ngai and Pissarides (2007)) or assume even identical sectoral production functions (e.g. Foellmi and Zweimueller (2008)). Hence, our assumption of only two (completely distinct) technologies is an improvement in comparison to some previous literature. Note that the empirical study of our paper (section 4) uses the more general assumption, i.e. each good is produced by a unique production function.

We assume that all goods can be consumed and used as intermediates. Furthermore, we assume that only the good *m* can be used as capital. (Note, that the model could be modified such that more than one good is used as capital e.g. in the manner of Ngai and Pissarides (2007).) This assumption implies:

$$Y_i = C_i + h_i, \quad \forall i \neq m \qquad (5)$$

$$Y_m = C_m + h_m + \dot{K} + \delta K \qquad (6)$$

where C_i denotes consumption of good *i*; δ denotes the constant depreciation rate of capital; h_i is the amount of good *i* that is used as intermediate input.

We assume that the intermediate-inputs-index *Z* is a Cobb-Douglas function of h_i's which is necessary for the existence of an ABGP (see Ngai and Pissarides 2007):

$$Z = \prod_{i=1}^{n} h_i^{\varepsilon_i} \quad (7)$$

where $\varepsilon_i > 0, \forall i; \quad \sum_{i=1}^{n} \varepsilon_i = 1$

Utility function

We assume the following utility function, which is quite similar to the utility function used by Kongsamut et al. (1997, 2001):

$$U = \int_0^{\infty} u(C_1,...C_n) e^{-\rho t} dt, \quad \rho > 0 \quad (8)$$

where

$$u(C_1,...C_n) = \ln\left[\prod_{i=1}^{n} (C_i - \theta_i)^{\omega_i}\right] \quad (9)$$

$$\sum_{i=1}^{m} \theta_i = 0 \quad (10)$$

$$\sum_{i=m+1}^{n} \theta_i = 0 \quad (11)$$

where U denotes the life-time utility of the representative household and ω_i, θ_i and ρ are constant parameters. In contrast to the model by Ngai and Pissarides (2007), the assumption of logarithmic utility function (equation (9)) is not necessary for our results, i.e. we could have assumed a constant intertemporal elasticity of substitution function of the consumption composite in equation (9).

We can see that this utility function is based on the Stone-Geary preferences. Without loss of generality we assume that θ_i s are not equal to zero and that they differ across goods i. The key reason why we use this utility function is that it features non-unitary income elasticity of demand and non-unitary-price elasticity of demand. That is, each good has its own income elasticity of demand and its own price elasticity of demand (as long as θ_i differs across goods). For example, the good i=4 has another price elasticity of demand than good i=7 (provided that $\theta_4 \neq \theta_7$). Due to this feature, we can determine the own price elasticity and the own income elasticity for groups of goods. For example, by setting the θ_i in a specific pattern we can determine that the (average) price elasticity of demand for the goods i=7,…14 is larger than for goods i= 56,…79.

This is the key to our argumentation about preference and technology correlation later: By setting parameter restrictions (10) and (11) we determine that

1.) on average, the income elasticity of demand for technology-1-goods is not larger or smaller in comparison to the income elasticity of demand for technology-2-goods.

2.) on average, the price elasticity of demand between technology-1-goods and technology-2-goods is equal to one.

Hence, the preferences and technologies are not correlated on average. This means for example, that demand for some of the goods that are produced by technology 1 can be price-inelastic and for some of the technology-1-goods price-elastic, while at the same time the demand for some goods that are produced by technology 2 can be price-elastic and for some of the technology-2-goods price inelastic. However, on average, the price elasticity of demand between technology-1-goods and technology-2-goods is equal to unity.

This restriction (equations (10) and (11)) reduces the generality of our model. Nevertheless, for our further argumentation it does not matter. It is simply a technical assumption in order to show in the simplest manner the existence of neutral-cross-capital-intensity structural change. That is, due to this assumption we can pursue our analysis along an ABGP, which is technically simple. Without this assumption, we would have to numerically solve the model and the distinction between neutral and non-neutral cross-capital-intensity structural change would be quite difficult. Nevertheless, we will discuss theoretical reasonability of this restriction later and we will show empirically that the largest part of structural change is in line with this restriction.

Overall, our utility function allows for structural change caused by all structural change determinants: In general the goods have a price elasticity of demand that is different from one (as discussed above). Hence, changing relative prices can cause structural change in this model (see also Ngai and Pissarides 2007 on price elasticity and structural change). Intertemporal elasticity of substitution differs across goods i and is *not* equal to unity, despite of the fact that equation (9) is logarithmic. Equations (8)-(11) imply that the utility function is *non-homothetic across goods i,* i.e. income elasticity of demand differs across goods i (depending on the parameterization of the θ_i's).

Aggregates and Sectors

We define aggregate output (*Y*), aggregate consumption expenditures (*E*) and aggregate intermediate inputs (*H*) as follows:

$$Y \equiv \sum_{i=1}^{n} p_i Y_i \; ; \qquad E \equiv \sum_{i=1}^{n} p_i C_i \; ; \qquad H \equiv \sum_{i=1}^{n} p_i h_i \qquad (12)$$

where p_i denotes the price of good *i.* We chose the good *m* as numéraire, hence:

$$p_m = 1 \qquad (13)$$

Note that in reality the manufacturing sector is not the numérarire in the real GDP calculations. Hence, our aggregate output Y is not the same as real GDP. However, the choice of numérarie is irrelevant when discussing ratios or shares (see e.g. Ngai and Pissarides (2004, 2007)), since the numérarire of the numerator and the denominator of a ratio offset each other. Therefore, we focus our discussion on the shares and ratios in our paper (e.g. aggregate capital-intensity, capital-to-output ratio, income-share of capital and labor), where the numérarire choice is irrelevant. Our results regarding the other Kaldor-facts, which are dealing with the development of the real-GDP-growth rate and the real interest rate, should be considered with caution. However, as discussed by Barro and Sala-i-Martin (2004), the constancy of the real interest rate (as a Kaldor fact) may anyway be questionable. Furthermore, as shown by Ngai and Pissarides (2004, 2007) the real GDP as measured in reality and the real GDP in manufacturing terms seem to behave quite similar. Therefore, possibly our results regarding the real GDP development may be to some extent related to the real GDP as measured in reality.

Last but not least we have to define the sectors of our economy. Without loss of generality we assume here that there are three sectors which we name for reasons of convenience (according to the tree sector hypothesis): agriculture, manufacturing and services. Furthermore, we assume that without loss of generality

- agricultural sector $i = 1,...a;\ 1 < a < m$
- manufacturing sector includes goods $i = a + 1,...s;\quad m < s < n$
- services sector includes goods $i = s + 1,...n$.

Hence, the agricultural sector uses only technology 1, the manufacturing sector uses technology 1 and 2 and the services sector uses only technology 2. Note, that this whole division is not necessary for our argumentation, neither the naming of the sectors. We could also assume that the capital-producing manufacturing sector uses only one technology (and the services sector both technologies). We could even assume that there are more sectors (and more technologies). In all these cases our key results would be the same. Furthermore, note that the assumption that a sector uses both technologies is plausible. For example, the service sector includes services that feature high TFP-growth and/or high capital intensity, e.g. ICT-based services, as well as services that feature low TFP-growth and/or low capital intensity, e.g. some personal services like counseling and consulting (for discussion and empirical evidence see e.g. Baumol et al. 1985 and Blinder 2007). Similar examples can be found in the manufacturing sector (e.g. a traditional clock maker vs. a car producer). Furthermore, our sector-division implies that only sector M (the manufacturing sector) produces capital. This is consistent with the empirical evidence, which implies that most capital goods are produced by the manufacturing sector (see e.g. Kongsamut et al. 1997).

According to our classification, we can define the outputs of the agricultural, services and manufacturing sector ($Y_{agr.}$, $Y_{man.}$ and $Y_{ser.}$) and the consumption expenditures on agriculture, manufacturing and services ($E_{agr.}$, $E_{man.}$ and $E_{ser.}$) as follows:

$$Y_{agr.} \equiv \sum_{i=1}^{a} p_i Y_i; \quad Y_{man.} \equiv \sum_{i=a+1}^{s} p_i Y_i; \quad Y_{ser.} \equiv \sum_{i=s+1}^{n} p_i Y_i \qquad (14)$$

$$E_{agr.} \equiv \sum_{i=1}^{a} p_i C_i; \quad E_{man.} \equiv \sum_{i=a+1}^{s} p_i C_i; \quad E_{ser.} \equiv \sum_{i=s+1}^{n} p_i C_i \qquad (15)$$

Furthermore, note that employment shares ($l_{agr.}$, $l_{man.}$ and $l_{ser.}$), capital shares ($k_{agr.}$, $k_{man.}$ and $k_{ser.}$) and intermediate shares ($z_{agr.}$, $z_{man.}$ and $z_{ser.}$) of sectors agriculture, manufacturing and services are given by:

$$\begin{aligned} l_{agr.} &\equiv \sum_{i=1}^{a} l_i; \; l_{man.} \equiv \sum_{i=a+1}^{s} l_i; \; l_{ser.} \equiv \sum_{i=s+1}^{n} l_i; \\ k_{agr.} &\equiv \sum_{i=1}^{a} k_i; \; k_{man.} \equiv \sum_{i=a+1}^{s} k_i; \; k_{ser.} \equiv \sum_{i=s+1}^{n} k_i; \\ z_{agr.} &\equiv \sum_{i=1}^{a} z_i; \; z_{man.} \equiv \sum_{i=a+1}^{s} z_i; \; z_{ser.} \equiv \sum_{i=s+1}^{n} z_i \end{aligned} \qquad (16)$$

3.2 Model Equilibrium

Optimality conditions

We have now specified the model completely. The intertemporal and intratemporal optimality conditions can be obtained by maximizing the utility function (equations (8)-(11)) subject to the equations (1)-(7) and (12)-(16) by using e.g. the Hamiltonian. When there is free mobility of factors across goods and sectors these optimality conditions are given by:

$$p_i = \frac{\partial Y_m / \partial(l_m L)}{\partial Y_i / \partial(l_i L)} = \frac{\partial Y_m / \partial(k_m K)}{\partial Y_i / \partial(k_i K)} = \frac{\partial Y_m / \partial(z_m Z)}{\partial Y_i / \partial(z_i Z)} = \frac{\partial Y_m}{\partial(z_m Z)} \frac{\partial Z}{\partial h_i}, \quad \forall i \quad (17)$$

$$p_i = \frac{\partial u(.) / \partial C_i}{\partial u(.) / \partial C_m}, \quad \forall i \quad (18)$$

$$-\frac{\dot{u}_m}{u_m} = r - \delta - \rho \quad (19)$$

where $u_m \equiv \partial u(.) / \partial C_m$ and $r \equiv \partial Y_m / \partial(k_m K)$ is the real interest rate.

