HORIZONTAL VERSUS VERTICAL FISCAL EQUALIZATION

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Abstract

We analyze a model in which the provision of regional public goods by regional governments leads to spillover effects and in which the central government can establish a vertical equalization scheme while the regional governments can set up a horizontal equalization scheme. The two levels of government decide in different chronological order. It turns out that, regardless of the timing, the central government always prevails. Horizontal equalization does not take place – nor is it necessary in order to achieve constrained Pareto efficiency. Moreover, if in the model economy the goal of achieving equality in living conditions across the regions is pursued, the only suitable candidate for reaching this goal is vertical equalization.

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1. Introduction

Since the early seventies of the last century there has been an ongoing debate on the efficiency of fiscal equalization between the regions of a federation. Fiscal equalization can be carried out either by the regional governments (horizontal equalization) or by the federal government (vertical equalization). The purpose of an efficiency oriented fiscal equalization scheme is to achieve an efficient distribution of the entire mobile population across the different regions by means of income transfers from one region to another.

Moreover, fiscal equalization between the regions of a federation also serves as a tool to achieve the goal of equal standards of living in the entire federation which is enshrined in the constitutions of various federations: For example, the Canadian constitution (section 36(1) states: "…, Parliament and the legislatures, together with the government of Canada and the provincial governments, are committed to (a) promoting equal opportunities for the well-being of Canadians; (b) …". Similarly, Art. 72(2) of the German Basic Law demands that the central (federal) government shall establish and maintain "equal living conditions in the federal territory". We will address this issue in chapter 5.

The debate on efficiency oriented fiscal equalization began with the contributions of Buchanan and Goetz (1972) and Flatters *et al.* (1974), with the latter being the first to put forward a closed model. In this model a private good and, in each region, a public good is provided. The provision of the private good is determined by competitive markets whereas the provision of the public goods is decided on by the regional governments, who have to stand for re-election. The workers of the federation can move from one region to another without any barriers to mobility. In order to reach, in migration equilibrium, an efficient distribution of the workers of the federation, an interregional income transfer is generally necessary. In Flatters *et al.* effecting this transfer is the task of the central government within the framework of a vertical equalization scheme. The problem of assigning this task to the central

government is not discussed. "The prescribed solution is a central authority making transfers of the private good from the overpopulated to the underpopulated region" (Mansoorian and Myers, 1993, p. 118).

In 1990, Myers demonstrated that vertical equalization is not required in this model to achieve an efficient migration equilibrium – horizontal equalization suffices. "While it is true that interregional transfers are generally required to achieve a Pareto optimum, it is also true that the Nash competing regional authorities will make these transfers in their own interest" (Myers, 1990, p. 114). The "region purchases a preferred regional population size" (ibid.).

The fact that an efficient distribution of the population may be achieved by means of horizontal or vertical equalization had already been hinted at earlier by Boadway and Flatters (1982): "This inefficiency can be eliminated by a particular system of interregional transfers of private goods either voluntarily arranged by the provinces or imposed by the central government" (Boadway and Flatters, 1982, p. 622).

The debate was intensified by Krelove (1992), who developed a model in which a separate fiscal equalization scheme is not necessary at all. By assuming that the ownership of land is evenly distributed among the whole population, he ensured that with "tax exporting" the necessary monetary flows occurred.

In 1993, Mansoorian and Myers extended Myers' model (1990) with a barrier to mobility in the form of varying degrees of attachment to home. They showed that even in such an extended model, a horizontal equalization scheme is sufficient to guarantee Pareto efficiency. "The primary implication of this result is that there is no efficiency role for a central authority in a fiscal externality economy with or without attachment to home, …" (Mansoorian and Myers, 1993, p.128).

In the following year, Wellisch (1994) supplemented the model of Mansoorian and Myers (1993) with spillover effects caused by the provision of regional

public goods. He was able to demonstrate that if imperfect mobility and spillover effects are present, and interregional transfers are made by regional authorities, the migration equilibrium that arises may well be efficient, but not the provision of public goods. Consequently, the resulting Nash equilibrium is not Pareto efficient. On the other hand, if the central authority establishes a vertical equalization scheme, the resulting public goods provision is also inefficient. As far as the efficiency of the migration equilibrium is concerned, however, Wellisch could not obtain a clear result. We will show that the migration equilibrium is efficient in this case as well.

In 2000, Caplan et al. replaced the regional public goods that generated spillover effects in Wellisch's (1994) model with a federal public good provided additively by regional governments. The task of organizing vertical equalization is assigned to the central government. These authors obtain results that do not appear to be compatible with those of Wellisch[1]. This will be dealt with in more detail below (see section 4.2).

Köthenbürger presented a model in 2007 in which spillover effects are present but which does not comprise interregional migration. He analyzes amongst other things the effects of equalizing transfers that serve not to manage migration but rather to achieve equal living conditions within the federation through redistribution. Since a conflict of interest between regional governments arises in this process, this fiscal equalization can only be effected by the central government.

In a recently published paper, Duran-Vigneron (2012) replaces the barrier to mobility, attachment to home, with individuals' heterogeneous preferences for local public goods, thus following up on the earlier work of Wrede (1997). With this modified assumption, horizontal equalization does not lead to an efficient result, making it necessary for the central government to intervene in order to ensure efficiency.

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[1] Cf. proposition 3 of Caplan *et al.* (2000) and proposition 3 of Wellisch (1994).

The contributions of Caplan et al. (2000), Köthenbürger (2007) and Duran-Vigneron (2012) introduce a further innovation in this context. The decisions of regional governments, on the one hand, and of the central government, on the other hand, are no longer taken at the same time. Instead, the timing of these decisions varies.

This paper builds on the work of Wellisch (1994) and extends it as follows:

(1) the timing of decisions is incorporated, and

(2) decisions concerning fiscal equalization are no longer taken either by the regional governments or by the central government, but instead, both the regional governments and the central government can plan a fiscal equalization system autonomously. This allows us to analyze the assignment problem.

2. The Model

Let there be a federation consisting of a central government and two regions, each with a regional government. The total population N is normalized to unity $(N = 1)$ and can migrate between the two regions, with the population of region i $(i \in \{1, 2\})$ denoted by N_i. In each region, firms competing with each other produce the numéraire good G. The quantity G_i of the goods produced in each region depends on the fixed endowment of region i with the immobile production factor $\overline{L_i}$ (*e.g.* land), which is owned by the regional government i [2]. Moreover, it is assumed that each resident of a region supplies one unit of labor perfectly inelastically there. Production thus also depends on the region's population N_i, which is variable. Production G_i can be described using the following production function:

$$G_i = F_i\left(N_i, \overline{L_i}\right), \tag{1}$$

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[2] This simplification will be commented on further below.

which has the usual neoclassical characteristics.

Production G_i may be used for private consumption as well as for the provision of a regional public good. We assume that all inhabitants of region i have the same *per capita* private consumption x_i. The amount of the regional public good provided in region i is labeled as Z_i. Thus:

$$G_i = N_i x_i + Z_i. \tag{2}$$

The competitive firms pay a wage equal to the respective marginal product of labor[3]:

$$w_i = F_i^{N}\left(N_i, \overline{L}_i\right). \tag{3}$$

The firms pay the following rent to the land owners, *i.e.* the regional governments

$$R_i = F_i - N_i w_i. \tag{4}$$

The regional governments use these earnings to finance the provision of the regional public goods. If a surplus remains, it is distributed equally among the inhabitants of the respective region. If, in contrast, a deficit arises it is financed by a tax levied *per capita* and equal for inhabitants of the respective region. Both situations may be summarized by setting the *per capita* subvention or tax, respectively, to be τ_i ($\tau_i > 0$, or $\tau_i < 0$, as the case may be; if $\tau_i = 0$, we have a Henry-George world).

