Order Sequencing in the Automobile Industry

A Rule Based Approach with Color Change Reduction

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A Framework for Order Sequencing

Rules for Order Sequencing

A Greedy Approach for Sequence Construction

Order Clustering to Reduce Color Changes
A Typical Production Plant

Body Shop
- press, weld, and mount the car body

Paint Shop
- enamel the body-in-white

Assembly Shop
- implement optional components
A Typical Production Plant

For all zones a set of rules must be respected

Storage systems allow color change reduction by short term interchanges prior to the paint shop
Orders and Commodities

Commodities:
- central locking
- wheels (rim type?)
- car glass (toned?)
- seats (standard / leather / sport)
- sun roof (slide/lift)
- engine (div. types)
- air conditioner
- power steering

Order:
- color
Manufacturing Errors

- Production process is essentially probabilistic
- Commodities are delayed due to manufacturing errors

- An exact optimization model is not sensible

- Three major goals:
  - Robustness
  - Transparency
  - Performance

- Instead of objectives we use rules with priority to evaluate the result
- We model the production process in deterministic way
What happens at the split point of zones?
- Zones have different velocity / capacity
- The time an order needs to pass a zone is equal for parallel zones

Parallel zones

Non-parallel zone

Deterministic Routing of Order Positions
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Deterministic Routing of Order Positions

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**Diagram:**

- Non-parallel zone
- Parallel zones

**Pattern:** (1,1,2)(2,1,1)(1,1,2)(1,1,1)

(to read from right)
There is more than one split point

(1,1,2)(2,1,1)(1,1,2)(1,1,1)

(1,2)(1,1)(1,2)(1,1)(1,2)(1,1)
It suffices to consider the master sequence and a set of patterns

\[(1,1,2)(2,1,1)(1,1,2)(1,1,1)
(1,2)(1,1)(1,2)(1,1)(1,2)(1,1)(1,1)\]
Manufacturing errors could occur during the production process.

- Use statistical data to determine the expected delay of a commodity on a zone.

 Zone z:

- Expected delay of c in z: 5

Order with commodity c

- c will appear later in zones succeeding z (respecting quotas)

- This enables to compute a more realistic order sequence.
The Rule Set

- **Rules** can be applied to each pair \((c, z)\) of commodity and zone.
- Any order containing \(c\) routed through zone \(z\) must respect this rule.
- A **priority** \(p = 1, \ldots, 10\) is assigned to each rule.
- Achieve a sequence with lexicographically minimal number of rule breaches.
Rules can be applied to each pair \((c,z)\) of commodity and zone

Any order containing \(c\) routed through zone \(z\) must respect this rule

A priority \(p=1,\ldots,10\) is assigned to each rule

Achieve a sequence with lexicographically minimal number of rule breaches

**Clustering Rule:** A commodity is allowed to occur in \(s\) out of \(S\) slots of the master sequence

cluster \(c\) in 2 out of 4 slots

orders with commodity \(c\)
**Clustering Rule:** A commodity is allowed to occur in $s$ out of $S$ slots of the master sequence.

**Banning Rule:** A commodity is banned from an interval of a zone.

ban $c$ from 2,...,7

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orders with commodity $c$
The Rule Set

- **Clustering Rule**: A commodity is allowed to occur in \( s \) out of \( S \) slots of the master sequence

- **Banning Rule**: A commodity is banned from an interval of a zone

- **Ratio Rule**: A commodity is allowed in at most \( x \) out of \( y \) consecutive sequence positions of a zone

allow \( c \) in 2 out of 8 positions

orders with commodity \( c \)
**The Rule Set**

- **Clustering Rule:** A commodity is allowed to occur in $s$ out of $S$ slots of the master sequence.

- **Banning Rule:** A commodity is banned from an interval of a zone.

- **Ratio Rule:** A commodity is allowed in at most $x$ out of $y$ consecutive sequence positions of a zone.

- **Grouping Rule:** Orders that hold a commodity $c$ should occur in groups with minimal and maximal size, where groups must keep a minimal distance.

  - Size of groups: 2,...,3
  - Minimal distance: 2

  ![Diagram](image-url)
**The Rule Set**

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- **Banning Rule**: A commodity is banned from an interval of a zone
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- **Spacing Rule**: Orders that hold a commodity \( c \) should occur with a minimal distance

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- **Grouping Rule:** Orders that hold a commodity \( c \) should occur in groups with minimal and maximal size, where groups must keep a minimal distance

- **Spacing Rule:** Orders that hold a commodity \( c \) should occur with a minimal distance

- **Spreading Rule:** Orders with a commodity should be evenly spread

  average distance: 1.5

  ![Diagram showing the spreading rule](image_url)
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- **Spacing Rule**: Orders that hold a commodity \( c \) should occur with a minimal distance.

- **Spreading Rule**: Orders with a commodity should be evenly spread.