Development of Aggregates in Equilibrium

By inserting equations (1) to (16) into optimality conditions (17) to (19), the following equations, that describe the optimal aggregate structure of the economy, can be obtained:

$$Y = \dot{K} + \delta K + E + H \quad (20)$$

$$\tilde{Y} = \left(\frac{k_m}{l_m}\right)^q G L^{1-q} K^q \quad (21)$$

$$\frac{\dot{E}}{E} = \left(\frac{l_m}{k_m}\right)^{1-q} \beta G L^{1-q} K^{q-1} - \delta - \rho \quad (22)$$

$$H = \gamma \tilde{Y} \left(c_1 + c_2 \frac{l_m}{k_m} \right) \quad (23)$$

$$\frac{l_m}{k_m} = 1 - c_3 \frac{E}{\tilde{Y}} - c_4 \frac{H}{\tilde{Y}} \quad (24)$$

where

$$\tilde{Y} \equiv \frac{Y}{c_5 + c_6 \frac{l_m}{k_m}} \qquad (25)$$

$$q \equiv \frac{\beta(1-\bar{\varepsilon}\mu) + \bar{\varepsilon}\gamma\nu}{1-\gamma(1-\bar{\varepsilon}) - \bar{\varepsilon}\mu} > 0 \qquad (27)$$

$$G \equiv A\left\{ A^{1-\bar{\varepsilon}} B^{\bar{\varepsilon}} \gamma \left[\frac{\chi}{\alpha} \left(\frac{\alpha\nu}{\chi\beta} \right)^{\nu} \left(\frac{\alpha\mu}{\chi\gamma} \right)^{\mu} \right]^{\bar{\varepsilon}} \prod_{i=1}^{n} \varepsilon_i^{\varepsilon_i} \right\}^{\frac{\gamma}{1-\gamma(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}} \qquad (28)$$

$$\bar{\varepsilon} \equiv \sum_{i=m+1}^{n} \varepsilon_i \qquad (29)$$

and

$c_1 \equiv 1 - c_2$, $c_2 \equiv \frac{1-\frac{\alpha\mu}{\chi\gamma}}{1-\frac{\alpha\nu}{\chi\beta}}$, $c_3 \equiv \left(1-\frac{\alpha\nu}{\chi\beta}\right)\frac{\chi}{\alpha}\frac{\sum_{m+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}$, $c_4 \equiv \left(1-\frac{\alpha\nu}{\chi\beta}\right)\frac{\chi}{\alpha}\sum_{i=m+1}^{n}\varepsilon_i$, $c_5 \equiv 1 - c_6$ and

$c_6 \equiv \frac{1-\frac{\alpha}{\chi}}{1-\frac{\alpha\nu}{\chi\beta}}$.

Note that *G* grows at positive constant rate, *q* is positive and $\bar{\varepsilon} < 1$.[5]

Equations (20)-(28) look actually more complicated than they are. As we will see soon they are quite the same as in the standard one-sector Ramsey-Cass-Koopmans-model[6] or Solow-model. The key difference is that our equations feature the term l_m / k_m, which reflects the impact of cross-capital intensity structural change on the development of aggregates.

[5] The term within the {}-brackets in equation (28) grows at constant positive rate since $\bar{\varepsilon}$ is positive and smaller than one (see equation (29)). Furthermore, the exponent of the {}-brackets is positive as well, since $\gamma(1-\bar{\varepsilon}) + \mu\bar{\varepsilon} < 1$ (a weighted average of numbers that are smaller than one (γ and μ) is always smaller than one). As well, *q>0,* since $\gamma(1-\bar{\varepsilon}) + \mu\bar{\varepsilon} < 1$.

[6] For a discussion of the Ramsey-model see e.g. Barro, Sala-i-Martin (2004) pp. 85ff.

However, before discussing these facts we start with the definition of our equilibrium growth path which is quite the similar as the definition used by Ngai and Pissarides (2007).

Definition 1: *An aggregate balanced growth path (ABGP) is an equilibrium growth path where aggregates (Y,* $\tilde{Y}$ *, K, E and H) grow at a constant rate.*

Note that this definition does require balanced growth for aggregate variables. However, it does not require balanced growth for sectoral variables (e.g. for sectoral outputs). Hence, it allows for structural change.

Lemma 1: *Equations (20) to (28) imply that there exists a unique ABGP, where aggregates (Y,* $\tilde{Y}$ *, K, E and H) grow at constant rate* g^* *and where* l_m / k_m *is constant. The ABGP-growth rate is given by* $g^* = \frac{(1-\mu\bar{\varepsilon})g_A + \gamma\bar{\varepsilon}g_B}{(1-\mu\bar{\varepsilon})\alpha + \gamma\bar{\varepsilon}\chi} + g_L$.

Proof: See APPENDIX A.

Proposition 1: *The ABGP is locally saddle-path stable.*

Proof: See APPENDIX B.

Proposition 1 ensures that the economy will approach to the ABGP even if the initial capital level is not such that the economy starts on the ABGP, provided that the initial capital level is not to far away from the ABGP-capital-level at the starting time.

Proposition 2: *Along the ABGP the aggregate dynamics of the economy are represented by the following equations:* $\hat{Y} = \dot{K} + \delta K + E$ *;* $\hat{Y} = \tilde{G}L^{1-q}K^q$ *and* $\frac{\dot{E}}{E} = \lambda\frac{\hat{Y}}{K} - \delta - \rho$*, where* $\tilde{G}$ *is a* *parameter growing at constant rate ("Hicks-neutral technological progress"), $\hat{Y}$ denotes aggregate output without intermediates production (i.e. Y-H) and λ is a constant (see APPENDIX A for details of these parameters).*

Proof: *See APPENDIX A.*

In fact Proposition 2 implies that the aggregate structure of our economy is quite the same as the structure of the standard Ramsey-Cass-Koopmans- or Solow-model (with Cobb-Douglas production function and logarithmic utility).

Now, the question arises, whether structural change takes place along the ABGP. We discuss this question in the following.

Development of Sectors in Equilibrium

By inserting equations (1) to (16) into optimality conditions (17) to (19), the following equations that describe the optimal sector structure of the economy (represented by the employment shares) can be obtained:

$$l_{agr.} = \Lambda_{agr.} + \frac{1}{\tilde{Y}} \sum_{i=1}^{a} \theta_i \qquad (30a)$$

$$l_{man.} = \Lambda_{man.} + \frac{1}{\tilde{Y}} \sum_{i=a+1}^{m} \theta_i + \Gamma \sum_{i=m+1}^{s} \theta_i \qquad (30b)$$

$$l_{ser.} = \Lambda_{ser.} + \Gamma \sum_{i=s+1}^{n} \theta_i \qquad (30c)$$

where

$$\Lambda_{agr.} \equiv \frac{\sum_{i=1}^{a}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\sum_{i=1}^{a}\varepsilon_i \qquad \text{(31a)}$$

$$\Lambda_{man.} \equiv \frac{\sum_{i=a+1}^{m}\omega_i + \frac{\chi}{\alpha}\sum_{i=m+1}^{s}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\left(\sum_{i=a+1}^{m}\varepsilon_i + \frac{\chi}{\alpha}\sum_{i=m+1}^{s}\varepsilon_i\right) + \frac{\dot{K}+\delta K}{\tilde{Y}} \qquad \text{(31b)}$$

$$\Lambda_{ser.} \equiv \frac{\chi}{\alpha}\frac{\sum_{i=s+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}\frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}}\frac{\chi}{\alpha}\sum_{i=s+1}^{n}\varepsilon_i \qquad \text{(31c)}$$

$$\Gamma \equiv \frac{1}{\left(\frac{\alpha\nu}{\chi\beta}\right)^{\nu}\left(\frac{\alpha\mu}{\chi\gamma}\right)^{\mu} B\left(\frac{G}{A}\right)^{\mu/\gamma} L\left(\frac{k_m K}{l_m L}\right)^{\nu+\frac{\mu\beta(1-\bar{\varepsilon})+\nu\bar{\varepsilon}}{1-(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}}} \qquad \text{(31d)}$$

Note that $\Lambda_{agr.}, \Lambda_{man.}, \Lambda_{ser.}$ and Γ can be easily derived as functions of exogenous parameters along the ABGP.[7] However, we omit here the explicit proof, since it is trivial and irrelevant for further discussion (for a sketch of the proof see footnote 7).

Lemma 2: *Structural change takes place along the ABGP. That is, the employment shares of sectors agriculture ($l_{agr.}$), manufacturing ($l_{man.}$) and services ($l_{ser.}$) are changing along the ABGP.*

---

[7] In APPENDIX A (equation (A.17) we have derived l_m / k_m as function of exogenous model parameters. This function can be used to derive $\tilde{Y}$ and Y as functions of exogenous model parameters by using equations (21) and (25). Then, when we have $\tilde{Y}$ and l_m / k_m as functions of exogenous model parameters, we can derive H as a function of exogenous model parameters by using equation (23). Finally, we can use Y and H to derive E as function of exogenous model parameters (via equation (20); note that the initial capital endowment K_0 is exogenously given; hence K can be calculated by using K_0 and the equilibrium growth rate of capital g^*, where g^* is a function of exogenous model parameters as shown in Lemma 1). When we have l_m / k_m, $\tilde{Y}$, K

Proof: This Lemma is implied by equations (30) and (31). Note that $\Lambda_{agr.}$, $\Lambda_{man.}$, and $\Lambda_{ser.}$ are constant along the ABGP (due to Lemma 1); $\tilde{Y}$ grows at rate g^* along the ABGP (see Lemma 1). Γ decreases at constant rate along the ABGP. The latter fact comes from Lemma 1 and equation (28). Note that *G/A* grows at positive constant rate; see equation (28) and footnote 5. Furthermore, note that the exponent $v + \frac{\mu\beta(1-\bar{\varepsilon})+v\bar{\varepsilon}}{1-(1-\bar{\varepsilon})\gamma-\mu\bar{\varepsilon}}$ is positive, since $\gamma(1-\bar{\varepsilon})+\mu\bar{\varepsilon}<1$ as explained in footnote 5. ***Q.E.D.***

Now, the remaining exercise is to show that along the ABGP our model is indeed consistent with all the stylized facts mentioned in the introduction and section 2 of our paper.

Consistency with Stylized Facts

Lemma 3: *The ABGP of our model satisfies the Kaldor facts regarding the development of the great ratios. That is, the aggregate capital intensity (K/L) is increasing; the aggregate capital-income-share (rK/Y or $rK/(Y-H)$), the aggregate labor-income-share (wL/Y or $wL/(Y-H)$) and the aggregegate capital-to-output ratio (K/Y or K/(Y-H)) are constant (where r is the real rate of return on capital and w is the real wage rate).*

Proof: The constancy of *K/Y* and *K/(Y-H)* as well as the increasing capital-intensity (*K/L*) are directly implied by Lemma 1. Since we assume perfect polypolisitic markets, the marginal productivity of capital (of labor) in a sub-sector i is equal to the real rate of return on capital (real wage rate) for all i. This implies for example for $i = m$:

$$r = \frac{\partial Y_m}{\partial(k_m K)} = \beta \frac{l_m}{k_m} \frac{\tilde{Y}}{K} \qquad (32)$$

and E as functions of exogenous model parameters, we can derive $\Lambda_{agr.}$, $\Lambda_{man.}$, $\Lambda_{ser.}$ and Γ as functions of exogenous model parameters.

$$w = \frac{\partial Y_m}{\partial (l_m L)} = \alpha \frac{\tilde{Y}}{L} \text{ (33)}$$

Hence, Lemma 1 and equations (32) and (33) imply that $\frac{rK}{Y}$, $\frac{rK}{Y-H}$, $\frac{wL}{Y}$ and $\frac{wL}{Y-H}$ are constant. ***Q.E.D.***

Note, that there are two further Kaldor-facts: namely Kaldor stated that the aggregate volume of production grows at a non-decreasing rate and that the real rate of return on capital is constant. As discussed in section 3.1, due to numéraire choice we cannot say whether these two Kaldor-facts are satisfied approximately in our model. However, as mentioned before, the constancy of the real interest rate seems to be rather not a fact in reality. Furthermore, the results by Ngai and Pissarides (2004, 2007) imply that the aggregate output expressed in manufacturing terms (as in our model) behaves quite similar as the aggregate output that is measured in reality (by using some compound numéraire). Hence, our model could be consistent with a constant real rate of aggregate output.