Based on the above assumptions, the *per capita* consumption in region i amounts to:

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[3] Partial differentiation of a function f *w.r.t.* x is denoted with a superscript throughout this paper, *i.e.* $f^{x} \equiv \partial f / \partial x$

$$x_i = w_i + \tau_i \,. \tag{5}$$

Concerning the framework of fiscal equalization, we make the following assumptions: Region i makes a voluntary transfer S_{ij} to the other region j (horizontal equalization). As commonly done in the literature, we assume that the central government finances a transfer T_2 to region 2 by a corresponding negative transfer T_1 from region 1. The central government's budget constraint is thus: $-T_1=T_2$ [4] (for simplicity, the subscripts will be omitted in the following and the transfer T_2 to region 2 will be denoted as T).

The budget constraint of regional government i is given by:

$$R_i + S_{ji} = S_{ij} - (-1)^i T + Z_i + \tau_i N_i \,, \tag{6}$$

which implies that the central government obtains the funds for the transfer T from regional government 1, and then forwards these funds to regional government 2.

Using relation (6) and equation (4), the *per capita* consumption (5) can be rewritten as

$$x_i = \frac{1}{N_i}\left[F_i - S_{ij} + S_{ji} + (-1)^i T - Z_i \right]. \tag{7}$$

A further remark concerning the earnings of the regional governments is perhaps required at this point. The assumption that the regional governments own the immobile production factor $\overline{L_i}$ of the respective regions and thus earn the rent R_i seems to be critical at first sight. An alternative assumption could be that the immobile production factor $\overline{L_i}$ is privately owned and that the rents R_i are earned by the residents.

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[4] Flatters *et al.* (1974), p. 104; Wellisch (1994), 178; Caplan *et al.* (2000), p. 270; Köthenburger (2007), p. 485; Duran-Vigneron (2012), p. 106/107.

Along these lines, Krelove (1992) assumes that the land of a federation is distributed equally among all inhabitants of the federation. Thus, each inhabitant of a certain region also earns a rent based on land in the respective other region. If both regions levy a source-based tax on these earnings, a portion of the taxes paid by the inhabitants of region i are paid to the regional government of region j and *vice versa*. This is called "tax-exporting" by Krelove. He then shows that such "cross-border" net earnings lead to an efficient distribution of the federation's population among the two regions. Thus, fiscal equalization is not required.

In contrast, a different conclusion is reached if the land of each region is equally distributed solely among the inhabitants of the respective region (Wellisch, 1994, p. 173). Based on this assumption, an efficient distribution of the population of the federation among the regions cannot be achieved without fiscal equalization. However, it should be noted that this assumption leads to the following problem: If, after a change in fiscal equalization, citizens move from one region to the other and take their property rights with them, then there are again inhabitants of a certain region who own land in the respective other region. This is in contradiction with the initial assumption of all land of a region being the exclusive property of the inhabitants of that region. Wellisch solves this problem by assuming that the property rights regarding land are conferred only *after* the completion of migration (Wellisch, 1994, p. 174, FN 5).

However, the same result is also obtained if one assumes that the land is owned by the regional governments (Mansoorian and Myers, 1993, p. 122)[5]. Generally, fiscal equalization is required if "cross-border" earnings on immobile production factors (*e.g.*, land) which flow from one region to the other are excluded.

In our model we follow the approach that the immobile production factor $\overline{L_i}$ is owned by the respective regional government, and the corresponding rent is

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[5] Similarly also Boadway and Keen (1996, p. 140) in a different context.

then used to reduce the *per capita tax* τ_i that needs to be paid by the inhabitants of that region. This simplifies the analysis.

The residents of each region derive utility from the consumption of both the private good x_i and the public good Z_i provided in their region. Furthermore, there is a spillover effect in the provision of the regional public good, i.e. residents of region i derive utility from public good Z_j provided in region j (Wellisch, 1994). Finally, there is also a varying degree of attachment to home (Mansoorian and Myers, 1993, 1997). The extent of a resident's attachment to region 2 is described by coefficient $n\left(n \in [0,1]\right)$, with a higher value of n indicating a higher level of attachment. The utility function of a resident with coefficient n can therefore be written in total as

$$V^n = \begin{cases} U_1\left(x_1, Z_1, Z_2\right) + k\left(1-n\right), & \text{if living in region 1} \\ U_2\left(x_2, Z_1, Z_2\right) + kn, & \text{if living in region 2,} \end{cases} \qquad (8)$$

with *k>0* expressing the intensity of the attachment to a region.
Function U_i is concave in all variables[6]. Since the population is mobile, an equilibrium in the population distribution across regions arises, and an individual who is indifferent between the regions then exists. For this individual the following therefore holds:

$$U_1\left(x_1, Z_1, Z_2\right) + k\left(1-n_1\right) = U_2\left(x_2, Z_1, Z_2\right) + kn_1. \qquad (9)$$

Individuals for whom $n > n_1$ live in region 2, whereas individuals for whom $n < n_1$ live in region 1. Thus, n_1 at the same time represents the number of inhabitants of region 1, *i.e.*, $n_1 = N_1$ and $1 - n_1 = N_2$ (Mansoorian and Myers, 1997, p. 269).

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[6] $U_i^y > 0, U_i^{yy} < 0$ for $y \in \{x_i, Z_i\}$.

Using this and equation (7), equation (9) can be rewritten as the following migration equilibrium condition:

$$U_1\left[\frac{F_1(N_1,\overline{L}_1)-S_{12}+S_{21}-T-Z_1}{N_1},Z_1,Z_2\right]+k(1-N_1)=$$

$$U_2\left[\frac{F_2(1-N_1,\overline{L}_2)-S_{21}+S_{12}+T-Z_2}{1-N_1},Z_1,Z_2\right]+kN_1. \qquad (10)$$

This relationship represents an implicit function $N_1 = N_1(S_{12},S_{21},T,Z_1,Z_2)$ if the derivative of (10) with respect to N_1 is not equal to zero. We assume for the stability of the migration equilibrium (Wellisch, 1994)[7]:

$$D \equiv \frac{U_1^x}{N_1}\left(F_1^N - x_1\right)+\frac{U_2^x}{N_2}\left(F_2^N - x_2\right)- 2k < 0. \qquad (11)$$

Differentiating equation (10) yields the following expressions:

$$\frac{dN_i}{dS_{ij}} = \frac{U_1^x/N_1+U_2^x/N_2}{D} < 0, \qquad (12)$$

$$\frac{dN_i}{dT} = \frac{dN_i}{dS_{ij}} < 0, \qquad (13)$$

$$\frac{dN_1}{dZ_1} = \frac{U_1^x/N_1-U_1^{Z_1}+U_2^{Z_1}}{D}, \qquad (14)$$

$$\frac{dN_1}{dZ_2} = -\frac{U_2^x/N_2-U_2^{Z_2}+U_1^{Z_2}}{D}. \qquad (15)$$

These four reaction functions are needed in the following considerations.

3. Efficiency Conditions

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[7]The stability problem has been investigated extensively by Stiglitz (1977). See also Boadway (1982).

In this section, we will specify the efficiency conditions for the above model from the point of view of a benevolent planner. These apply to both the population distribution and the provision of public goods and are identical to those in Wellisch (1994).

The necessary first-order condition for the efficient provision of public goods is

$$N_i \frac{U_i^{Z_i}}{U_i^x} + N_j \frac{U_j^{Z_i}}{U_j^x} = 1; \quad i,j = 1,2; \quad i \neq j.$$ (16)

This is the generalized Samuelson condition for the provision of public consumer goods in the presence of spillover effects.

