Anti-triangular forms for Hermitian pencils

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Hermitian pencils, i.e., pairs of Hermitian matrices, arise in many applications, such as the linear quadratic optimal control problem. This is the problem of minimizing the cost functional

$$\frac{1}{2}\int_{t_0}^{\infty}\Big(x(t)^*Qx(t)+u(t)^*Ru(t)+u(t)^*S^*x(t)+x(t)^*Su(t)\Big)dt \tag{1}$$

subject to the dynamics

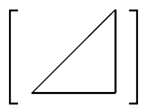
$$\begin{aligned} E\dot{x}(t) &= Ax(t)+Bu(t), \qquad t_0<t \qquad (2)\\ x(t_0) &= x_0, \qquad (3) \end{aligned}$$

where $A,E,Q\in\mathbb{C}^{n\times n}, B,S\in\mathbb{C}^{n\times m}, R\in\mathbb{C}^{m\times m}$, Q and R Hermitian, $x_0,x(t),u(t)\in\mathbb{C}^n$, and $t_0,t\in\mathbb{R}$. It is known that solutions of (1)–(3) can be obtained via the solution of a boundary value problem. For the solution of this boundary value problem one has to compute deflating subspaces of the matrix pencil

$$\lambda\begin{bmatrix} E & 0 & 0\\ 0 & -E^* & 0\\ 0 & 0 & 0\end{bmatrix}-\begin{bmatrix} A & 0 & B\\ Q & A^* & S\\ S^* & B^* & R\end{bmatrix}. \tag{4}$$

Permuting some rows and columns one can easily see that this pencil is equivalent to a Hermitian pencil.

If one is interested in finding condensed forms for Hermitian pencils under unitary transformations (for the sake of numerical stability), anti-triangular forms seem to be good forms to look for. A matrix X is called anti-triangular if

$X \hat{=}$  .

We derive conditions which anti-triangular forms for Hermitian pencils can be obtained under unitary equivalence transformations. Also the case that the pencil (4) is singular will be included.