Lemma 4*: Along the ABGP the development of sectoral employment shares over time (equations (30)-(31)) can be monotonous (monotonously increasing, monotonously decreasing or constant) or non-monotonous ("hump-shaped" or "U-shaped"), depending on the parameterization of the model.*

Proof: This Lemma is implied by equations (30)-(31). In the proof of Lemma 2 we have shown that $\Lambda_{agr.}$, $\Lambda_{man.}$, and $\Lambda_{ser.}$ are constant along the ABGP, $\tilde{Y}$ grows at rate g^* along the ABGP (see Lemma 1) and Γ decreases at constant rate along the ABGP. Hence, since $1/\tilde{Y}$ and Γ grow at different rates, equation (30b) implies that the development of the manufacturing-employment-share over time ($l_{man.}$) can be non-monotonous, provided that

$\sum_{i=a+1}^{m}\theta_i$ has not the same sign as $\sum_{i=m+1}^{s}\theta_i$. That is, it can be hump-shaped or U-shaped depending on the parameterization. Hence, the model can reproduce a "hump-shaped" development of the manufacturing-employment share over time, which has been emphasized by Ngai and Pissarides (2007) and Maddison (1980). Note that only sectors, which use at least two technologies, can feature non-monotonous development of their employment share over time. However, as discussed in section 3.1 the manufacturing sector (i.e. the capital producing sector) need not using two technologies, i.e. the model could be set up such that the agricultural sector or the services sector uses two technologies. Hence, in fact any of the sectors could feature non-monotonous dynamics of its employment-share over time. The proof that

- $l_{agr.}$ can be monotonously increasing, monotonously decreasing or constant,
- $l_{man.}$ can be monotonously increasing or monotonously decreasing, and
- $l_{ser.}$ can be monotonously increasing, monotonously decreasing or constant

is obvious when taking into account that $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$ can be negative, positive or equal to zero respectively. ***Q.E.D.***

Lemma 5: *Agriculture, manufacturing and services have different production functions in our model. Especially, the optimal capital intensity differs across these sectors.*

Proof: Since we assumed that agriculture (services) uses only technology 1 (2) its production function is represented by technology 1 (2). Hence, we know that the technology (especially the TFP-growth-rate and the capital-intensity) differ across agriculture and services. Furthermore, manufacturing uses both technologies. Hence, the average manufacturing technology is a mix of technology 1 and 2. Hence, the representative production function of the manufacturing sector is different in comparison to the services sector or the agricultural sector which each use only one technology. Nevertheless, since we have an emphasis on the cross-capital-intensity structural change, let us have a close look on the capital-intensity ($\frac{k_{agr.}K}{l_{agr.}L}$, $\frac{k_{man.}K}{l_{man.}L}$ and $\frac{k_{ser.}K}{l_{ser.}L}$), the output-elasticity of labor ($\lambda_{agr.}$, $\lambda_{man.}$ and $\lambda_{ser.}$) and the output-elasticity of capital ($\kappa_{agr.}$, $\kappa_{man.}$ and $\kappa_{ser.}$) in each sector

$$\frac{k_{agr.}K}{l_{agr.}L} = \frac{k_m K}{l_m L} \neq \frac{k_{man.}K}{l_{man.}L} = \frac{k_m K}{l_m L}\left(1+\frac{\alpha\nu}{\chi\beta}\right) \neq \frac{k_{ser.}K}{l_{ser.}L} = \frac{k_m K}{l_m L}\frac{\alpha\nu}{\chi\beta} \qquad (34)$$

$$\lambda_{agr.} = \frac{wl_{agr.}L}{Y_{agr.}} = \alpha \neq \lambda_{man.} = \frac{wl_{man.}L}{Y_{man.}} = \frac{\alpha l_{man.}}{\sum_{i=a+1}^{m} l_i + \frac{\alpha}{\chi}\sum_{i=m+1}^{s} l_i} \neq \lambda_{ser.} = \frac{wl_{ser.}L}{Y_{ser.}} = \chi \quad (35)$$

$$\kappa_{agr.} = \frac{rk_{agr.}K}{Y_{agr.}} = \beta \neq \kappa_{man.} = \frac{rk_{man.}K}{Y_{man.}} = \beta\frac{\sum_{i=a+1}^{m} l_i + \frac{\alpha\nu}{\chi\beta}\sum_{i=m+1}^{s} l_i}{\sum_{i=a+1}^{m} l_i + \frac{\alpha}{\chi}\sum_{i=m+1}^{s} l_i} \neq \kappa_{ser.} = \frac{rk_{ser.}K}{Y_{ser.}} = \nu \quad (36)$$

(Note output elasticity of factors is equal to the factor-income shares due to the assumption of perfect markets and perfect factor mobility.) Overall, capital intensities and output-elasticities of inputs differ across sectors agriculture, manufacturing and services. ***Q.E.D.***

Lemma 6: *Along the ABGP the factor reallocation across the agricultural, manufacturing and services sector is determined by cross-sector-TFP-growth disparity, by cross sector capital-intensity-disparity and by non-homothetic preferences.*

Proof: As discussed above, the *TFP-growth rates* and the *capital-intensities* differ across the sectors agriculture, manufacturing and services; see also Lemma 5. Equations (30)-(31) (and equations (21) and (28)) imply that cross-sector-differences in TFP-growth-rates and cross-sector-differences in output-elasticities of inputs (which determine the capital-intensities) determine the strength of the factor reallocation between the sectors agriculture, manufacturing and services. Especially, they affect the sectoral employment shares ($l_{agr.}$, $l_{man.}$

and $l_{ser.}$) via the terms $\tilde{Y}$ and Γ, which are among others functions of the parameters that determine the sectoral TFP-growth rates and sectoral capital intensities (see equations (21), (31d) and (28) and Lemma 5).

Furthermore, equations (8) to (11) imply that preferences are *non-homothetic* across sectors agriculture, manufacturing and services. A detailed proof is in APPENDIX C, where we show among others that the terms $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$ determine the pattern of non-homotheticity across sectors agriculture, manufacturing and services. Equations (30)-(31) imply that this non-homotheticity determines the strength and direction of structural change (via terms $\sum_{i=1}^{a}\theta_i$, $\sum_{i=a+1}^{m}\theta_i$, $\sum_{i=m+1}^{s}\theta_i$ and $\sum_{i=s+1}^{n}\theta_i$). ***Q.E.D.***

Lemma 7: *Intersectoral outsourcing (i.e. shifts in intermediates production across sectors) takes place along the ABGP. That is, along the ABGP manufacturing-sector-producers shift more and more intermediates production to services-sector-producers (i.e.* h_i / h_j *changes), provided that services-sector-production becomes cheaper and cheaper (or less and less expensive) in comparison to manufacturing-sector-production (i.e. provided that relative prices change), and vice versa. Any direction of relative price changes (and hence any direction of intermediate-production shifts between the manufacturing and the services sector) can be generated along the ABGP, depending on the parameterization.*

Proof: See Appendix D.

Theorem 1: *The ABGP satisfies simultaneously the following stylized facts:*

- *Kaldor-facts regarding the development of the great ratios,*
- *Kuznets facts regarding structural change patterns,*

- *“stylized facts regarding cross-sector-heterogeneity in production-technology” (see section 2 as well), and*
- *empirical evidence on structural change determinants in industrialized countries (see section 2).*

Proof: The consistency of the ABGP with the *Kaldor facts* is implied by Lemma 3.

Note that empirical evidence on structural change between agriculture, manufacturing and services in industrial countries implies the following stylized facts for the development of the employment shares over the last century:

- the agricultural sector featured a monotonously decreasing employment share,
- the services sector featured a monotonously increasing employment share, and
- the manufacturing sector featured a constant or “hump-shaped” employment share (depending on the length of the period considered).

These stylized facts have been formulated by Kongsamut et al. (1997, 2001); on the “humped shape” of the manufacturing-employment share see e.g. Ngai and Pissarides (2004, 2007) and Maddison (1980). In the proof of Lemma 4 we have shown that our model can reproduce these stylized facts regarding the development of the agricultural, manufacturing and services employment shares. Hence, the ABGP is consistent with the *Kuznets-facts*.

The consistency of the ABGP with the *“stylized facts regarding cross-sector-heterogeneity in production-technology”* is shown in Lemma 5, where we show that production technology differs across agriculture, manufacturing and services in our model.

Finally the consistency of the ABGP with the *empirical evidence on structural-change-determinants in industrialized countries* is shown in Lemmas 6 and 7. ***Q.E.D.***

The Relationship between Structural Change and Aggregate-Dynamics

Now we turn to the question about the relationship between structural change and aggregate growth, i.e. we ask how structural change affects aggregate growth, which is important for understanding the Kuznets-Kaldor-puzzle. In the following we will show that there are two types of cross-capital-intensity structural change, which are distinguished according to their impact on the aggregate structure of the economy.

Definition 2: *The term "cross-capital-intensity structural change" stands for factor reallocation across sectors that differ by capital intensity.*

It can be shown that

$$\bar{l} \equiv \frac{l_m}{k_m}\frac{\beta_m}{\alpha_m} = \left(\frac{\kappa_{agr.}}{\lambda_{agr.}} l_{agr.} + \frac{\kappa_{man.}}{\lambda_{man.}} l_{man.} + \frac{\kappa_{ser.}}{\lambda_{ser.}} l_{ser.} \right) \qquad (37)$$

where $\lambda_{agr.}$ $(\kappa_{agr.}), \lambda_{man.}$ $(\kappa_{man.})$ and $\lambda_{ser.}$ $(\kappa_{ser.})$ are respectively the income-share of labor (capital) in sectors agriculture, manufacturing and services. Equation (37) follows from the assumption of factor mobility across sectors and from the assumption of perfect markets.

Equation (37) and Lemma 1 imply that there are two sorts of cross-capital-structural change:

(1) Cross-capital-intensity structural change where $\bar{l}$ *is not constant.* Lemma 1 implies that the economy is on an ABGP, only if l_m / k_m is constant; furthermore, equation (37) implies that the constancy of $\bar{l}$ is required for the constancy of l_m / k_m. Hence, as long as $\bar{l}$ is not constant, the economy is not on an ABGP and the Kaldor-facts are not satisfied (exactly). That is, this type of structural change is not compatible with the Kaldor facts (unless structural change is very weak such that its impact via $\bar{l}$ is weak which would imply that Kaldor facts are approximately satisfied).

(2) Cross-capital-intensity structural change that is compatible with a constant $\bar{l}$. Hence, an economy can be on an ABGP, even when cross-capital-intensity factor reallocation takes place, provided that this factor reallocation is such that $\bar{l}$ = const. (see also Lemma 1).

So we can give the following definition and theorem:

Definition 3: *"Neutral cross-capital-intensity structural change" stands for cross-capital-intensity structural change that satisfies the following condition:*

$$\bar{l} \equiv \left(\frac{\kappa_{agr.}}{\lambda_{agr.}} l_{agr.} + \frac{\kappa_{man.}}{\lambda_{man.}} l_{man.} + \frac{\kappa_{ser.}}{\lambda_{ser.}} l_{ser.} \right) = const. \qquad (38)$$

Theorem 2: *Along the ABGP, the cross-capital-intensity structural change (between agriculture, manufacturing and services) is "neutral" in the sense of Definition 3.*

Proof: Note that we have shown in Lemma 5 that sectors agriculture, manufacturing and services differ by technology, and especially by capital intensity and by output-elasticities of inputs/income-shares of inputs. Lemma 2 implies that structural change takes place across these sectors. Equation (37), Definition 3 and Lemma 1 (necessity of a constant l_m / k_m for an ABGP) imply the rest of the theorem. ***Q.E.D.***

Theorem 3*: Neutral cross-capital-intensity structural change is an explanation for the Kuznets-Kaldor-Puzzle in our model.*

Proof: Remember that the Kuznets-Kaldor-puzzle was about the empirical question why cross-capital-intensity structural change is compatible with the stability of the great ratios (Kaldor facts). Theorem 2 implies that neutral-cross-capital-intensity structural change takes place along the ABGP, while Theorem 1 shows that the ABGP is consistent with the Kaldor facts. Thus, Kaldor-facts are satisfied, since cross-technology structural change needs not necessarily to contradict the Kaldor facts, which is satisfied in our model only neutral cross-capital-intensity structural change patterns. Furthermore, Theorem 1 shows the generality of our proof: neutral cross-technology structural change is not only consistent with the Kaldor facts about the great ratios but also with the other stylized facts which are relevant for the analysis of the relationship between structural change and aggregates. Hence, Theorem 1 shows that we solved the Kuznets-Kaldor-puzzle under consideration of the most important structural change determinants and under assumption of sectoral cross-technology disparities observed in reality. ***Q.E.D.***

The convenient feature regarding latter two theorems is that we can use them to test our theory empirically: We can calculate $\bar{l}$, and then decompose which share of structural change does not change the value of $\bar{l}$ and which share of structural change changes the value of $\bar{l}$. In this way we can evaluate the quantitative significance of our model-explanation for the Kuznets-Kaldor-Puzzle, since our explanation focuses only on structural change that does not change $\bar{l}$ (due to Theorem 2).