The first-order condition for the efficient population distribution is provided by

$$-\frac{2kN_2}{U_2^x} \leq \left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) \leq \frac{2kN_1}{U_1^x}.$$ (17)

It is identical to that used in the case without spillovers (Mansoorian and Myers, 1993).

Pareto efficiency, i.e. a situation characterized by both an efficient population distribution and an efficient provision of public goods, exists when both conditions (16) and (17) are simultaneously fulfilled.

4. Different Timing of Decisions

t In this section, we will derive the equilibria resulting from different timings of the decisions of the governments within the framework of the model presented in section 2 and examine their efficiency. In this context, the

regional governments always set the transfers S_{ij} and the quantity of public goods Z_i autonomously, while the central government always determines the transfer T. This leads to the decisive difference between the models of Caplan et al. (2000), Köthenbürger (2007) and Duran-Vigneron (2012) and the model used here. In the papers mentioned, it is the central government that decides on the interregional transfer and the regional governments decide on the quantity of the public goods they provide. Here, by contrast, both levels of government can organize a transfer autonomously allowing the assignment problem to be analyzed.

4.1 The Central Government Acts First

In this section, we determine the equilibrium that arises given the following sequence of decisions:

Stage 1: The central government sets the interregional transfer T (vertical fiscal equalization), taking into consideration the anticipated response of the regional governments S_{ij} (horizontal fiscal equalization) and Z_i.

Stage 2: The regional governments simultaneously decide on the horizontal transfers S_{12} and S_{21} and on the provision of public goods Z_1 and Z_2.

The central government is thus the Stackelberg leader of a two-stage game in this case, which is solved by backwards induction.

For each $\bar{T}$ specified by the central government, the regional governments choose Z_i and S_{ij} in the second stage such that U_i is maximized in the respective region:

$$\max_{S_{ij}, Z_i} U_i \left\{ \frac{F_i\left[N_i\left(S_{ij}, \bar{S}_{ji}, \bar{T}, Z_i, \bar{Z}_j\right), \bar{L}_i\right] - S_{ij} + \bar{S}_{ji} + (-1)^i \bar{T} - Z_i}{N_i\left(S_{ij}, \bar{S}_{ji}, \bar{T}, Z_i, \bar{Z}_j\right)}, Z_i, \bar{Z}_j \right\}. \quad (18)$$

Differentiating U_i *w.r.t.* S_{ij} yields:

$$\frac{dU_i}{dS_{ij}} = \frac{U_i^x}{N_i}\left[\frac{dN_i}{dS_{ij}}\left(F_i^N - x_i\right) - 1\right]. \quad (19)$$

Taking (12) into account, we obtain the following Kuhn-Tucker condition for the optimal choice of S_{ij} in (18):

$$\left(F_i^N - x_i\right) - \left(F_j^N - x_j\right) + \frac{2kN_j}{U_j^x} \begin{array}{l} = 0, \quad \text{if } S_{ij} > 0, \\ \geq 0, \quad \text{if } S_{ij} = 0. \end{array} \quad (20)$$

All in all we obtain:

$$-\frac{2kN_2}{U_2^x} \leq \left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) \leq \frac{2kN_1}{U_1^x}. \quad (21)$$

Thus, efficiency condition (17) is fulfilled in equilibrium for all given $\bar{T}$.

Following Wellisch (1994), region 1 (2) is referred to as being transfer-constrained if the left (right) inequality in (21) strictly holds. This implies $S_{12} = 0 (S_{21} = 0)$. Only if there is equality in the left (right) inequality, transfer $S_{12}(S_{21})$ can be non-zero. With imperfect mobility ($k > 0$), the case that both horizontal transfers are non-zero, i.e. that neither region is transfer-constrained, is therefore ruled out[8]. In the following, only the case $S_{12} > 0$ and $S_{21} = 0$ will be considered.

Differentiating U_i with *w.r.t.* Z_i yields:

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[8] This statement results from (21) with $U_i^x > 0$ and $N_i \neq 0$ for $i \in \{1,2\}$.

$$\frac{dU_i}{dZ_i} = U_i^{Z_i} + \frac{U_i^x}{N_i}\left[\frac{dN_i}{dZ_i}\left(F_i^N - x_i\right) - 1\right]. \tag{22}$$

Noting (14), we thus obtain the following first-order condition for a maximum for (18) *w.r.t.* $Z_i > 0$ (Wellisch, 1994, p. 177):

$$N_i \frac{U_i^{Z_i}}{U_i^x} + N_j \frac{U_j^{Z_i}}{U_j^x} \frac{F_i^N - x_i}{F_j^N - x_j - \dfrac{2kN_j}{U_j^x}} = 1. \tag{23}$$

A comparison with efficiency condition (16) shows that this can only be fulfilled if region i is not transfer-constrained. However, since at least one region must be transfer-constrained, the provision of the public good is inefficient in at least one region.

Therefore, the resulting Nash equilibrium – as described by (21) and (23) – is not Pareto-efficient.

This analysis of the regional governments' decisions on the second stage of the game corresponds to that of Wellisch (1994). We now add the analysis of the decisions of the central government:

In stage 1, the central government takes the above responses of the regional governments into account when setting transfer T. It optimizes for $\theta \in [0,1]$ a utilitarian welfare function:

$$\max_T \left\{ \theta\, U_1\left[\frac{F_1\left[N_1(\cdot), \bar{L}_1\right] - S_{12}(T) + S_{21}(T) - T - Z_1(T)}{N_1\left[T, S_{12}(T), S_{21}(T), Z_1(T), Z_2(T)\right]}, Z_1(T), Z_2(T)\right]\right.$$

$$\left. + (1-\theta)U_2\left[\frac{F_2\left[1 - N_1(\cdot), \bar{L}_2\right] + S_{12}(T) - S_{21}(T) + T - Z_2(T)}{1 - N_1\left[T, S_{12}(T), S_{21}(T), Z_1(T), Z_2(T)\right]}, Z_1(T), Z_2(T)\right]\right\}, \tag{24}$$

where the symbol „·" represents the abbreviated form of the arguments of N_1. These are written out in full in the denominators.

Noting (22), $dN_1/dZ_2 = -dN_2/dZ_2$, and $S_{21} = 0$, the optimization of (24) provides the following first-order condition:

$$\theta\left\{\frac{U_1^x}{N_1}\left(1+\frac{dS_{12}}{dT}\right)\left[\frac{dN_1}{dT}\left(F_1^N - x_1\right)-1\right]-\frac{2kU_1^{Z_2}}{\frac{U_1^x}{N_1}\left(F_1^N - x_1\right)-2k}\frac{dZ_2}{dT}\right\}$$

$$+(\theta-1)\left\{\frac{U_2^x}{N_2}\left(1+\frac{dS_{12}}{dT}\right)\left[\frac{dN_1}{dT}\left(F_2^N - x_2\right)-1\right]-\frac{2kU_2^{Z_1}}{\frac{U_2^x}{N_2}\left(F_2^N - x_2\right)-2k}\frac{dZ_1}{dT}\right\}=0. \quad (25)$$

Since in the optimization conditions for regional government 1 only the sum $S_{12} + T$ appears as an argument, the optimal reaction of this government is in fact determined by the value of this sum. Therefore, if region 1's government offers a positive transfer S_{12}, and the central government marginally changes its transfer T, the government of region 1 will seek to hold this sum at constant value in order to continue to achieve maximum utility. Thus $dS_{12}/dT = -1$ [9]. A rigorous proof for this relation is given in Appendix 6.1. From (25) it follows for this case that:

$$\theta U_1^{Z_2}\frac{2k}{\frac{U_1^x}{N_1}\left(F_1^N - x_1\right)-2k}\frac{dZ_2}{dT}-(1-\theta)U_2^{Z_1}\frac{2k}{\frac{U_2^x}{N_2}\left(F_2^N - x_2\right)-2k}\frac{dZ_1}{dT}=0. \quad (26)$$

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[9] This corresponds to the result obtained by Warr (Warr, 1982) according to which „donors respond to incremental fiscal redistributions by reducing their voluntary contributions by exactly a dollar for every dollar transferred in this way" (Warr, (1982), p. 131).

