Condition Numbers – a Geometric Approach

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Abstract

For a nonsingular matrix A, the classical definition of a "condition number"

$$\kappa(A) := \|A^{-1}\| \cdot \|A\|.$$

is usually introduced in the context of perturbation theory for linear equations $Ax = b$, but is frequently used in other contexts, like eigenvalue and SVD computations, as well.

Of course, matrix A has many different "condition numbers", depending on the problem and the type of perturbations being considered. Furthermore, many recent perturbation and accuracy results suggest that the traditional notion of a single "condition number" should be extended and revised to provide a deeper insight into the sensitivity of computations in the floating–point arithmetic environment.

On the other hand, the accumulated work and experience in the last 10 (or, perhaps, more than 40) years, show that relative gaps and angles are important factors, while absolute scales (like norms) are not (as) important for relative accuracy. This indicates that a geometric approach should be quite appropriate and natural in consideration of "condition numbers".

Therefore, we go back to the basic principles and review the fundamental definition of condition numbers, from a geometric perspective. In this framework, some of the recent results can be naturally interpreted in terms of matrix volumes. Finally, we discuss some possible extensions to the indefinite product spaces with applications to the indefinite matrix factorizations.

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