However, before doing so we show two further interesting results

Proposition 3: *The output-elasticity of inputs* $(\lambda_{man.}, \kappa_{man.})$ *is not constant in the manufacturing sector along the ABGP, but changes according to the amount of inputs used in this sector.*

Proof: This is implied by equations (35) and (36). Note, that any sector that uses two technologies has a non-constant output-elasticity of inputs in our model setting. ***Q.E.D.***

This result is interesting: in fact it implies that observed technology changes in sectors need not necessarily resulting form technological progress at sector level, but can also result from structural change. Of course this requires that sectors use several technologies, which seems to be a reasonable assumption. This fact could be of importance for further research, especially when analyzing endogenous technological progress at sector level. That is, Proposition 3 implies that such research will require considering technology change at sector level with caution, since some technology change may not result from technological progress at sector level.

As argued in section 3.1 we assume that there are only two technologies in our model, but that there is an arbitrary number of subsectors. Hence, some subsectors have to use identical technologies. As explained there, we use this assumption to explain the concept of "uncorrelated preferences and technologies" in a traceable way, which will be of interest later in this paper. However, the assumption of partly identical production functions is not necessary for the key results of the actual section: the following proposition shows that our key result of this section (namely for the existence of neutral cross-capital-intenstity structural change) can be derived even all (sub-)sectors have completely different production functions.

Proposition 4: Generalization of our results: *In a framework where*

- *all sub-sectors (i) have sub-sector-specific production functions,*
- *sub-sector production functions are general neoclassical production functions*
- *and intermediate production is omitted*

a necessary condition for neutrality of cross-capital-intensity structural change and for the satisfaction of Kaldor-facts is

$$\tilde{l} \equiv \sum_i \frac{l_i}{\lambda_i} = const. \quad (39)$$

where λ_i is the output-elasticity of labor in subsector i which is equal to the labor-income share in sector i.

Proof: *See APPENDIX E.*

4. A Measure of Neutrality of Cross-Capital-Intensity Structural Change

In the previous section, we have presented a model that explains the Kuznets-Kaldor-puzzle with a certain structural change pattern which we name "NCCI structural change". In Theorem 2 and Proposition 4 we have shown that this structural change pattern must satisfy condition (38). Due to lack of data we cannot consider intermediates production explicitly. Therefore, we assume that capital and labor are the only inputs in the production function in this section. In this case condition (38) transforms into condition (39).

In proposition 4 we have generalized the validity of condition (39) to a more general framework than that that of section 3. Hence, the development of this condition is not only of interest for our model, but for all models that analyze ABGP's.

We can use condition (39) to asses to what extent NCCI structural change takes place in reality.

For the calculations in this section we use the data for the U.S.A., which is available at the web-site of the U.S. Department of Commerce (Bureau of Economic Analysis). We use the U.S.-Gross-Domestic-Product-(GDP)-by-Industry-Data, which is based on the sector-definition from the "Standard Industrial Classification System", which defines the following sectors:

(1) Agriculture, forestry, and fishing

(2) Mining

(3) Construction

(4) Manufacturing

(5) Transportation and public utilities

(6) Wholesale trade

(7) Retail trade

(8) Finance, insurance, and real estate

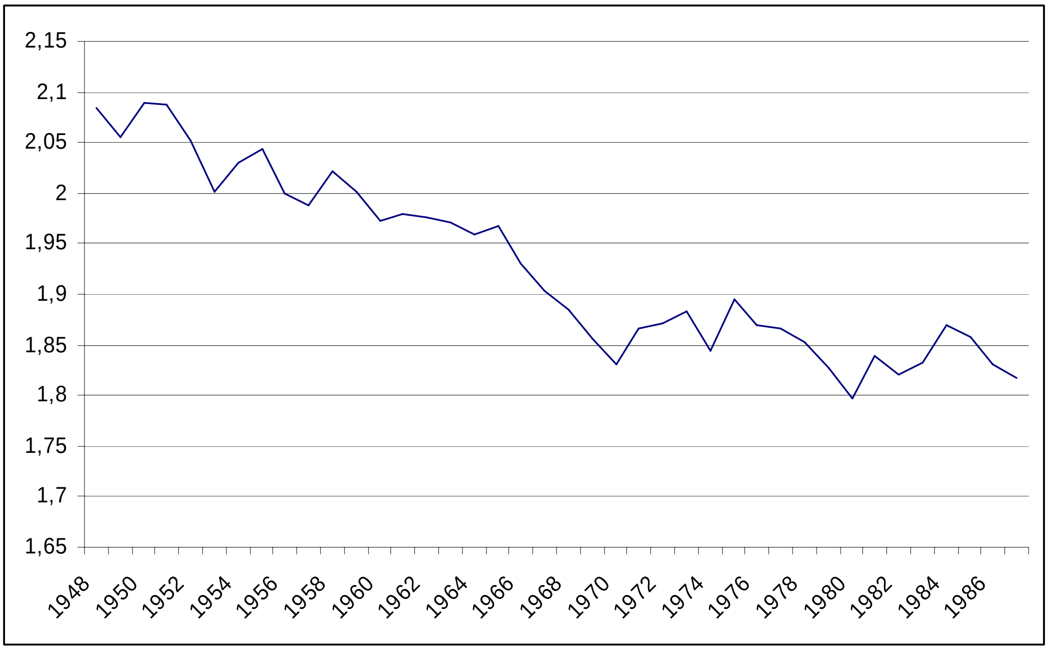
(9) Services

Our calculations are based on the data for the period 1948-1987. Uniform data for longer time-periods is not available, since the "Standard Industrial Classification System" has been modified over time (hence, the sector definition after 1987 is not the same as the sector definition before 1987).

To calculate the sectoral labor income shares (λ_i) we divided "(Nominal) Compensation of Employees" by "(Nominal) Value Added by Industry" in each sector. The sectoral employment shares (l_i) are calculated by using the sectoral data on "Full-time Equivalent Employees". (This approach is similar to that used by Acemogu and Guerrieri (2008)).

Figure 1 depicts the development of $\tilde{l}$, calculated by these data:

Figure 1: Development of $\tilde{l}$ over time



We can see that $\tilde{l}$ is decreasing and not constant. Hence, in conjunction with our results from the previous section (see also Definition 3) we can conclude that cross-capital-intensity structural change was not exclusively neutral during our sample period. That is, neutral cross-capital-intensity structural change has been accompanied by non-neutral cross-capital-intensity structural change. In other words, some (part) of the cross-sector factor reallocation did not change $\tilde{l}$, but some factor reallocation caused a decrease in $\tilde{l}$. A simple way to understand this view is the following: By grouping sectors in a certain manner one could create two groups of sectors. Group 1 would include technologically distinct sectors and if we calculated $\tilde{l}$ for this group, it would be (nearly) constant. Group 2 would include technologically distinct sectors and if we calculated $\tilde{l}$ for this group, it would not be constant; in fact the change in $\tilde{l}$ of this group would be (nearly) equal to the change in $\tilde{l}$ of the overall economy. Hence, by calculating the employment shares of the two groups, we could say how much of the economy wide factor reallocation was neutral and how much not. The change in the employment share of group 1 would represent the NCCI structural change and the change in the employment share of the group 2 would represent non-neutral structural change.

However, due to data restrictions we do not know the exact reallocation patterns of labor, i.e. when an employee leaves a sector we do not know to which sector he/she is reallocated. Hence, our method of creating groups seems somewhat arbitrary, since probably there are several different combinations of sectors that could be included into group 1, while ensuring that $\tilde{l}$ of this group is nearly constant. Hence, the employment share of group 1 would differ (strongly) depending on which sector-combination is used. Hence, it would be difficult to choose one combination. (We could use the criterion that the combination, which has the most constant $\tilde{l}$, should be used; however, this criterion seems as well somewhat arbitrary.)

Therefore, we use the following concept to assess neutrality: Any actual $\tilde{l}$ can be expressed as a unique combination of neutral and "maximally non-neutral structural change". "Maximally non-neutral structural change" is the pattern of factor reallocation that causes the maximal change in $\tilde{l}$ for a given amount of reallocated labor over a period. Hence, maximally non-neutral structural change is a diametric concept of NCCI structural change: while NCCI structural change is defined upon no change in $\tilde{l}$, maximally non-neutral structural change is defined upon maximal change in $\tilde{l}$. This allows us to create an index that shows us whether a given amount of reallocated labor has been reallocated rather in the neutral way or rather in the maximally non-neutral way. According to this discussion the following relation must be true:

$$(\Delta\tilde{l})^{actual} = (1 - I_N)(\Delta\tilde{l})^{neutral} + I_N(\Delta\tilde{l})^{\max} \qquad (39)$$

where I_N is a weighting factor between NCCI and maximally non-neutral structural change, i.e. it indicates whether structural change was rather neutral or non-neutral; if I_N =1, structural change is maximally non-neutral over the observation period; if I_N =0 structural change is neutral over the observation period. $(\Delta\tilde{l})^{actual}$ measures the change in $\tilde{l}$ that really took place between 1948 and 1987; $(\Delta\tilde{l})^{\max}$ measures the maximal change in $\tilde{l}$, that would be (hypothetically) possible with the given amount of cross-sector factor reallocation between 1948 and 1987, i.e. $(\Delta\tilde{l})^{\max}$ stands for "completely non-neutral structural change". $(\Delta\tilde{l})^{neutral}$ measures the change in $\tilde{l}$ that is caused by NCCI structural change. Since per definition $(\Delta\tilde{l})^{neutral}$ is equal to zero, we can rearrange the condition from above as follows:

$$I_N \equiv \frac{(\Delta\tilde{l})^{actual}}{(\Delta\tilde{l})^{\max}} \qquad (40)$$

$(\Delta\tilde{l})^{\max}$ and $(\Delta\tilde{l})^{actual}$ are defined as follows:

$$(\Delta\tilde{l})^{actual} \equiv \tilde{l}_{1987} - \tilde{l}_{1948} = \sum_i \frac{l_i^{1987}}{\lambda_i^{1987}} - \sum_i \frac{l_i^{1948}}{\lambda_i^{1948}} \qquad (41)$$

$$(\Delta\tilde{l})^{\max} \equiv \tilde{l}_{1987}^{\max} - \tilde{l}_{1948} = \sum_i \frac{l_i^{1987\max}}{\lambda_i^{1987}} - \sum_i \frac{l_i^{1948}}{\lambda_i^{1948}} \qquad (42)$$

where l_i^{1948}, l_i^{1987}, λ_i^{1948} and λ_i^{1987} denote respectively the employment share of sector i in 1948, the employment share of sector i 1987, the labor-share of income in sector i 1948 and the labor-share of income in sector i 1987. $l_i^{1987\max}$ stands for the employment share of sector i, which would result, if the labor, which has been reallocated between 1948 and1987, were reallocated in such a manner that the maximal decrease in $\tilde{l}$ is accomplished between 1948 and 1987. That is, the $l_i^{1987\max}$'s stand for the hypothetical factor allocation in 1987, which yields the maximally non-neutral structural change between 1948 and 1987.

Last but not least, since our definition of $l_i^{1987\max}$ requires knowing how much labor has been reallocated between 1948 and 1987, we measure the observable amount of factor reallocation that took place between 1948 and 1987 by:

$$\Delta l \equiv \frac{1}{2}\sum_i \left| l_i^{1987} - l_i^{1948} \right|$$

This measure is set up as follows: First, the change in the employment share in each sector is measured. The absolute values (modulus) of these changes are summed up (otherwise, without taking absolute values, that sum of the sectoral changes would always be equal to zero, since $\sum_i l_i = 1$ per definition). Since the change in the employment share in one sector has always a corresponding change in the employment shares of the other sectors (labor is reallocated across sectors), the sum of the absolute values of the changes must be divided by two to avoid double-counting.