$Z_i = Z_i(T)$, $i \in \{1,2\}$, are the reaction functions resulting from (21) and (23) of regional governments 1 and 2 for the provision of regional public good Z_i as a function of transfer T, which is chosen by the central government.

Two cases have to be distinguished:

In the *first case*, when regional government 1 offers a positive transfer, i.e. when $dS_{12}/dT = -1$ we have:

$$\frac{dZ_1}{dT} = \frac{dZ_2}{dT} = 0. \tag{27}$$

A proof for this result is provided in Appendix 6.2. On the other hand, this result is also obvious. If the central government marginally changes transfer T by dT, regional government 1 will adjust its transfer S_{12} by just $-dT$, since $dS_{12} = -dT$. The sum $S_{12} + T$, i.e. the net interregional transfer, thereby remains constant.

Consequently, the central government can change neither the value of its objective function nor the supply of regional good Z_i as long as $dS_{12}/dT = -1$.

If, however, the central government chooses a transfer greater than the optimal transfer S_{12}^* regional government 1 would be prepared to implement when $T = 0$, this regional government can no longer prevent a further increase in T by reducing S_{12}. As soon as $dS_{12}/dT = 0$ (*second case*), regional government 1 will make no transfers S_{12}, and the central government's optimal transfer fulfills $T^* \geq S_{12}^*$. This is illustrated in Figure 1.
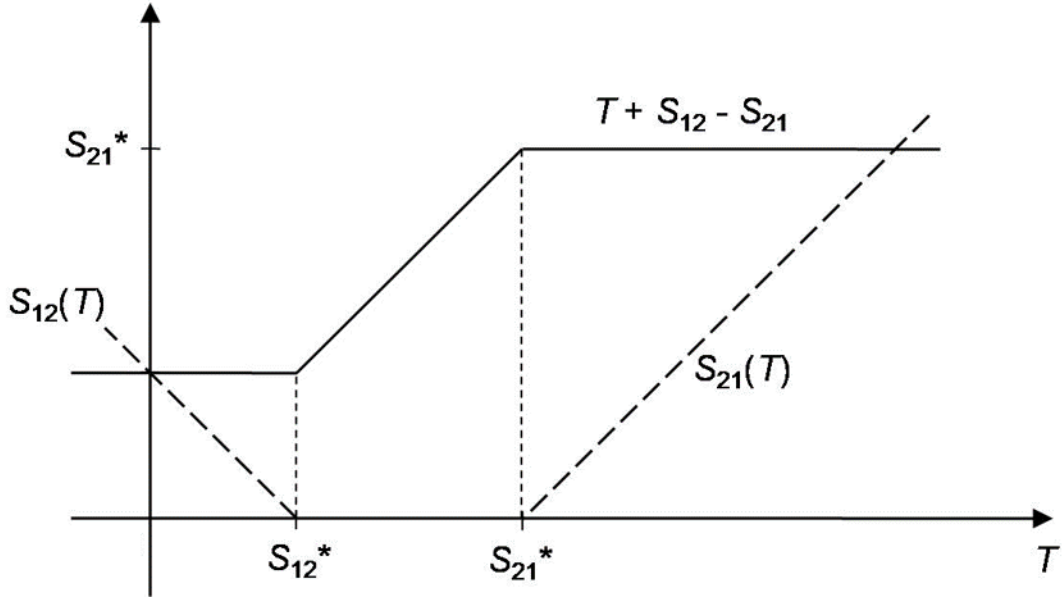
Figure 1: Net Transfer

Figure 1 shows the net interregional transfer $T + S_{12} - S_{21}$. As long as $T < S_{12}^*$ and thus $dS_{12}/dT = -1$ (*first case*), regional government 1 is able to enforce a net transfer of S_{12}^* (which is the optimum for regional government 1) by adjusting S_{12}. However, if the central government chooses a transfer for which $T > S_{12}^*$, and hence $dS_{12}/dT = 0$ holds (*second case*), regional government 1 is no longer able to enforce a net transfer of S_{12}^*. Analogous arguments lead to the conclusion that regional government 2 cannot affect the central government's decision for $T < S_{21}^*$. Therefore, the central government can autonomously set the net transfer between the values S_{12}^* and S_{21}^* by choosing T.

For the *second case*, i.e. for $dS_{12}/dT = 0$, $dS_{21}/dT = 0$, and a net transfer between the values S_{12}^* and S_{21}^*, we obtain from (25), after replacing dN_1/dT by using equation (13):

$$\left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) + \frac{2kN_2}{U_2^x}\left[\theta - (1-\theta)\frac{N_1}{N_2}\frac{U_2^x}{U_1^x}\right]$$

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$$= 2kD \frac{N_1}{U_1^x} \frac{N_2}{U_2^x} \left[\theta \frac{U_1^{Z_2}}{\frac{U_1^x}{N_1}\left(F_1^N - x_1\right) - 2k} \cdot \frac{dZ_2}{dT} + \left(1-\theta\right) \frac{U_2^{Z_1}}{\frac{U_2^x}{N_2}\left(F_2^N - x_2\right) - 2k} \cdot \frac{dZ_1}{dT} \right] . \quad (28)$$

Taking the extreme values $\theta = 1$ and $\theta = 0$ into account, we obtain the following inequality:

$$-\frac{2kN_2}{U_2^x}\left(1 - D\alpha_2 \frac{dZ_2}{dT}\right) \le \left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) \le \frac{2kN_1}{U_1^x}\left(1 + D\alpha_1 \frac{dZ_1}{dT}\right), \quad (29)$$

with

$$\alpha_1 \equiv \frac{N_2}{U_2^x} \frac{U_2^{Z_1}}{2k - (F_2^N - x_2)\frac{U_2^x}{N_2}}, \quad (30)$$

$$\alpha_2 \equiv \frac{N_1}{U_1^x} \frac{U_1^{Z_2}}{2k - (F_1^N - x_1)\frac{U_1^x}{N_1}} . \quad (31)$$

Thus, the central government chooses transfer T such that inequality (29) holds.

Without spillover effects, the expressions in brackets on the right and on the left hand sides of condition (29) would be equal to 1, since, for $U_1^{Z_2} = U_2^{Z_1} = 0$, $\alpha_1 = \alpha_2 = 0$. Optimization condition (29) of the central government would then be identical to efficiency condition (17). Since, for $U_i^{Z_j} = 0$, optimization conditions (23) of the regional governments would be reduced to the Samuelson conditions $N_i U_i^{Z_1} - U_i^x = 0$, and since this would also hold for efficiency conditions (16), the subgame perfect equilibrium would be Pareto-efficient in this case.

