Perturbation theory for generalized Lyapunov equations

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Perturbation theory for generalized Lyapunov equations $E^T X A + A^T X E = G$ is presented. It is well-known that the reciprocal of $\mathrm{Sep}(E, A) = \min\{\|E^T X A + A^T X E\|_F \ : \ \|X\|_F = 1\}$ is an important quantity to measure the sensitivity of solutions of generalized Lyapunov equations to perturbations in the data. However, in the case of singular E we have that $\mathrm{Sep}^{-1}(E, A) = \infty$.

We consider the constrained generalized Lyapunov equation $E^T X A + A^T X E = -P_r^T G P_r$, $X = X P_l$, where P_r and P_l are the spectral projections onto the right and left deflating subspaces of the matrix pencil $\lambda E - A$ corresponding to the finite eigenvalues. This equation arises in control problems and stability analysis for descriptor systems. The constrained generalized Lyapunov equation has a unique solution if and only if all finite eigenvalues of $\lambda E - A$ lie in the open half-plane. We present a spectral norm based condition number that may be used as a measure of the sensitivity of the solution of this equation. A generalized Schur-Bartels-Stewart method for solving the constrained generalized Lyapunov equation is presented and rounding error analysis for this method is discussed.