It is possible that between 1948 and 1987 in some sectors the employment share increased first and decreased then. Hence, the pure difference $l_i^{1987} - l_i^{1948}$ would indicate less reallocation than actually took place. Our index of factor reallocation (Δl) neglects such non-monotonousity in sectoral employment shares. Hence, it underestimates the real amount of labor reallocated between 1948 and 1987. Therefore, our index I_N underestimates the neutrality of structural change: if more labor were reallocated during the period, the hypothetical maximal change in $\tilde{l}$ $((\Delta\tilde{l})^{\max})$ would be larger; hence, I_N would be smaller, which would imply more neutrality. Overall, for these reasons, our index I_N indicates less neutrality than actually is.

The data that we need for our calculations is given in the following table:

Table 1

Sector	$1/\lambda_i^{1948}$	$1/\lambda_i^{1987}$	l_i^{1948}	l_i^{1987}
(8)	5.248981966	3.997781119	0.039609477	0.077711379
(1)	6.874359747	3.921756596	0.05019623	0.019310549
(2)	2.62541713	3.240100098	0.024056398	0.008630482
(5)	1.632072868	2.20691581	0.099835263	0.063265508
(6)	1.937362752	1.72651328	0.062648384	0.070192118
(7)	1.988458748	1.649066345	0.141770435	0.191092947
(3)	1.495168451	1.505702087	0.056228499	0.059919498
(4)	1.505805486	1.447391372	0.376011435	0.229516495
(9)	1.681140684	1.444831355	0.149643878	0.280361023

Now, by using these data, we have to do the following steps to calculate I_N :

1.) Calculate the amount of reallocated labor between 1948 and (1987), which results in $\Delta l \approx$ 0.23.

2.) Calculate $\tilde{l}_{1987}^{\max}$. According to our definition of $\tilde{l}_{1987}^{\max}$, we have to do the following steps:

a.) Find the sector that has the smallest $1/\lambda_i^{1987}$. This is actually sector (9).

b) Make a ranking of the *remaining* sectors according to their $1/\lambda_i^{1987}$. This ranking is given by (8)-(1)-(2)-(5)-(6)-(7)-(3)-(4), where sector (8) has the largest $1/\lambda_i^{1987}$ and sector (4) has the smallest $1/\lambda_i^{1987}$ in this ranking.

c) By using the ranking from b) reallocate the labor from the *sectors* with the largest $1/\lambda_i^{1987}$ to sector (9). We first use the whole amount of labor, that has been employed in sector (8) in 1948, then the whole amount of labor, that has been employed in sector (1) in 1948, and so on, stepping up in the ranking until we have hypothetically reallocated the whole $\Delta l \approx$ 0.23. Hence, we obtain the following maximally non-neutral factor allocation for the year 1987

Table 2

Sector	$l_i^{1987\ \max}$
(8)	= 0
(1)	= 0
(2)	= 0
(5)	= 0
(6)	$= l_{(6)}^{1948} - (\Delta l - l_{(1)}^{1948} - l_{(2)}^{1948} - l_{(5)}^{1948} - l_{(8)}^{1948}) = 0.046969461$
(7)	$= l_{(7)}^{1948} = 0.141770435$
(3)	$= l_{(3)}^{1948} = 0.056228499$
(4)	$= l_{(4)}^{1948} = 0.376011435$
(9)	$= l_{(9)}^{1948} + \Delta l = 0.37902017$

3.) The rest of the calculations is quite simple: by inserting the data from Tables 1 and 2 into equations (40)-(42), we can obtain I_N.

Our calculations imply an index I_N = 0.45. This implies that actual structural change was slightly closer to its neutral extreme than to its non-neutral extreme. In other words, the actual structural change between 1948 and 1987 was by 55% neutral and by 45% maximally non-neutral.

In this sense, our model can explain 55% of the structural change between 1948 and 1987.

Note that our measure underestimates the neutrality of structural change. That is, in reality more than 55% of structural change can be regarded as neutral. There are two reasons: as discussed above, our measure assumes monotonousity of factor reallocation; furthermore, as will be discussed close to the end of next section, our period is quite short and structural change is more neutral over very long periods of time.

5. On Correlation between Preferences and Technologies

In section 3.1 we have assumed that preferences and technologies are uncorrelated in our model. In detail, we have assumed that

- on average the income elasticity of demand is equal when comparing technology-A-goods and technology-B-goods
- on average the relative price elasticity of demand is equal to unity when comparing technology-A-goods and technology-B-goods.

In the following we will discuss the rationale for these assumptions. We focus here on relative price elasticity of demand, but the corresponding arguments apply for the income elasticity of demand.

Assuming that the relative price-elasticity of demand between two goods is different from unity implies that the household has a certain preference for the one good over the other: Imagine that there are only two goods (good A and good B). If the relative price of the good A increases by one percent and the relative demand for this good decreases by less than one percent, good A is regarded as more important than good B by the household in the dynamic context. That is, the price change causes a weaker reaction than it would be if the two goods were regarded as equivalents. Only if two goods are regarded as equivalents, a one-percent-change in the relative price between these goods would yield a one-percent-change in the demand-relation between these goods.

Now, the same argument could be applied to two groups of goods (group A and group B): if the household regards the two groups as equivalents, the average (relative) price elasticity between the two groups is equal to unity. Otherwise, we would have to postulate that on average group A includes goods that are preferred over group B (or the other way around).

Now, imagine that the whole range of products in an economy is divided into two groups according to their production technology. Group A includes goods that are regarded as technologically progressive and group B includes goods that are produced by a backward technology. Furthermore, let us make the following assumptions:

(a) The household doesn't know anything about the production process, i.e. the household's preference depends only on the "objective taste" of the goods (but not on the knowledge that the good is produced at e.g. high-capital-intensity). "Objective taste" means the taste which depends only on the physical/chemical properties of the good or on the basic properties (i.e. actual quality) of the service, but not on the knowledge about the production process of the good or service. For example, if two goods are produced by different capital intensities, but if the two goods are basically the same (i.e. have the same physical and chemical properties), the objective taste of the two goods is the same. A further example is the following experiment: imagine that a live concert is recorded and then later replayed as a playback to a similar audience (while the original musicians pretend performing music). The labor-intensity of the original concert is higher in comparison to the playback concert, since pretending is easier (i.e. less labor-intense) in comparison to performing live music. The objective taste of the two concerts would be the same. (However, the "subjective taste" of the two concerts would differ, if the audience knew that the second concert is only a playback.)

(b) The "objective taste" of a good is on average not dependent on the technology that is used to produce it. That is, some very tasty goods are produced by progressive technology and some very tasty goods are produced by backward technology; as well, some less tasty goods are produced by progressive technology and some less tasty goods are produced by backward technology.

With these assumptions we would conclude that on average group-A-goods are not preferred over group-B-goods and group-B-goods are not preferred over group-A-goods. That is, the groups are regarded as equivalents; hence, on average the relative price-elasticity of demand between these two groups will be *close to* one (according to our discussion above).

Now let us make a further assumption:

(c) We look only on the averages over very long periods of time and we assume that there are many technologies and goods.

Hence, from this perspective due to the law of large numbers the price elasticity between the two groups is equal to unity.

In other words, if preferences and technologies are uncorrelated (i.e. if the taste does not depend on production technology), the household behavior will not display any preference for the technology-level (group A or group B), provided that very long periods of time are considered and provided that there are many goods.

This is what we assumed in section 3.1: we assumed that there are two technologies and that there are many goods that are produced with these technologies and that the preference structure does not display any preference for a certain technology. This is what we did by assumptions (10) and (11). These assumptions ensure that on average the relative price-elasticity between technology-A-goods and technology-B-goods is equal to unity.

Now the question is whether the assumptions (a), (b) and (c) are suitable in long run growth models.

Assumption (c) seems not to be problematic, since the long-run growth theory is anyway based on analyzing long-run-averages (e.g. the time preference rate is assumed to be constant in standard neoclassical growth models). Furthermore, since we look at very long run, any accidental correlations between technology and preferences that may arise from a relatively low number of products may as well offset each other over the period's average.

Assumption (b) is less problematic in comparison to assumption (a). In fact, the technological progress during the last century has implicitly shown that the basic physical/chemical properties of a good are not necessarily dependent on its capital-intensity. In industrialized countries nearly all goods featured some technological progress that substituted labor by capital, while the basic physical properties of the goods remained the same basically. The most obvious example is agriculture. Food has for the most part the same basic physical properties today as earlier in the century, while the capital-intensity of agriculture increased significantly. Such developments are also apparent in manufacturing (e.g. regarding the increasing capital-intensity of car-production) and services (e.g. cash-teller-machines). Furthermore, today we can imagine for nearly every good or service a relatively realistic technology that could substitute the labor by capital, without changing the basic physical properties of the good. It is not plausible to assume that in the very long run technological progress is restricted to certain types of goods. In the last two decades many service-jobs, which were regarded as labor-intensive, were replaced by computer-machines and the substitutability of human by machines in services is increasing. Hence, when developing a long run theory of structural change, the dependency between technology and certain types of goods (and hence certain preferences) seems to be difficult to defend. Therefore, overall, the assumption that the "objective taste" of a good is independent of the capital-intensity of the production process seems to be acceptable to some degree, especially when assuming (c).

It is more difficult to evaluate *assumption (a)* a priori. Assumption (a) requires that the representative household behaves like he doesn't know about the actual capital intensity of a good, i.e. it is required that the household's demand reaction to a price and/or income change is based only on physical/chemical properties of a good. What we know from basic microeconomics (e.g. form the discussion about "Giffen-goods") is that the price elasticity (and income elasticity) depends on the basic physical/chemical properties of the good, i.e. whether the physical/chemical properties of a good are such that it is feasible to satisfy the basic needs of a household. (The price elasticity for such goods is low.) On the other hand, there is also a discussion about a “snob” effect, where some high labor-intensity services (like a full time servant) are used to signal the wealth of the household. Such services have a relatively high income-elasticity and price-elasticity. However, as well, there are many high-capital-intensity-goods that have high price-elasticity of demand and high income-elasticity of demand, like very expensive cars. Hence, there is both: capital-intensive and labor-intensive goods that feature a relatively high price-elasticity and a relative high income-elasticity. Our model requires that *on average* (i.e. when looking on the average of at all consumption goods) the income (price) elasticity of demand does not depend on the capital-intensity of a good.

Last not least, the increasing complexity of the products and of the production process, international outsourcing and increasing variety of products make it increasingly unlikely that the household has information about the capital-intensity of a large part of its consumption bundle.

All in all, our empirical evidence from the previous section implies that the assumption of no/low correlation between technology and preferences can explain a part of the Kuznets-Kaldor-puzzle. The fact that there is some correlation between technology and preferences results probably from the fact that assumption (a) has not been satisfied over the time-period of our sample. That is, probably high labor-intensity of a service has been regarded as an aspect of quality and/or luxury. Hence, high-labor-intensity services have probably had high income-elasticity of demand on average, which caused the correlation between technology and preferences in the past.

The fact that there has been some correlation between preferences and technologies in our sample does not necessarily imply that we can presume such correlation in future:

We analyzed only a 40 year period. This is a very short period to satisfy our assumption (c) and to study growth theory empirically in general. Remember that Kaldor-facts (which we seek to explain in our paper) do not necessarily apply to such a short period. The probability is very high that over such a short period "accidental" correlation between technology and preferences arises, which does not persist over the long run. It seems that this was the case: The technological innovation between 1940 and 1980 allowed to a big part an increase in capital-intensity in non-service-sectors (such as manufacturing and agriculture). That is, the technological break-throughs were such that they were easy to implement in non-services sectors but they were hardly implementable in the services sector. Hence, if services have high income-elasticity of demand, some correlation between technology and preferences may have been arisen due to such biased technological progress. However, new sorts of technological break-through occurred after this period, especially in the information and communication technology. Such break-throughs have increased the capital-intensity in the services sector and have a high potential for increasing the capital-intensity of the services sector drastically (e.g. by progress in computers and robotics, which is implementable in services).

