A Subspace Approximation Method for the Perturbed Quadratic Eigenvalue Problem

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The quadratic eigenvalue problem, $(\lambda^2 \mathrm{M} + \lambda \mathrm{C} + \mathrm{K})\mathrm{x} = 0$, arises often in engineering applications, for example in the study of the vibration of damped or gyroscopic systems in structural engineering as well as in the solution of quadratically constrained least square problems. We consider the perturbed quadratic eigenvalue problem $\mathrm{P}(\lambda(t), t)\mathrm{x}(t) = (\lambda^2(t)\widehat{\mathrm{M}}(t) + \lambda(t)\widehat{\mathrm{C}}(t) + \widehat{\mathrm{K}}(t))\mathrm{x}(t) = 0$, where $\widehat{\mathrm{M}}(t) = \mathrm{M} + t\Delta\mathrm{M}$, $\widehat{\mathrm{C}}(t) = \mathrm{C} + t\Delta\mathrm{C}$ and $\widehat{\mathrm{K}}(t) = \mathrm{K} + t\Delta\mathrm{K}$, $0 \le t \le 1$. Assuming the perturbation is small and the eigenvalues $\lambda(t)$ of interest are nondefective, the problem is to compute a subspace which contains approximations to the associated eigenspaces.

For small perturbations of nondefective quadratic eigenvalue problems, low order derivatives of the eigenvector matrices exist and their span can be considered as a perturbation subspace. We show that this perturbation subspace is contained in a certain generalized Krylov subspace, and can be computed within that subspace in a very straightforward way. The method, thus, involves finding eigenmatrix derivatives in a generalized Krylov subspace and solving reduced quadratic problems in the resulting subspaces spanned by the derivatives. Convergence of the method is at least as fast as that of the corresponding Taylor series. Combined with inverse iteration as a finishing procedure, this approach is often a faster, cheaper alternative to solving the perturbed problem. We discuss extending the idea to problems with larger perturbations using a homotopy approach. Numerical experiments are drawn from structural dynamics problems to demonstrate the applicability of the method.