However, if the expressions in brackets on the right and/or on the left hand sides of condition (29) are not equal to 1, the bounds of this optimization condition no longer correspond to those of efficiency condition (17). The values of these expressions may be derived as follows:

D is negative *per definitionem*. The signs of α_1 and α_2 may be determined as follows:

For $S_{12} = 0$, (20) yields:

$$\left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) + \frac{2kN_2}{U_2^x} \geq 0, \tag{32}$$

and thus

$$\left(F_1^N - x_1\right) = \left(F_2^N - x_2\right) - \frac{2kN_2}{U_2^x} + \delta, \tag{33},$$

wherein $\delta \geq 0$.

If (33) is inserted into (11), one obtains:

$$\frac{U_1^x}{N_1}\left[\left(F_2^N - x_2\right) - \frac{2kN_2}{U_2^x} + \delta\right] + \frac{U_2^x}{N_2}\left(F_2^N - x_2\right) - 2k < 0, \tag{34},$$

and after some rearrangement:

$$\frac{U_2^x}{N_2}\left(F_2^N - x_2\right) - 2k + \delta \frac{\dfrac{U_2^x}{N_2}}{1 + \dfrac{N_1}{U_1^x}\dfrac{U_2^x}{N_2}} < 0, \tag{35}.$$

Hence:

$$2k - \frac{U_2^x}{N_2}\left(F_2^N - x_2\right) > 0,$$ (36).

Thus, $\alpha_1 > 0$. Analogous considerations yield $\alpha_2 > 0$.

The response of each regional government to a change of the central government's transfer T, of dZ_1 / dT and of dZ_2 / dT can be obtained by implicitly differentiating condition (23). Therein, one obtains $dZ_i / dT \neq 0$. However, the signs of the corresponding expressions cannot generally be determined (Wellisch, 1994, p. 179). This is different if no spillover effects are present. Then, $dZ_1 / dT < 0$ and $dZ_2 / dT > 0$ (see Appendix 6.3). This can also be easily understood as follows: If the central government marginally increases its transfer, the income available in region 1 will decrease whereas the income available in region 2 will increase. Since the private and public consumer goods are normal goods, their provision will increase in region 2, whereas it will decrease in region 1. Therefore, the indefiniteness of the signs in case of spillover effects is solely caused by the derivatives $U_j^{Z_i}$ being nonzero. Here, we assume that the spillover effects are small enough such that the signs which are obtained without spillovers remain unchanged. We then obtain:

$$D\alpha_1 \frac{dZ_1}{dT} > 0$$ (37),

$$D\alpha_2 \frac{dZ_2}{dT} < 0$$ (38).

Therefore, the bounds of inequality (29) are greater than those in efficiency condition (17).

The fact that maximizing the social welfare function (24) by the central government does not lead to Pareto-efficiency may be surprising at first sight.

However, this situation corresponds to a "second best" problem. The regional governments realize optimization conditions (23). These are different from efficiency conditions (16). Therefore, it is not anymore Pareto-efficient if efficiency condition (17) is realized by the central government[10]. The second best solution is rather given by optimization conditions (23) and (29). However, also this second best solution cannot be realized. The central government is not able to surpass the bounds of (21). If it was able to do so, this would lead to a contradiction to the assumption of $dS_{ij}/dT = 0$. This is due to the fact that at the bounds of inequality (21) transfer T just equals S_{12}^* (S_{21}^*), i.e. the transfer that is optimal for region 1 (region 2). Once the transfer drops below this value (or rises above it), the government of region 1 (region 2) will respond by paying transfer $dS_{12} = -dT$ ($dS_{21} = dT$) which results in $T + S_{12} = S_{12}^*$ ($T - S_{21} = S_{21}^*$), see Figure 1. Consequently, transfer T must be within the interval $\left[S_{12}^*, S_{21}^*\right]$.

As a result, the subgame perfect equilibrium described by conditions (21) and (23) is only a "third best" solution.

If, in addition to the goal of achieving an efficient population distribution, the central government pursues the goal of establishing equal living conditions across the whole federation, it will in general choose a weight θ in (24) with $0 < \theta < 1$. If there are no spillover effects $\left(U_1^{Z_2} = U_2^{Z_1} = 0\right)$, it can be demonstrated that $dT/d\theta < 0$ holds[11]. Assuming that the spillover effects are small enough not to change this result, we obtain a unique value of T for each θ. For all θ such that $0 < \theta < 1$, a value of T within the bounds of inequality (21) is then obtained. As a consequence, $S_{12} = S_{21} = 0$, i.e. the central government prevails with its policy and horizontal equalization does not occur. Thus we have the following results.

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[10] „…, given that one of the Paretian optimum conditions cannot be fulfilled, then an optimum situation can be achieved only by departing from all the other Paretian conditions" (Lipsey and Lancaster, 1956/57, p. 11).
[11] See Appendix 6.4

Proposition 1. *If the possibility of horizontal and vertical equalization exists, it is the central government alone that sets the size of the transfer. Horizontal equalization then does not take place. The population distribution is efficient as well, but an inefficient provision of the public goods results in both regions. Consequently, the subgame perfect equilibrium is not Pareto efficient.*

Using conditions (21), (23) and (29), it can be shown that the results of Flatters et al. (1974), Myers (1990) and Mansoorian and Myers (1993) mentioned in the introduction are special cases of these general conditions.

In Flatters et al. (1974), the regional governments do not have an instrument to manage migration $\left(S_{ij}=0\right)$ [12]. Only the central government has such an instrument at its disposal $\left(T \geq 0\right)$. Furthermore, there are neither barriers to mobility $\left(k=0\right)$ nor spillover effects $\left(U_j^{Z_i}=0\right)$. Regional government i then provides the optimal quantity of the public good $\left(N_i U_i^{Z_i}/U_i^x =1\right)$ in stage 2 of the game according to (23), while, in the first stage, the central government chooses a transfer that leads to an efficient population distribution in line with inequality (21) $\left(F_1^N - x_1 = F_2^N - x_2\right)$. And so there is Pareto efficiency.

Myers (1990) analyzes the case in which $k=0$, $U_j^{Z_i}=0$, $S_{ij} \geq 0$ and $T=0$. In the second stage of the game, regional government 1 provides a quantity of the public good and a transfer S_{12} such that (23) $\left(N_i U_i^{Z_i}/U_x^i =1\right)$ and (21) $\left(F_1^N - x_1 = F_2^N - x_2\right)$ fulfill the resulting Nash equilibrium. This equilibrium is thus also Pareto efficient without a transfer by the central government.

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[12] "….the regions are assumed not to have an instrument to make transfers to other regions even though these transfers will affect regional population …". Myers (1990, p. 108).

Mansoorian and Myers (1993) assume that $S_{ij} > 0$, $k > 0$, $U_j^{Z_i} = 0$ and $T = 0$.

In the second stage of the game, regional government 1 again fulfills conditions (23) and (21), which is why there is also Pareto efficiency in the corresponding Nash equilibrium without the central authority having to intervene.

4.2 The Regional Governments Act First

In this section, we determine the equilibrium that arises when decisions are made in the following order:

Stage 1: The regional governments set the horizontal transfers S_{12} and S_{21} as well as the extent of the provision of the public goods Z_1 and Z_2, anticipating the response of the central government.

Stage 2: The central government sets transfer T.

The problem is again solved using backwards induction.