Hence, our empirical results probably over-estimate the long-run degree of correlation between preferences and technologies; the long-run correlation between preferences and technologies is probably very low or even inexistent. In this sense, our model of independent preferences and technologies predicts quite well the future structural change impacts on aggregates.

6. Concluding Remarks

In this paper we have searched for a solution of the Kuznets-Kaldor-puzzle. In fact, the Kuznets-Kaldor-puzzle states that aggregate ratios behaved in a quite stable manner in industrialized countries, while at the same time massive factor reallocation took place across sectors, which differ by technology (and especially by capital-intensity). (We have shown that previous literature implies that cross-sector-differences in capital intensity were the real obstacle regarding the Kaldor-facts, while e.g. cross-sector-differences in TFP-growth are quite compatible with Kaldor-facts.)

For the first time in the literature, we have shown that an ABGP can exist even when factors are reallocated across sectors that differ by capital intensity. We name the cross-capital-intensity structural change that is compatible with an ABGP "neutral structural change".

To test the actual neutrality of structural change we developed an index of neutrality. In fact, our measure of neutrality indicates the weighting between two measures $\left(\Delta\tilde{l}\right)^{neutral}$ and $\left(\Delta\tilde{l}\right)^{\max}$. $\left(\Delta\tilde{l}\right)^{neutral}$ measures the hypothetical change in $\tilde{l}$ that would result, if the empirically observed amount of reallocated labor (Δl) were reallocated in the neutral way. $\left(\Delta\tilde{l}\right)^{\max}$ measures the hypothetical change in $\tilde{l}$ that would result, if Δl were reallocated in the maximally non-neutral way. Hence, the weighting between these two measures implies how much labor has been reallocated in the neutral way and how much labor has been reallocated in the non-neutral way between 1948 and 1987. This index implies that 55% of structural change can be regarded as neutral. We provided also some theoretical arguments which imply that over the (very) long run significantly more than 55% of the structural change is neutral (see section 5).

We also provided a sort of "micro-foundation" for the neutrality of structural change, by showing that NCCI structural change can arise if preferences and technologies are uncorrelated. Therefore, our neutrality index could also be interpreted as an index of correlation between technology and preferences. In this sense, our empirical findings imply that the correlation between preferences and technologies is rather low. (Exactly speaking, the actual correlation was closer to the extreme of "no correlation" than to the extreme of "maximal correlation").

Note that we could try to assess the degree of correlation between preferences and technologies in an alternative way: First we would have to estimate the price elasticity of demand, the income elasticity of demand and the production functions for all sectors and then we would have to try to somehow figure out the degree of correlation between the estimated preference and technology parameters. This approach would be problematic for two reasons:

(1) Estimation of preference parameters (and especially of income elasticity of demand) is very difficult, since there are problems in measuring the changes in quality of goods and services. Hence, it is difficult to isolate whether demand for a good increased due to relatively high income-elasticity of demand or due to an increase in quality of the service. See e.g. Ngai and Pissarides (2007).

(2) Even if we could measure the preference and technology parameters exactly there would be a problem in defining a measure of correlation between preferences and technologies, since we have actually two sorts of preference parameters (income elasticity of demand and price elasticity of demand). Hence, if we have two economies (A and B), which are identical except for their correlation between income elasticity and technology and between price elasticity and technology, it would be difficult to say in which economy the correlation between preferences and technologies is lower: For example, if the correlation between *income elasticity* and technology is slightly lower in country A in comparison to country B and if the correlation between *price elasticity* and technology is slightly lower in country B in comparison to country A, we could not say whether preferences and technologies are more or less correlated in country A in comparison to country B. Our approach omits this problem by focusing on the factor reallocation across technology which as modeled in our paper reflects the degree of correlation between preferences and technologies.

Furthermore, note that our empirical findings are valid for all the literature that analyses structural change along ABGP's (and where capital is included into analysis): we have shown in Proposition 4 that every ABGP, that satisfies the Kaldor-facts (exactly), must feature NCCI structural change. Hence, we can say that the papers by Kongsamut et al. (2001), Ngai and Pissarides (2007) and Foellmi and Zweimueller (2008) are compatible with 55% of structural change observed.

Overall, our explanation for the Kuznets-Kaldor-puzzle is the following: There is a certain degree of independency between technologies and preferences. As discussed in the previous section, over the very long run such independency comes from the assumption that the household's consumption decisions are based on the physical and chemical properties of the goods, but not on the capital-intensity (i.e. households are not interested in the production process of the consumption goods but only on the "taste" of the goods). If preferences and technologies are uncorrelated (or independent) structural change patterns can arise that satisfy all the empirical observations associated with the Kuznets-Kaldor-puzzle (especially factors are reallocated across sectors that differ by capital intensity). We show that this explanation is compatible with 55% of the structural change.

The remaining task is to answer the question why the remaining 45% of the structural change are compatible with the Kuznets-Kaldor-Puzzle. One answer may be that these 45% are quantitatively small hence their aggregate impact is relatively low (in comparison to the other aggregate-growth determinants, e.g. technological progress). In fact, this is implied by the paper by Acemogly and Guerrieri (2008). However, there may be other explanations as well. For example, the aggregate effect of these 45% of structural change may be offset by the aggregate effects of other growth determinants, e.g. some sort of "economy-wide technological progress" may have accelerated between 1948 and 1987 which would have offset the (negative) impacts of non-neutral structural change. Further research could analyze this question in more detail. Furthermore, it seems interesting to search for other micro-foundations of NCCI structural change: we explained the parameter restrictions, which are necessary for the existence of NCCI structural change, by uncorrelated preferences and technologies; however, there are certainly other micro-foundations that can explain these parameter restrictions.

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APPENDIX A

Equations (20) to (29) are relevant for aggregate analysis. Now let us search, like in the "normal" Ramsey model, for a growth path where E and K grow at constant rate, i.e.

$$\frac{\dot{E}}{E} \overset{!}{=} g_E \qquad \text{(A.1)}$$

$$\frac{\dot{K}}{K} \overset{!}{=} g_K \qquad \text{(A.2)}$$

Equations (A.1) and (22) imply that

$$\frac{\dot{\Omega}_m}{\Omega_m} = const. \qquad \text{(A.3)}$$

where $\Omega_m \equiv \frac{k_m K}{l_m L}$.

Requirement (A.3) and equation (21) imply that

$$\frac{\dot{\tilde{Y}}}{\tilde{Y}} = const. \quad \text{(A.4)}$$

(A.2) and (A.3) imply

$$\frac{\left(k_m \dot{/} l_m\right)}{k_m / l_m} = const. \qquad \text{(A.5)}$$

(A.1), (A.2) and (A.4) imply

$$\frac{\left(E \dot{/} \tilde{Y}\right)}{E / \tilde{Y}} = const. \text{ and } \frac{\left(K \dot{/} \tilde{Y}\right)}{K / \tilde{Y}} = const. \qquad \text{(A.6)}$$

Equations (A.2), (20) and (25) imply

$$c_5 + c_6 \frac{l_m}{k_m} = \frac{E}{\tilde{Y}} + \frac{H}{\tilde{Y}} + (g_K + \delta) \frac{K}{\tilde{Y}} \qquad \text{(A.7)}$$

Solving equation (24) for $\frac{H}{\tilde{Y}}$ and inserting it into equation (A.7) yields after some algebra:

$$c_5 - \frac{1}{c_4} = \left(1 - \frac{c_3}{c_4}\right)\frac{E}{\tilde{Y}} - \left(c_6 + \frac{1}{c_4}\right)\frac{l_m}{k_m} + (g_K + \delta)\frac{K}{\tilde{Y}} \qquad \text{(A.8)}$$

Remember that c_3, c_4, c_5, c_6, g_K and δ are constants. Furthermore, note that (A.5) and (A.6) imply that $\frac{E}{\tilde{Y}}, \frac{l_m}{k_m}$ and $\frac{K}{\tilde{Y}}$ grow at constant rate. Hence, equation (A.8) can be satisfied at any point of time only if $\frac{E}{\tilde{Y}}, \frac{l_m}{k_m}$ and $\frac{K}{\tilde{Y}}$ are constant (i.e. they grow at the constant rate zero), i.e.

$$\frac{E}{\tilde{Y}} = const., \quad \frac{l_m}{k_m} = const., \quad \frac{K}{\tilde{Y}} = const. \qquad \text{(A.9)}$$

Equations (A.9), (23), (25) imply

$$\frac{\dot{E}}{E} = \frac{\dot{\tilde{Y}}}{\tilde{Y}} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{H}}{H} = const. \qquad \text{(A.10)}$$

Q.E.D.

Let g^* denote the constant growth rate from equation (A.10). Hence, (A.9), (A.10), (21) and (26) imply

$$g^* = \frac{\frac{\dot{G}}{G}}{1-q} + g_L \qquad \text{(A.11)}$$

Inserting equations (27) and (28) into equation (A.11) yields after some algebra:

$$g^* = \frac{(1-\mu\bar{\varepsilon})g_A + \gamma\bar{\varepsilon}g_B}{(1-\mu\bar{\varepsilon})\alpha + \gamma\bar{\varepsilon}\chi} + g_L \qquad \text{(A.12)}$$

Q.E.D.

Note that in all the calculations from above we searched for an equilibrium growth path where E and K grow at constant rate. As a result we obtained that H grows at constant rate along this growth path. Hence, we can treat H like exogenous technological progress along this growth path. Let $\hat{Y} \equiv Y - H$. In this case equation (20) can be written as follows:

$$\hat{Y} = \dot{K} + \delta K + E \qquad \text{(A.13)}$$

Q.E.D.

Inserting equations (21), (23) and (25) into $\hat{Y} \equiv Y - H$ yields:

$$\hat{Y} = \tilde{G} L^{1-q} K^{q} \qquad \text{(A.14)}$$

where $\tilde{G} \equiv G\left(\frac{k_m}{l_m}\right)^{q}\left(c_5 - \gamma c_1 + (c_6 - \gamma c_2)\frac{l_m}{k_m}\right)$ grows at constant positive rate due to (A.9) ($\tilde{G}$ grows at constant positive rate). ***Q.E.D.***

Inserting equation (A.14) into equation (22) yields:

$$\frac{\dot{E}}{E} = \lambda \frac{\hat{Y}}{K} - \delta - \rho \qquad \text{(A.15)}$$

where $\lambda \equiv \beta \frac{l_m}{k_m}\left(c_5 - \gamma c_1 + (c_6 - \gamma c_2)\frac{l_m}{k_m}\right)$ is constant due to (A.9). ***Q.E.D.***

Equations (A.14) and (A.15) include the term l_m / k_m. This term is constant along the equilibrium growth path and can be derived as function of model parameters by setting equation (22) equal to g^* and solving afterwards for l_m / k_m:

$$\frac{k_m}{l_m} = \left(\frac{g^* + \delta + \rho}{\beta}\right)^{\frac{1}{q-1}} G^{\frac{1}{1-q}} \frac{L}{K} \qquad \text{(A.16)}$$

Note that the term $G^{\frac{1}{1-q}} L$ is a function of exogenous parameters and grows at rate g^* (see equation (A.11) for g^*). K grows at rate g^* along the equilibrium growth path as well (see

Lemma 1). Hence, the term $G^{\frac{1}{1-q}}\frac{L}{K}$ is constant along the equilibrium growth path, so that we can rewrite equation (A.16) in terms of initial values of exogenous parameters (the index zero denotes the initial value of the corresponding variable):

$$\frac{k_m}{l_m} = \left(\frac{g^* + \delta + \rho}{\beta}\right)^{\frac{1}{q-1}} (G_0)^{\frac{1}{1-q}} \frac{L_0}{K_0} \qquad \text{(A.17)}$$

where q, G_0 and g^* are given by equations (27), (28) and (A.12). ***Q.E.D.***

We have shown now that along an equilibrium growth path where E and K grow at constant rate H grows at constant rate as well and k_m / l_m is constant. When this fact is taken into account, the economy in aggregates is represented by equations (A.13)-(A.15). These equations are similar to the Ramsey-model regarding all relevant features; hence, they imply that this equilibrium growth path exists and is unique. ***Q.E.D.***

APPENDIX B

First we rearrange our aggregate equation system (20)-(29) as follows:

$$\dot{\hat{K}} = \left(\frac{l_m}{k_m}\right)^{-q} (\alpha + \beta \frac{l_m}{k_m})\hat{K}^q - \hat{E} - (\delta + g_L + \frac{g_G}{1-q})\hat{K} \qquad \text{(B.1)}$$

$$\frac{\dot{\hat{E}}}{\hat{E}} = \beta \left(\frac{l_m}{k_m}\right)^{1-q} \hat{K}^{q-1} - \delta - \rho - g_L - \frac{g_G}{1-c} \quad \text{(B.2)}$$

$$\lambda_m = \frac{1 - \mu\bar{\varepsilon} + \gamma\bar{\varepsilon}\frac{\nu}{\beta} - \frac{\chi\beta - \alpha\nu}{\alpha\beta} \frac{\hat{E}}{\hat{K}^q \left(\frac{l_m}{k_m}\right)^{-q}}}{1 - \mu\bar{\varepsilon} + \gamma\bar{\varepsilon}\frac{\chi}{\alpha}} \qquad \text{(B.3)}$$

where aggregate variables are expressed in "labor-efficiency units", i.e. they are divided by $LG^{\frac{1}{1-q}}$; hence $\hat{K} \equiv \frac{K}{LG^{\frac{1}{1-q}}}$ and $\hat{E} \equiv \frac{E}{LG^{\frac{1}{1-q}}}$.

