Stability of Structured Hamiltonian Eigensolvers

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Various applications give rise to eigenvalue problems for which the matrices are Hamiltonian or skew-Hamiltonian and also symmetric or skew-symmetric. We define structured backward errors that are useful for testing the stability of numerical methods for the solution of these four classes of structured eigenproblems. We introduce the symplectic quasi-QR factorisation and show that for three of the classes, it enables the structured backward error to be efficiently computed. We also give a detailed rounding error analysis of some recently developed Jacobi-like algorithms of Faßbender, Mackey & Mackey for these eigenproblems. Based on the direct solution of 4×4 , and in one case 8×8 , structured subproblems these algorithms produce a complete basis of symplectic orthogonal eigenvectors. We prove that, when the rotations are implemented using suitable formulae, the algorithms are strongly backward stable and we show that the QR algorithm does not have this desirable property.

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