The central government sets the size of transfer T for given values of $\bar{S}_{12}$, $\bar{S}_{21}$, $\bar{Z}_1$ and $\bar{Z}_2$ in stage 2. It chooses T such that social welfare W will be maximized:

$$W = \theta U_1 \left\{ \frac{F_1\left[N_1\left(T, \bar{S}_{12}, \bar{S}_{21}, \bar{Z}_1, \bar{Z}_2\right), \bar{L}_1 \right] - \bar{S}_{12} + \bar{S}_{21} - T - \bar{Z}_1}{N_1\left(T, \bar{S}_{12}, \bar{S}_{21}, \bar{Z}_1, \bar{Z}_2\right)}, \bar{Z}_1, \bar{Z}_2 \right\}$$

$$+ \left(1 - \theta\right) U_2 \left\{ \frac{F_2\left[1 - N_1\left(T, \bar{S}_{12}, \bar{S}_{21}, \bar{Z}_1, \bar{Z}_2\right), \bar{L}_2 \right] - \bar{S}_{21} + \bar{S}_{12} + T - \bar{Z}_2}{1 - N_1\left(T, \bar{S}_{12}, \bar{S}_{21}, \bar{Z}_1, \bar{Z}_2\right)}, \bar{Z}_2, \bar{Z}_1 \right\}. \quad (39)$$

The first-order condition for a maximum is characterized by:

$$\theta \frac{U_1^x}{N_1}\left[\frac{dN_1}{dT}\left(F_1^N - x_1\right) - 1\right] = (1-\theta)\frac{U_2^x}{N_2}\left[\frac{dN_1}{dT}\left(F_2^N - x_2\right) - 1\right]. \tag{40}$$

Inserting the derivative dN_1/dT from (13) in (40) yields

$$\left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) + \frac{2kN_2}{U_2^x}\left[\theta - (1-\theta)\frac{N_1}{N_2}\frac{U_2^x}{U_1^x}\right] = 0. \tag{41}$$

Taking the extreme values $\theta = 1$ und $\theta = 0$ into account, we obtain the inequality

$$-\frac{2kN_2}{U_2^x} \le \left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) \le \frac{2kN_1}{U_1^x}, \tag{42}$$

which corresponds to efficiency condition (17). Therefore, we can again limit our considerations to the case $S_{12} \ge 0$ and $S_{21} = 0$ (i.e. that region 2 is transfer-constrained).

Since the transfers only appear as a sum $-\left(S_{12} + T\right)$ or $S_{12} + T$ in the optimization condition (41), the central government will, regardless of the choice of S_{12}, set transfer T in such a way that this sum preserved, i.e. $dT/dS_{12} = -1$ as long as $T < S_{12}^*$. The proof can be found in Appendix 6.5. Since regional government 1 therefore cannot change the net transfer from the outset, carrying out a voluntary transfer is not meaningful, hence $S_{12} = 0$.

So here too it is solely the central government that sets transfer T within the bounds of inequality (42). The regional governments are left only with the task of optimizing their choice of z_i.

In stage 1, regional government i therefore solves the following maximization problem:

$$\max_{Z_i} U_i \left\{ \frac{F_i\left\{N_i\left[T\left(Z_i,\bar{Z}_j\right),Z_i,\bar{Z}_j\right],\bar{L}_i\right\}+(-1)^i\,T\left(Z_i,\bar{Z}_j\right)-Z_i}{N_i\left[T\left(Z_i,\bar{Z}_j\right),Z_i,\bar{Z}_j\right]},Z_i,\bar{Z}_j \right\}. \text{ (43)}$$

This yields the following first-order condition for the provision of Z_1:

$$\frac{dT}{dZ_1}\left[\left(F_1^N-x_1\right)-\left(F_2^N-x_2\right)+\frac{2kN_2}{U_2^x}\right]$$

$$-\left(1-N_1\frac{U_1^{Z_1}}{U_1^x}\right)\left[\left(F_2^N-x_2\right)-\frac{2kN_2}{U_2^x}\right]+N_2\frac{U_2^{Z_1}}{U_2^x}\left(F_1^N-x_1\right)=0, \qquad \text{(44)}$$

or

$$\frac{dT}{dZ_1}\left(\frac{F_1^N-x_1}{\left(F_2^N-x_2\right)-\dfrac{2kN_2}{U_2^x}}-1\right)-1+N_1\frac{U_1^{Z_1}}{U_1^x}+N_2\frac{U_2^{Z_1}}{U_2^x}\cdot\frac{F_1^N-x_1}{\left(F_2^N-x_2\right)-\dfrac{2kN_2}{U_2^x}}=0. \text{ (45)}$$

An analogous condition exists for Z_2:

$$\frac{dT}{dZ_2}\left(\frac{F_2^N-x_2}{\left(F_1^N-x_1\right)-\dfrac{2kN_1}{U_1^x}}-1\right)+1-N_2\frac{U_2^{Z_2}}{U_2^x}+N_1\frac{U_1^{Z_2}}{U_1^x}\cdot\frac{F_2^N-x_2}{\left(F_1^N-x_1\right)-\dfrac{2kN_1}{U_1^x}}=0. \text{ (46)}$$

The central government's response to a marginal change in the provision of the public consumer goods, i.e. dT/dZ_1 and dT/dZ_2 can be obtained by implicitly differentiating (41). Therein, it turns out that in general $dT/dZ_i \neq 0$. The signs of the expressions dT/dZ_i, however, cannot be determined in general.

What can be determined from a comparison of optimization conditions (45)/(46) and (23) is the following: in addition to the inefficiency evident from

(23), another inefficiency results from (45)/(46) due to the non-vanishing values of dT/dZ_i if one of the expressions $\left(F_i^N - x_i\right)/\left[\left(F_j^N - x_j\right) - \dfrac{2kN_j}{U_j^x}\right] - 1$ ($i \neq j$) in (45)/(46), i.e. one of the multipliers of dT/dZ_i, is nonzero. This is actually the case for at least that of dT/dZ_2 as shown in the following:

Since at least region 2 is transfer constrained, one has:

$$\left(F_2^N - x_2\right) - \left(F_1^N - x_1\right) + \frac{2kN_1}{U_1^x} > 0 \tag{47}.$$

Thus:

$$F_2^N - x_2 > \left(F_1^N - x_1\right) - \frac{2kN_1}{U_1^x} \tag{48},$$

and therefore

$$\left(F_2^N - x_2\right)/\left[\left(F_1^N - x_1\right) - \frac{2kN_1}{U_1^x}\right] \neq 1.$$

The same result is obtained for the multiplier of dT/dZ_1 only if also region 1 is transfer-constrained.

The additional inefficiencies present in (45)/(46) do in general not compensate the first inefficiency already present in (23).

Hence, also here, the subgame perfect equilibrium is not Pareto-efficient.

This leads to

Proposition 2. *If the regional governments act first, in this case as well, there is only the vertical equalization effected by the central*

government. Horizontal equalization does not take place. The resulting population distribution is efficient, but the provision of public goods is inefficient. The subgame perfect equilibrium is therefore Pareto-inefficient.

This latter inefficiency does not arise in Caplan et al. (2000) although they only replace both regional public goods with a federal public good whose provision also continues to be determined by the two regional governments independently of each other. This can be explained as follows. In the model used here, each regional government decides on its provision of the public good without taking into account the associated effects thereof in the other region. In Caplan et al., however, the two regional governments are linked to each other in the task of providing the federal public good via the reaction functions $Z_i = Z_i(Z_j), \quad i \neq j$. The game in Caplan et al. thus exhibits a structure corresponding to that of the Cournot duopoly model and should not be confused with the game played in the literature forming the basis of this paper.

4.3 Interim Result

The results of sections 4.1 and 4.2 can be summarized as follows.

Regardless of the order in which the regional governments, on the one hand, and the central government, on the other, decide, in the framework of the model used here, the central government determines the extent of the redistribution across regions solely by means of the vertical equalization scheme it implements.