These equations imply that $\hat{K}$, $\hat{E}$ and l_m / k_m have the following values along the ABGP

$$\hat{K}^* = \sigma^{\frac{1}{1-q}} \left(\frac{l_m}{k_m} \right)^* \quad \text{(B.4)}$$

$$\hat{E}^* = \alpha\sigma^{\frac{q}{1-q}} + \rho\sigma^{\frac{1}{1-q}} \left(\frac{l_m}{k_m} \right)^* \quad \text{(B.5)}$$

$$\left(\frac{l_m}{k_m} \right)^* = \frac{\alpha}{\beta} \frac{\beta + (\nu\gamma - \mu\beta)\bar{\varepsilon} - (\chi\beta - \alpha\nu)}{\alpha + (\chi\gamma - \mu\alpha)\bar{\varepsilon} + (\chi\beta - \alpha\nu)\frac{\rho}{\beta}\sigma} \quad \text{(B.6)}$$

where $\sigma \equiv \frac{\beta}{\delta + \rho + g_L + \frac{g_G}{1-q}}$

where an asterisk denotes the ABGP-value of the corresponding variable.

The proof of convergence to the ABGP is analogous to the proof by Acemoglu and Guerrieri (2008) (see there for details and see also Acemoglu (2009), pp. 269-273, 926).

First, we have to show that the determinant of the Jacobian of the differential equation system (B.1)-(B.2) (where l_m / k_m is given by equation (B.3)) is different from zero when evaluated at the ABGP (i.e. for $\hat{K}^*$, $\hat{E}^*$, $\left(\frac{l_m}{k_m} \right)^*$ from equations (B.4)-(B.6)). This implies that this differential equation system is hyperbolic and can be linearly approximated around $\hat{K}^*$, $\hat{E}^*$, $\left(\frac{l_m}{k_m} \right)^*$ (Grobman-Hartman-Theorem; see as well Acemoglu (2009), p. 926, and Acemoglu and Guerrieri (2008)). The determinant of the Jacobian is given by:

$$\left|J\right| = \begin{vmatrix} \dfrac{\partial \dot{\hat{K}}}{\partial \hat{K}} & \dfrac{\partial \dot{\hat{K}}}{\partial \hat{E}} \\ \dfrac{\partial \dot{\hat{E}}}{\partial \hat{K}} & \dfrac{\partial \dot{\hat{E}}}{\partial \hat{E}} \end{vmatrix} = \frac{\partial \dot{\hat{K}}}{\partial \hat{K}} \frac{\partial \dot{\hat{E}}}{\partial \hat{E}} - \frac{\partial \dot{\hat{E}}}{\partial \hat{K}} \frac{\partial \dot{\hat{K}}}{\partial \hat{E}} \qquad \text{(B.7)}$$

The derivatives of equations (B.1)-(B.2) are given by:

$$\begin{aligned}
\frac{\partial \dot{\hat{K}}}{\partial \hat{K}} &= q\hat{K}^{q-1}\left(\alpha \left(\frac{l_m}{k_m} \right)^{-q} + \beta \left(\frac{l_m}{k_m} \right)^{1-q} \right) \\
&\quad + \hat{K}^{q}\left(-q\alpha \left(\frac{l_m}{k_m} \right)^{-q-1} + (1-q)\beta \left(\frac{l_m}{k_m} \right)^{-q} \right) \frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{K}} - \left(\delta + g_L + \frac{g_G}{1-q} \right) \\
\frac{\partial \dot{\hat{K}}}{\partial \hat{E}} &= \hat{K}^{q}\left(-q\alpha \left(\frac{l_m}{k_m} \right)^{-q-1} + (1-q)\beta \left(\frac{l_m}{k_m} \right)^{-q} \right) \frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{E}} - 1 \\
\frac{\partial \dot{\hat{E}}}{\partial \hat{K}} &= \beta \hat{E} \left((q-1)\hat{K}^{q-2} \left(\frac{l_m}{k_m} \right)^{1-q} + (1-q)\hat{K}^{q-1} \left(\frac{l_m}{k_m} \right)^{-q} \frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{K}} \right) \\
\frac{\partial \dot{\hat{E}}}{\partial \hat{E}} &= \left(\beta \left(\frac{l_m}{k_m} \right)^{1-q} \hat{K}^{q-1} - \delta - \rho - g_L - \frac{g_G}{1-q} \right) \\
&\quad + \beta \hat{E} \hat{K}^{q-1} (1-q) \left(\frac{l_m}{k_m} \right)^{-q} \frac{\partial \left(\frac{l_m}{k_m} \right)}{\partial \hat{E}}
\end{aligned} \qquad \text{(B.8)}$$

where the derivatives of equation (B.3) are given by

$$\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial\hat{E}} = -\frac{\frac{\chi\beta-\alpha\nu}{\alpha\beta}\frac{\left(\frac{l_m}{k_m}\right)^q}{\hat{K}^q}}{\left(1+\bar{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)\left(1+\left(1+\bar{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)^{-1}\frac{\chi\beta-\alpha\nu}{\alpha\beta}q\frac{\hat{E}}{\hat{K}^q\left(\frac{l_m}{k_m}\right)^{1-q}}\right)}$$

$$\frac{\partial\left(\frac{l_m}{k_m}\right)}{\partial\hat{K}} = \frac{\frac{\chi\beta-\alpha\nu}{\alpha\beta}q\frac{\hat{E}}{\hat{K}^{q+1}\left(\frac{l_m}{k_m}\right)^{-q}}}{\left(1+\bar{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)\left(1+\left(1+\bar{\varepsilon}\frac{\chi\gamma-\alpha\mu}{\alpha}\right)^{-1}\frac{\chi\beta-\alpha\nu}{\alpha\beta}q\frac{\hat{E}}{\hat{K}^q\left(\frac{l_m}{k_m}\right)^{1-q}}\right)} \quad \text{(B.9)}$$

Inserting the derivatives (B.8) and (B.9) into (B.7) and inserting the ABGP-values from equations (B.4)-(B.6) yields after some algebra the following value of the determinant of the Jacobian evaluated at the ABGP:

$$|J|^* = \frac{-(1-q)(\chi\beta-\alpha\nu)\frac{\hat{E}^*}{\hat{K}^*}\left[\rho+\frac{\bar{\alpha}}{\chi\beta-\alpha\nu}\frac{\beta}{\sigma}\right]}{\bar{\alpha}+\frac{\chi\beta-\alpha\nu}{\beta}q\frac{\hat{E}^*}{(\hat{K}^*)^q\left[\left(\frac{l_m}{k_m}\right)^*\right]^{1-q}}} \quad \text{(B.10)}$$

where $\bar{\alpha} \equiv \alpha + \bar{\varepsilon}(\chi\gamma - \alpha\mu) > 0$ and q is given by equation (27).

This equation can be transformed further by inserting using equations (27) and (B.4)-(B.6)

$$|J|^* = \frac{-\frac{\hat{E}^*}{\hat{K}^*}\frac{\beta}{\sigma}\bar{\alpha}\left[\bar{\beta}-(\chi\beta-\alpha\nu)\right]}{\bar{\alpha}\left[\bar{\beta}-(\chi\beta-\alpha\nu)\right]+\bar{\beta}^2} \quad \text{(B.11)}$$

where $\bar{\beta} \equiv \beta + \bar{\varepsilon}(\nu\gamma - \beta\mu) > 0$. Note that $\frac{\hat{E}^*}{\hat{K}^*}$ is positive and is given by equations (B.4)-(B.6).

We can see that the determinant evaluated at PBGP is different form zero. Hence, the ABGP is hyperbolic. Furthermore, equations (B.10) and (B.11) imply that $\left|J\right|^* < 0$. (Equation (B.10) implies that $\left|J\right|^* < 0$, if $\chi\beta - \alpha\nu > 0$; equation (B.11) implies that $\left|J\right|^* < 0$, if $\chi\beta - \alpha\nu < 0$ as well.)

Our differential equation system consists of two differential equations ((B.1) and (B.2)) and of two variables ($\hat{E}$ and $\hat{K}$), where we have one state and one control-variable. Hence, saddle-path-stability of the ABGP requires that there exist one negative (and one positive) eigenvalue of the differential equation system when evaluated at ABGP (see also Acemoglu and Guerrieri (2008) and Acemoglu (2009), pp. 269-273). Since $\left|J\right|^* < 0$ we can be sure that this is the case. ($\left|J\right|^* < 0$ can exist only if one eigenvalue is positive and the other eigenvalue is negative. If both eigenvalues were negative or if both eigenvalues were positive, the determinant $\left|J\right|^*$ would be positive.) Therefore, the ABGP is saddle-path-stable. ***Q.E.D.***

APPENDIX C

It follows from the optimality condition (18) that

$$C_i = \frac{\omega_i}{\sum_{i=1}^{n} \omega_i} \frac{E}{p_i} + \theta_i \qquad \forall i \quad \text{(C.1)}$$

For the sake of simplicity we consider only the non-homotheticity between the services sector and the conglomerate of the agriculture and manufacturing sector. Inserting equation (C.1) into equations (15) yields (remember equation (10)):

$$E_{agr.+man.} = d_1 E + d_2 \qquad \text{(C.2)}$$

$$E_{ser.} = d_3 E + d_4 \qquad \text{(C.3)}$$

where $E_{agr.+man} = E_{agr.} + E_{ser.}$ $d_1 \equiv \frac{\sum_{i=1}^{s}\omega_i}{\sum_{i=1}^{n}\omega_i}$, $d_2 \equiv p\sum_{i=m+1}^{s}\theta_i$, $d_3 \equiv \frac{\sum_{i=s+1}^{n}\omega_i}{\sum_{i=1}^{n}\omega_i}$ and $d_4 \equiv p\sum_{i=s+1}^{n}\theta_i$. Note that p is given by $p = \frac{\partial Y_m / \partial(l_m L)}{\partial Y_n / \partial(l_n L)}$ and stands for the relative price of sectors $i = m+1,...n$.