Two concurrent strategies for rule evaluation

- Count the number of rule breaches for each priority (quantity of rule breaches)
- Increase a penalty value for a broken rule (quality of rule breaches)

- Clustering rule is considered separately
Greedy Master Sequence Construction

Simplification:

- Clustering rules are zone independent
- There is at most one clustering commodity per order (suitable for color clustering)
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- Orders are assigned to slots of the master sequence
- Compute each slot of the master sequence separately (Divide & Conquer)
Greedy Master Sequence Construction

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- Orders are assigned to slots of the master sequence
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- Known approach: Goal Chasing (Monden 1983)
  - Choose step by step an order which assures an even resource consumption
  - Only suitable for spreading

\[ |\text{desired resource consumption} - \text{current resource consumption}|^2 \rightarrow \text{min} \]
Greedy Master Sequence Construction

Simplification:
- Clustering rules are zone independent
- There is at most one clustering commodity per order (suitable for color clustering)

Orders are assigned to slots of the master sequence

Compute each slot of the master sequence separately (Divide & Conquer)

Modified Goal Chasing approach:
- Choose in every step a “best” order
  - with lexicographically minimized rule breach vector $b=(b_1,\ldots, b_{10})$
  - and minimized penalty (priority-weighted penalties)

Fully configurable by the choice of priorities
Order Clustering Algorithm

Let \( c_1, \ldots, c_u \) denote all clustering commodities and \( O(c) \) the set of all orders containing commodity \( c \).

1. Assign to each clustering commodity its preferred slots
   - Other banning rules could imply varying ratio of a commodity
   - Spreading commodities must be evenly distributed over the slots

![Diagram of order sets and commodity spreading](image-url)

[banning commodity spreading commodity](image-url)

[banning interval commodity](image-url)
Let $c_1, \ldots, c_u$ denote all clustering commodities and $O(c)$ the set of all orders containing commodity $c$.

1. Assign to each clustering commodity its preferred slots
   - Other banning rules could imply varying ratio of a commodity
   - Spreading commodities must be evenly distributed over the slots

2. Assign each order to a preferred slot
   - Consider banning and spreading rules

$O(c_j): \{o_1, o_2, o_3, \ldots, o_{|A|}\}$

banning commodity
spreading commodity

banning interval

commodity
Let $e_1, \ldots, e_r$ denote all spreading commodities.

For each set $O(c_i)$ introduce a vector $w_i$ ($U$ is the set of non-clustering orders):

$$w_{i,j} := \begin{cases} |O(c_i) \cap O(e_j)| & j = 1, \ldots, r \\ |O(c_i)| & j = r+1 \end{cases}$$

$$w_{u+1,j} := \begin{cases} |U \cap O(e_j)| & j = 1, \ldots, r \\ |U| & j = r+1 \end{cases}$$
Let $e_1, \ldots, e_r$ denote all spreading commodities.

For each set $O(c_i)$ introduce a vector $w_i$ ($U$ is the set of non-clustering orders)

\[
w_{i,j} \quad \begin{cases} |O(c_i) \cap O(e_j)| & j = 1, \ldots, r \\ |O(c_i)| & j = r + 1 \end{cases}
\]

\[
w_{u+1,j} \quad \begin{cases} |U \cap O(e_j)| & j = 1, \ldots, r \\ |U| & j = r + 1 \end{cases}
\]

Split these vectors according to the desired number of slots.
Let $e_1, \ldots, e_r$ denote all spreading commodities.

Slots contain vectors indexed from 1 to $r+1$.

The last component is reserved to ensure that a slot is filled completely.
Best Vector Packing into Bins Problem

- Let $e_1, \ldots, e_r$ denote all spreading commodities
- Slots contain vectors indexed from 1 to $r+1$
- The last component is reserved to ensure that a slot is filled completely

Desired slot contents: $$w^*_s,j := \begin{cases} \frac{|\Omega(e_j)|}{s} \cdot b(e_j, s) & j = 1, \ldots, r \\ \frac{n}{s} & j = r+1 \end{cases}$$
Best Vector Packing into Bins Problem

- Let $e_1, \ldots, e_r$ denote all spreading commodities

- Greedy approach: In each step choose a pair of vector and bin, so that the vector improves on the bins contents best possible

- This yields an assignment of an order $o$ to its set of preferred slots $S^*(o)$. 
Assignment of Orders to Slots

- Given for each order $o$ a set $S^*(o)$ of preferred slots
- Assign each order to a preferred slot (respect other banning / spreading rules)
- Binary variable $x_{o,s}$ to indicate that $o$ is assigned to slot $s$.

Spreading objective: $\sum_{o:o \in o} x_{o,s} = O(c) / S \cdot b(c,s)$, $\forall s = 1,\ldots,S$, $c \in C_{Spr}$

Clustering objective: $\sum_{o:o \in o, s \in S^*(o)} x_{o,s} = 0$, $\forall s = 1,\ldots,S$, $c \in C_{Clu}$

Slot size constraint: $\sum_{o} x_{o,s} = n / S$, $\forall s = 1,\ldots,S$

Banning constraint: $\sum_{o:o \in o} x_{o,s} \leq n / S - |B(c,s)|$, $\forall s = 1,\ldots,S$, $c \in C_{Ban}$

- Number of variables can be reduced significantly

add new constraints

Solve relaxation for $p=i$

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Results

- Easily configurable
  - simple rule definitions
  - transparent parameter tuning with priorities

- High performance
  - Operates with greedy subroutines and on small linear programs only
  - About 3 minutes on 1500 orders and realistic rules (Sun 450 MHz, 1GB)

- Color batch size increased by 50%

- The algorithm is currently used in all automobile plants of the Ford Motor Company across Europe
Thanks for your attention.