It is interesting to compare this result with the actual fiscal equalization system of the Federal Republic of Germany. There, in a first step, disparate financial capacities of the Länder are reduced within the framework of a complex horizontal equalization scheme. After that, in the framework of a vertical equalization scheme, the federal government provides grants to the

Länder which are still financial weak[13] . As a result, the financial capacities of the Länder are nearly equal. With that the Federal Government, after all, decides on fiscal equalization in Germany.

5. Equal standard of living throughout the federation

"But the primary justification for fiscal equalization must be on equity grounds" (Oates, 1999, p. 1128). The task of a fiscal equalization scheme across regions is not only to ensure an efficient distribution of population; it should, above all, also secure an equal standard of living throughout the federation. However, both demands can only be partially fulfilled simultaneously by a vertical equalization scheme.

It was shown in section 4.1 that when the government of region 1 decides on a horizontal transfer, it will only have the utility of its own residents in mind. The transfer S_{12}^* it wishes to give corresponds to the weight $\theta = 1$ in the social welfare function if we ignore inefficiencies associated with the provision of public goods (*cf.*, Wellisch, 1994, p. 176, Mansoorian and Myers, 1997, p. Lemma 1, 275). The other region is only of interest to the government of region 1 to the extent that it is available for admitting further residents from region 1 when it is overpopulated[14]. The weight granted to the residents of region 2 by the government of region 1 is $\theta = 0$.

The central government, on the other hand, will assign positive weights to the residents of both regions in order to maximize welfare and therefore choose a transfer $T^* > S_{12}^*$. A possible result of such a decision is shown in Figure 2.

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[13] "Such law (a federal law, A&A) … may also provide for grants to be made by the Federation to financially weak Länder from its own funds to assist them in meeting their general financial needs (supplementary grants)." Article 107 (2) of the basic Law for the Ferderal Republic of Germany.
[14]"…the region purchases a preferred regional population size" (Myers, 1990, 194).
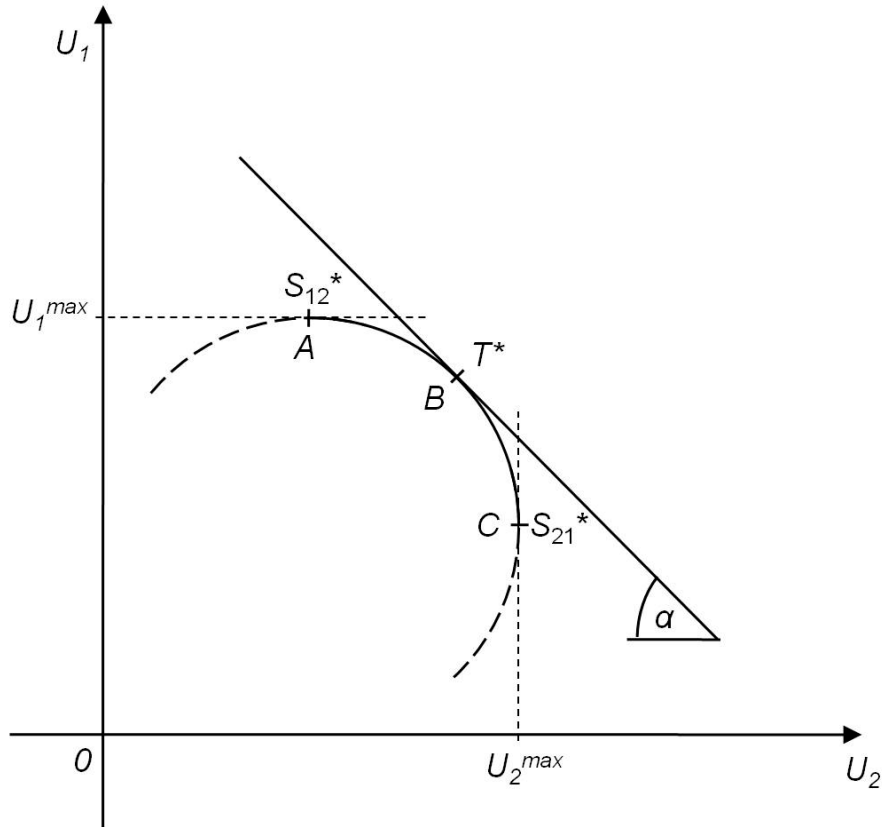
Figure 2: The Welfare Maximum

The utility-possibility frontier is indicated by the curve ABC. The downward sloping tangent to the utility-possibility frontier at point B is an iso-welfare line with a gradient of

$$\frac{dU_1}{dU_2} = -\frac{1-\theta}{\theta}.$$

This welfare maximum can be achieved with transfer T^* if a specific transfer can be assigned to each θ: $dT/d\theta < 0$ (see Appendix 6.4).

The possibility of influencing the utility distribution between the two regions by varying T disappears, however, when there is no attachment to home $(k = 0)$ since then $dT/d\theta = 0$ (see Appendix 6.4). Since in that case region i maximizes $U_i(x_i, Z_i, Z_j)$ subject to the constraint $U_i(x_i, Z_i, Z_j) = U_j(x_j, Z_i, Z_j)$,

it is immediately evident that maximizing U_i maximizes U_j[15]. It is, therefore, not possible to influence the utility distribution by means of a transfer – and such a transfer is also not necessary since residents' utility has already been equalized through migration. The utility-possibility frontier dwindles to a point in this case.

Proposition 3. *If the population is perfectly mobile $(k=0)$, a vertical equalization scheme carried out for reasons of equality is neither possible nor necessary. If migration barriers exist $(0<k<\infty)$, the central government can ensure that the migration equilibrium is efficient by using a vertical equalization scheme, and can at the same time pursue redistributive goals. If the barriers to migration are prohibitively high $(k \to \infty)$, the central government can be guided solely by redistributive considerations in its choice of a vertical equalization scheme.*

6. Conclusions

In this paper, a federation consisting of two regions is considered, between which migration is possible. In each of the regions, a regional government decides on the quantity of the regional public good to be provided and the size of the horizontal transfer to pay to the other region. The regional public goods from each region generate spillover effects. A central government decides on the size of a vertical transfer. The regional governments decide simultaneously on the quantity of the public goods to be provided. The two levels of government decide on horizontal and vertical fiscal equalization schemes in different chronological order.

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[15]"With no attachment, when region 2 maximizes $U(x_2)$ subject to $U(x_1)=U(x_2)$ it is in effect maximizing $U(x_1)$. Because of the free mobility assumption there is complete incentive equivalence and no disagreement." (Mansoorian and Myers, 1993, p. 125).

The central finding here is that the central government will always prevail with the vertical equalization scheme irrespective of the timing of the decisions, provided that barriers to mobility are present. Horizontal equalization is neither effected nor necessary to attain constraint efficiency. The vertical equalization scheme leads to an efficient distribution of the entire population of the federation across both regions. The provision of the regional public goods is inefficient, however, so long as spillover effects are present. This inefficiency can be eliminated by means of Pigouvian taxes.