If preferences are non-homothetic across sectors consumption expenditures on agriculture and manufacturing ($E_{agr.+man.}$) do not grow at the same rate as consumption expenditures on services ($E_{ser.}$), when treating relative prices as constants. Hence, we have to show that $E_{agr.+man.}$ and $E_{ser.}$ do not grow at the same rate when treating d_1-d_4 as constants. It follows from equations (C.2) and (C.3) that when treating d_1-d_4 as constants the following equations are true

$$\frac{\dot{E}_{agr.+man.}}{E_{agr.+man.}} = \frac{\dot{E}}{E}\frac{1}{1+\frac{d_2}{d_1}E} \qquad \text{(C.4)}$$

$$\frac{\dot{E}_{ser.}}{E_{ser.}} = \frac{\dot{E}}{E}\frac{1}{1+\frac{d_4}{d_3}E} \qquad \text{(C.5)}$$

which shows that $E_{agr.+man.}$ and $E_{ser.}$ do not grow at the same rate when treating d_1-d_4 as constants, i.e., preferences are non-homothetic between the services sector and the conglomerate of the agriculture sector. In the same way it can be shown that preferences are non-homothetic between the manufacturing sector and the agriculture sector. ***Q.E.D.***

APPENDIX D

The optimality condition (17) implies after some algebra that

$$h_i = \varepsilon_i \frac{H}{p_i}, \quad \forall i \qquad \text{(D.1)}$$

Hence,

$$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j}\frac{p_j}{p_i} \text{ for } i = a+1,...s \text{ and } j = s+1,...n \text{ (D.2)}$$

In equation (D.2) i stands for the manufacturing sector and j for the services sector. Let us now take a look at an arbitrary producer of the manufacturing sector, e.g. the producer i = 3, where a+1<3 < s. We rewrite equation (D.2) as follows to show the viewpoint of "producer 3":

$$\frac{h_3}{h_j} = \frac{\varepsilon_3}{\varepsilon_j}\frac{p_j}{p_3} \text{ for } j = x+1,...n \qquad \text{(D.3)}$$

From the view point of "producer 3" equation (D.3) determines the ratio between the input of own intermediates (i.e. the amount of intermediates that is produced by "producer 3" and used by "producer 3") and input of services-sector-produced intermediates (i.e. the amount of intermediates that is produced by "producer j" from the services sector and used by "producer 3"). (Remember that h_3 and h_j enter the production function of "producer 3" via equations (1) and (7).) Hence, for example, a decrease in $\frac{h_3}{h_j}$ means that "producer 3" increases the input of producer-j-intermediates relatively more strongly than the input of own intermediates, i.e. "producer 3" substitutes own intermediate inputs by external intermediate inputs, i.e. "producer 3" outsources additional intermediates production to producer j. Therefore, we can conclude from equation (D.3) that "producer 3" outsources more and more to "producer j" (i.e. $\frac{h_3}{h_j}$ decreases), provided that $\frac{\dot{p}_j}{p_j} - \frac{\dot{p}_3}{p_3} < 0$ (i.e. provided that the price for the good j in terms of the good 3 $(\frac{p_j}{p_3})$ decreases; or in other words: provided that the output of “producer j” becomes cheaper and cheaper (or less and less expensive) in comparison to the output of “producer 3”).

From this discussion and from equation (D.2) we can conclude the following: manufacturing-sector-producers ($i = a+1,...s$) shift more and more intermediates production to services-sector-producers ($j = s+1,...n$), i.e. $\frac{h_i}{h_j}$ decreases, provided that services-sector-production becomes cheaper and cheaper (or less and less expensive) in comparison to manufacturing-production, i.e. provided that $\frac{\dot{p}_j}{p_j} - \frac{\dot{p}_i}{p_i} < 0$, and vice versa. ***Q.E.D.***

Note that relative prices are determined by exogenous parameters. Hence, which producers outsource and whether outsourcing from manufacturing to services increases (or the other way around) depends on the parameterization of the model. In general both cases are possible. By using optimality condition (17) the relative prices can be calculated, so that we can reformulate equation (D.2) after some algebra as follows

$$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j}\frac{\alpha}{\chi}\frac{A}{B}\frac{1}{\vartheta}\left(\frac{k_m}{l_m}\frac{K}{L}\right)^{\beta-\nu+\psi(\gamma-\mu)} D^{\gamma-\mu} \quad \text{for } i = a+1,...m \quad \text{and} \quad j = s+1,...n \quad \text{(D.4)}$$

$$\frac{h_i}{h_j} = \frac{\varepsilon_i}{\varepsilon_j} \quad \text{for } i = m+1,...s \quad \text{and} \quad j = s+1,...n \qquad \text{(D.5)}$$

where $\vartheta \equiv \left(\frac{\alpha}{\chi}\frac{\nu}{\beta}\right)^{\nu}\left(\frac{\alpha}{\chi}\frac{\mu}{\gamma}\right)^{\mu}$, $\psi \equiv \frac{\beta-(\beta-\nu)\bar{\varepsilon}}{1+(\gamma-\mu)\bar{\varepsilon}-\gamma}$ and

$$D \equiv \left\{\gamma A\left[\frac{\chi}{\alpha}\frac{B}{A}\left(\frac{\alpha\nu}{\chi\beta}\right)^{\nu}\left(\frac{\alpha\mu}{\chi\gamma}\right)^{\mu}\right]^{\bar{\varepsilon}}\prod_{i=1}^{n}\varepsilon_i^{\varepsilon_i}\right\}^{\frac{1}{1-\gamma(1-\bar{\varepsilon})-\mu\bar{\varepsilon}}}.$$

From equation (D.5) we can see that some of the manufacturing sector producers (i.e. producers $i = m+1,...s$) do not change their outsourcing behavior (i.e. these producers keep their ratio of external to own intermediates production ($\frac{h_i}{h_j}$) constant). Equation (D.4) implies that the rest of the manufacturing sector producers (i.e. the producers $i = a+1,...m$) change their outsourcing behavior. Calculating the growth rate of equation (D.4) yields (remember Lemma 1):

$$\frac{(h_i \dot{/} h_j)}{h_i / h_j} = g_A - g_B + (\beta - \nu + \psi(\gamma - \mu))g^* + (\gamma - \mu)\frac{\dot{D}}{D} \text{ for } i = a+1,...m \text{ and } j = s+1,...n \tag{D.6}$$

ψ, g^* and $\dot{D}/D$ are positive. We omit here a detailed discussion of $\frac{(h_i \dot{/} h_j)}{h_i / h_j}$, since it is less relevant for our purposes. The only important thing is that $\frac{(h_i \dot{/} h_j)}{h_i / h_j}$ can be positive (e.g. if $g_A - g_B > 0$, $\beta - \nu > 0$ and $\gamma - \mu > 0$) or negative (e.g. if $g_A - g_B < 0$, $\beta - \nu < 0$ and $\gamma - \mu < 0$) depending on the parameterization of the model. Hence, the intermediates-production may be shifted from manufacturing to services or the other way around, depending on the parameterization of the model. ***Q.E.D.***

APPENDIX E

It is well known that balanced growth requires either labor-augmenting technological progress (or production function(s) of type Cobb-Douglas.) Furthermore, a standard assumption in macroeconomic models is that the production function has constant returns to scale. (Later, we will see that the aggregate production function has the same structure as the sectoral production functions.) Since we want to reassess the standard growth theory we do not depart from these assumptions. Therefore, we assume now that sectoral production functions are given by:

(1)' $Y_i = B_i l_i L f_i(\Omega_i) \quad \forall i = 1,...n$

where

(26)' $$\Omega_i \equiv \frac{k_i K}{l_i L B_i} \quad \forall i = 1,...n$$

B_i stands for the level of sector-specific and labor augmenting technological progress; $f_i(\Omega_i)$ is a sector-specific function of Ω_i; it is the intensive form of a "standard" constant returns to scale function, where in this appendix Ω_i denotes the capital-to-labor ratio in efficiency units in sector i.

The sectoral growth rates of labor-augmenting technological progress (g_i) are constant, i.e. $\dot{B}_i / B_i = g_i \quad \forall i$. The following equations remain the same as in the previous discussion:

(3)' $\sum_i k_i = 1$

(3)'' $$\sum_i l_i = 1$$

(12)' $$Y \equiv \sum_i p_i Y_i$$

We still assume that sector m is numéraire (m<n) (although we do not make here any assumptions about which sector produces capital). Hence, equation (13) holds.

When labor and capital are mobile across sectors and markets are polypolistic the following efficiency conditions must be true:

(17)' $$\frac{p_i}{p_j} = \frac{\partial Y_j / \partial(k_j K)}{\partial Y_i / \partial(k_i K)} = \frac{\partial Y_j / \partial(l_j L)}{\partial Y_i / \partial(l_i L)} \quad \forall i, j$$

(32)' $$r + \delta = p_i \partial Y_i / \partial(k_i K) \quad \forall i$$

Note, that we do not make here any assumption about the household behavior. The assumptions above are sufficient to derive Proposition 4.

The capital share of income in sector i (or: the elasticity of capital with respect to output in sector i) is given by:

(E.1) $$\kappa_i(\Omega_i) \equiv \frac{(\partial Y_i/\partial k_i K) k_i K}{Y_i} = \Omega_i \frac{f_i'(\Omega_i)}{f_i(\Omega_i)}$$

where $f_i'(\Omega_i) \equiv \frac{\partial f_i(\Omega_i)}{\partial \Omega_i}$.

By inserting equations (1)', (26)' and (13) into equation (32)' we obtain:

(E.2) $$r + \delta = f_m'(\kappa_m)$$

Inserting first equations (1)' and (26)' into equation (17)' and then inserting equation (E.1) into this term yields:

(E.3) $$\frac{k_i}{l_i}\frac{1-\kappa_i(\Omega_i)}{\kappa_i(\Omega_i)} = \frac{k_m}{l_m}\frac{1-\kappa_m(\Omega_m)}{\kappa_m(\Omega_m)} \quad \forall i$$

Solving this term for k_i and inserting it into equation (3)' yields (remember that $\frac{\kappa_i}{1-\kappa_i} = \frac{1}{1-\kappa_i} - 1$ and $\sum_i l_i = 1$):

(E.4) $$\frac{k_m}{l_m}\frac{1-\kappa_m(\Omega_m)}{\kappa_m(\Omega_m)} = \left(\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} - 1\right)^{-1}$$

Equations (13) and (17)' imply:

(E.5) $$p_i = \frac{B_m\left(f_m(\Omega_m) - \Omega_m f_m'(\Omega_m)\right)}{B_i\left(f_i(\Omega_i) - \Omega_i f_i'(\Omega_i)\right)} \quad \forall i$$

Inserting equations (1)', (E.1) and (E.5) into equation (12)' yields:

(E.6) $$Y = B_m L f_m(\Omega_m)(1-\kappa_m(\Omega_m))\sum_i \frac{l_i}{1-\kappa_i(\kappa_i)}$$

Inserting equation (E.4) into equation (E.6) yields equation

(E.7) $$\frac{Y}{K} = \frac{f_m(\Omega_m)}{\Omega_m}\left(\kappa_m(\Omega_m) + [1-\kappa_m(\Omega_t^m)]\frac{k_m}{l_m}\right)$$

Definition E.1: *An ABGP is a growth path where* $\frac{Y}{K}$ *and* $\frac{rK}{Y}$ *are constant.*

Definition E.1 is consistent with Definition 1 (and with the Kaldor facts). In fact both definitions yield the same equilibrium growth path (but Definition 1 is stronger than necessary). However, now we use Definition E.1 in order to demonstrate that the necessary condition for the ABGP is independent of the numéraire. (Remember that, since $\frac{Y}{K}$ and $\frac{rK}{Y}$ are ratios, they are always the same irrespective of the choice of the numérarire.)

Lemma E.1: *A necessary condition for the existence of an ABGP (according to Definition E.1) is* $l_m / k_m = const.$ *or equivalently* $\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} = const.$.

Proof: Definition E.1 requires that $\frac{Y}{K}$ and $\frac{rK}{Y}$ are constant; hence r must be constant; hence Ω_m must be constant (due to equation (E.2)). Due to equation (E.7), $\Omega_m = const.$ and $\frac{Y}{K} = const.$ require $l_m / k_m = const.$. $l_m / k_m = const.$ and $\Omega_m = const.$ require $\sum_i \frac{l_i}{1-\kappa_i(\Omega_i)} = const.$ (due to equation (E.4)). Note that $1-\kappa_i(\Omega_i) = \lambda_i$, since we assume that there are only two production factors capital and labor. (λ_i stands for the output-elasticity of labor in sector i or equivalently for the labor-income share in sector i.) ***Q.E.D.***