A geometric convergence theory for preconditioned inverse iteration

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The contribution presents a convergence analysis for *preconditioned inverse iteration* to determine the smallest eigenvalue together with an eigenvector of a symmetric positive definite matrix. Preconditioned inverse iteration derives from the well–known (scaled) inverse iteration procedure (also called inverse power method) in a way that the occurring system of linear equations is solved approximately by using a preconditioner.

Sharp convergence estimates for preconditioned inverse iteration are obtained under general conditions. The influence of the preconditioner is restricted to a single constant, i.e. the spectral radius of the error propagation matrix. Thus the choice of the preconditioner is clearly separated from the underlying iterative eigensolver. Furthermore, the convergence estimates do not depend on the largest eigenvalue of the given matrix. Hence multigrid preconditioners can be used to solve eigenvalue problems for elliptic differential operators with comparable efficiency as known from boundary value problems.

The convergence analysis of preconditioned inverse iteration is based on a predominantly geometric description: a central problem is to find the supremum of the Rayleigh quotient on certain balls, which result from application of preconditioned inverse iteration for all admissible preconditioners to a fixed iterate.

The convergence theory can also be extended to a subspace implementation of preconditioned inverse iteration. Such a method is designated to determine a modest number of the smallest eigenvalues and its corresponding invariant subspace of eigenvectors. Sharp convergence estimates are derived for the Ritz values associated with the actual iteration subspace.