Abstract for a poster

Block Newton methods for improving groups of singular values

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Consider a rectangular matrix $A \in \mathbb{R}^{m\times n}$, $m \leq n$, that has q positive singular values $\sigma_{i_1}, \dots, \sigma_{i_q} > 0$ which are separated from the remaining ones but may be clustered or multiple. For simplicity of notation we denote these singular values by $\sigma_1, \dots, \sigma_q$ and set $\Sigma_q := \mathrm{diag}(\sigma_1, \dots, \sigma_q)$, and let $\mathrm{im}\, U_q = \mathrm{span}\{u^1, \dots, u^q\}$, $\mathrm{im}\, V_q = \mathrm{span}\{v^1, \dots, v^q\}$ be the corresponding left and right singular subspaces, resp., i.e., we have $A\, V_q = U_q\, \Sigma_q$, $A^T\, U_q = V_q\, \Sigma_q^T$. We are interested in improving approximations $\mathrm{im}\, X \approx \mathrm{im}\, U_q$, $\mathrm{im}\, Y \approx \mathrm{im}\, V_q$ defined by matrices $X \in \mathbb{R}^{m\times q}$, $Y \in \mathbb{R}^{n\times q}$ with $X^T X + Y^T Y = I_q$ and $X^T A\, Y + Y^T A^T X = S = \mathrm{diag}(s_1, \dots, s_q)$ by a rapidly convergent method. The last property can be achieved by a Rayleigh-Ritz step of dimension q. For doing so we apply the quadratically convergent, generalized Rayleigh-Ritz procedure proposed by Lösche, Schwetlick, and Timmermann to the symmetric matrix $C = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$. This requires per step to solve q bordered linear systems with matrices

$$\begin{bmatrix} -s_i I & A & X \\ A^T & -s_i I & Y \\ X^T & Y^T & 0 \end{bmatrix} \in \mathbb{R}^{(m+n+q)\times(m+n+q)}, \quad i = 1, \dots, q,$$

followed by a q-dimensional Rayleigh-Ritz process.