Appendix 6.1

Proof of $dS_{12} / dT = -1$

Inserting (7) in the first order maximization conditions (21) and (23), and considering $S_{12} > 0$ as well as $S_{21} = 0$ one obtains:

$$\left[F_1^N - \frac{1}{N_1}\left(F_1 - S_{12} - T - Z_1\right)\right] - \left[F_2^N - \frac{1}{N_2}\left(F_2 + S_{12} + T - \overline{Z}_2\right)\right] \tag{49}$$

$$+ \frac{2kN_2}{U_2^x\left[\dfrac{1}{N_2}(F_2 + S_{12} + T - \overline{Z}_2), Z_1, \overline{Z}_2\right]} = 0$$

$$N_1 \frac{U_1^{Z_1}\left[\dfrac{1}{N_1}(F_1 - S_{12} - T - Z_1), Z_1, \overline{Z}_2\right]}{U_1^x\left[\dfrac{1}{N_1}(F_1 - S_{12} - T - Z_1), Z_1, \overline{Z}_2\right]} +$$

$$+ N_2 \frac{U_2^{Z_1}\left[\frac{1}{N_2}(F_2 + S_{12} + T - \overline{Z}_2), Z_1, \overline{Z}_2\right]}{U_2^{x}\left[\frac{1}{N_2}(F_2 + S_{12} + T - \overline{Z}_2), Z_1, \overline{Z}_2\right]} \times$$

$$\times \frac{F_1^{N} - \frac{1}{N_1}(F_1 - S_{12} - T - Z_1)}{\left[F_2^{N} - \frac{1}{N_2}(F_2 + S_{12} + T - \overline{Z}_2)\right] - \frac{2kN_2}{U_2^{x}\left[\frac{1}{N_2}(F_2 + S_{12} + T - \overline{Z}_2), Z_1, \overline{Z}_2\right]}} - 1 = 0 \qquad (50)$$

These are two implicit functions:

$$M_1(S_{12}, Z_1, T) = 0,$$

$$M_2(S_{12}, Z_1, T) = 0.$$

These can be explicitly solved if their Jacobian is nonzero:

$$S_{12} = S_{12}^{*}(T),$$

$$Z_1 = Z_1^{*}(T).$$

The rule for implicitly differentiating implicit functions leads to:

$$\begin{pmatrix} \partial M_1 / \partial S_{12} & \partial M_1 / \partial Z_1 \\ \partial M_2 / \partial S_{12} & \partial M_2 / \partial Z_1 \end{pmatrix} \times \begin{pmatrix} \partial S_{12}^{*} / \partial T \\ \partial Z_1^{*} / \partial T \end{pmatrix} = \begin{pmatrix} -\partial M_1 / \partial T \\ -\partial M_2 / \partial T \end{pmatrix},$$

and

$$\frac{dS_{12}^{*}}{dT} = -\frac{\partial M_1 / \partial T \times \partial M_2 / \partial Z_1 - \partial M_2 / \partial T \times \partial M_1 / \partial Z_1}{\partial M_1 / \partial S_{12} \times \partial M_2 / \partial Z_1 - \partial M_2 / \partial S_{12} \times \partial M_1 / \partial Z_1}.$$

Since the Jacobian is nonzero, the function $S_{12} = S_{12}^{*}(T)$ exists. Numerator and denominator are equal if

$\partial M_1 / \partial T = \partial M_1 / \partial S_{12}$, and $\partial M_2 / \partial T = \partial M_2 / \partial S_{12}$.

Since S_{12} and T are present in equations (49) and (50) in an exactly identical manner, this is the case. Thus:

$$\frac{dS_{12}^*}{dT} = -1.$$

Appendix 6.2

Proof of $dZ_1^* / dT = 0$ for $dS_{12}^* / dT = -1$

From appendix 6.1 one obtains:

$$\frac{dZ_1^*}{dT} = -\frac{\partial M_1 / \partial S_{12} \times \partial M_2 / \partial T - \partial M_2 / \partial S_{12} \times \partial M_1 / \partial T}{\partial M_1 / \partial S_{12} \times \partial M_2 / \partial Z_1 - \partial M_2 / \partial S_{12} \times \partial M_1 / \partial Z_1}.$$

For $dS_{12}^* / dT = -1$ (see Appendix 6.1) the following holds:

$\partial M_1 / \partial T = \partial M_1 / \partial S_{12}$ and $\partial M_2 / \partial T = \partial M_2 / \partial S_{12}$.

Inserting these latter equations in the former expression yields:

$$\frac{dZ_1^*}{dT} = -\frac{\partial M_1 / \partial T \times \partial M_2 / \partial T - \partial M_2 / \partial T \times \partial M_1 / \partial T}{\partial M_1 / \partial S_{12} \times \partial M_2 / \partial Z_1 - \partial M_2 / \partial S_{12} \times \partial M_1 / \partial Z_1} = 0.$$

Appendix 6.3

Proof of $dZ_1 / dT < 0$ for $U_j^{Z_i} = 0$

Condition (23) reduces to the Samuelson condition:

$$U_1^x \left[\frac{1}{N_1} (F_1 - T - Z_1), Z_1 \right] - N_1 U_1^{Z_1} \left[\frac{1}{N_1} (F_1 - T - Z_1), Z_1 \right] = 0.$$

This is an implicit function. Implicit differentiation leads to:

$$\frac{dZ_1}{dT} = -\frac{\partial M_3 / \partial T}{\partial M_3 / \partial Z_1} = -\frac{-\dfrac{1}{N_1}U_1^{xx} + N_1 U_1^{Z_1 x}}{-\dfrac{1}{N_1}U_1^{xx} + 2U_1^{xZ_1} - N_1 U_1^{Z_1 Z_1}} < 0.$$

Analogous considerations lead to

$$\frac{dZ_2}{dT} > 0.$$

Appendix 6.4

Without spillover effects we obtain from (28):

$$\left(F_1^N - x_1\right) - \left(F_2^N - x_2\right) + \frac{2kN_2}{U_2^x}\left[\theta - (1-\theta)\frac{N_1}{N_2}\frac{U_2^x}{U_1^x}\right] = 0.$$

With $x_1 = x_{11} - \dfrac{N_2 x_{12}}{N_1}$, $x_2 = x_{22} + x_{12}$ and $T = N_2 x_{12}$ this yields:

$$F_1^N - \left(x_{11} - \frac{T}{N_1}\right) - \left[F_2^N - \left(x_2 + \frac{T}{N_2}\right)\right] + \frac{2kN_2}{U_2^x}\left[\theta - (1-\theta)\frac{N_1}{N_2}\frac{U_2^x}{U_1^x}\right] = 0$$

or, because $N_1 + N_2 = 1$,

$$\frac{T}{N_1 N_2} - \left(F_1^N - x_{11}\right) - \left(F_2^N - x_{22}\right) + \frac{2kN_2}{U_2^x}\left[\theta - (1-\theta)\frac{N_1}{N_2}\frac{U_2^x}{U_1^x}\right] = 0.$$

Implicit differentiation yields:

$$\frac{dT}{d\theta} = -2kN_1 N_2\left(\frac{N_2}{U_2^x} + \frac{N_1}{U_1^x}\right) < 0.$$

For $k = 0$ we obtain

$$\frac{dT}{d\theta} = 0.$$

Appendix 6.5

Proof of $dT / dS_{12} = -1$

Inserting (7) into (41) yields:

$$\left[F_1^N - \frac{1}{N_1}\left(F_1 - \overline{S}_{12} - T - \overline{Z}_1\right)\right] - \left[F_2^N - \frac{1}{N_2}\left(F_2 + \overline{S}_{12} + T - \overline{Z}_2\right)\right] +$$

$$+ \frac{2kN_2\theta}{U_2^x\left[\frac{1}{N_2}\left(F_2 + \overline{S}_{12} + T - \overline{Z}_2\right),\overline{Z}_1,\overline{Z}_2\right]} - \frac{2kN_1(1-\theta)}{U_1^x\left[\frac{1}{N_1}\left(F_1 - \overline{S}_{12} - T - \overline{Z}_1\right),\overline{Z}_1,\overline{Z}_2\right]} = 0.$$

It can be seen immediately that T and S_{12} enter efficiency condition (41) in exactly the same manner. Therefore, implicitly differentiating yields:

$$\frac{dT}{dS_{12}} = -1.$$

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