Laser-Doppler velocity profile sensor with submicrometer spatial resolution that employs fiber optics and a diffractive lens

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We report a novel laser-Doppler velocity profile sensor for microfluidic and nanofluidic applications and turbulence research. The sensor’s design is based on wavelength-division multiplexing. The high dispersion of a diffractive lens is used to generate a measurement volume with convergent and divergent interference fringes by means of two laser wavelengths. Evaluation of the scattered light from tracers allows velocity gradients to be measured in flows with submicrometer spatial resolution inside a measurement volume of 700-μm length. Using diffraction optics and fiber optics, we achieved a miniaturized and robust velocity profile sensor for highly resolved velocity measurements. © 2005 Optical Society of America

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1. Introduction

For an increasing number of applications in the field of fluid mechanics, velocity measurement techniques with high spatial resolution are in demand. In turbulence research, velocity profile measurements down to the Kolmogorov microscale are required. In the field of biomedicine, measurements of flow through stents and blood pumps are of special interest. In the emerging field of microfluidics and nanofluidics there is a demand for flow measurements in miniaturized components such as micromixers, micronozzles, lab-on-a-chip applications, injection nozzles, etc.

Optical measurement techniques permit noninvasive investigations of flows. Naqwi and Reynolds and Naqwi et al. have developed an optical sensor that can measure near-wall velocity gradients by means of cylindrical laser waves generated by a double-slit configuration. However, this method is restricted to linear velocity gradients. A well-established technique for local velocity measurements with high precision is laser-Doppler anemometry (LDA). Often a differential Doppler technique is applied: Two coherent laser beams are brought to an intersection, such that in the intersection volume an interference fringe system with nearly constant fringe spacing \( d \) is generated. Small tracer particles, which pass through this fringe system, scatter light that is amplitude modulated with Doppler frequency \( f = \nu / d \), where \( \nu \) is the velocity component of the tracer particles perpendicular to the fringes. The spatial resolution is determined by the size of the measurement volume, i.e., by the extent of the interference fringe system, which is usually approximately 0.1 mm × 0.1 mm × 1 mm. A resolution of \(~50 \, \mu m\) can be achieved by strong focusing of the laser beams or by the use of beam stops in the receiving optics. However, a further increase in the spatial resolution is critical, as it will worsen the precision of velocity measurement. This increase is caused by the accompanying wave-front distortion that is due to self-diffraction of the (Gaussian) laser beam.

To overcome these limitations we have developed a laser-Doppler velocity profile sensor that is able to measure the position of a passing tracer particle in the measurement volume. The sensor design is based on the generation of two superposed interference fringe systems, one with convergent and the other with divergent fringes, by means of two wavelengths. Because the local fringe spacings are different for the two wavelengths, the Doppler frequencies...
are also different. The ratio of these two frequencies is characteristic for the position at which the particle passes the bichromatic interference fringe system, and therefore the position can be determined. A spatial resolution of 1.6 μm was demonstrated, and the sensor could successfully be applied for measuring boundary layers in fluid flows. Unlike the sensor of Naqwi, this method does not assume any velocity distribution but can be applied to flows with arbitrary velocity profiles.

The special arrangement of two superposed interference fringe systems with convergent and divergent fringes requires that the beam waist of one wavelength be placed before the crossing plane of the partial beams, and the beam waist of the other wavelength be placed behind the crossing plane. So far this has been done by use of two laser diodes of different wavelengths with separate collimation optics. The two beams were collinearly superposed by means of a dichroic mirror at 45° incidence. However, this rectangular arrangement as well as the necessity to provide each diode with individual x-, y-, and z-adjustment devices results in a relatively large measurement head and, therefore, strongly restricts the possibility for miniaturization. Furthermore, during transport or measurement, vibration may cause a lateral shift of the superposed fringe systems such that the necessary coincidence measurement is inhibited.

To make the sensor more compact and less sensitive, one can employ fiber-optics beam delivery. The advantages of fiber-optic setups, such as high flexibility and immunity to electromagnetic disturbances, have long been recognized and offer great potential and immunity to electromagnetic disturbances, advantages of fiber-optic setups, such as high flexibility, one can employ fiber-optics beam delivery. The setup and characterize the sensor. Section 5 summarizes the results.

This paper is organized as follows: In Section 2 we briefly recapitulate the method of operation of the velocity profile sensor. In Section 3 the new arrangement is described, and its requirements are discussed in detail. In Section 4 we report the experimental setup and characterize the sensor. Section 5 summarizes the results.

2. Operation of the Velocity Profile Sensor

The velocity profile sensor uses two superposed interference fringe systems, one with convergent fringes and the other with divergent fringes; see Fig. 1. To distinguish the two interference fringe systems physically, we generate them by means of two different laser wavelengths (wavelength-division multiplexing). Because of the wave-front curvatures of the Gaussian beams employed, fringe spacing d is not constant but is a function of the position along the optical axis (z axis): \( d_{1,2} = d_{1,2}(z) \). These distorted fringe systems are usually unwanted and occur, e.g., if the beam waists of the Gaussian beam do not coincide with the central crossing plane of the laser beams. The Doppler frequencies \( f = v/d \) (v is the particle velocity; d is the fringe spacing), measured from the amplitude-modulated signals caused by light scattering from tracer particles at both fringe systems, are therefore a function of position as well: \( f_{1,2} = f_{1,2}(z) \). (For inclined particle trajectories the Doppler frequency becomes a function of time as well and, consequently, a specific position cannot be identified. In what follows, we assume that the particle passes perpendicularly to the z axis, i.e., that there is a vanishing \( v_z \) component.) The position of the tracer in the measurement volume can be determined by a calibration function, defined as

\[
q(z) = \frac{f_2(v, z)}{f_1(v, z)} = \frac{v/d_2(z)}{v/d_1(z)} = \frac{d_1(z)}{d_2(z)}.
\]  

Because this function is independent of v but varies along the optical axis, its inverse function \( z = z(q) \) can be used for determination of the position at which the tracer passes the measurement volume. The velocity

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**Fig. 1.** Schematic of the velocity profile sensor: Two (superposed) interference fringe systems are generated by means of two laser wavelengths, one with convex wave fronts that result in divergent fringes (top) and the other with concave wave fronts that result in convergent fringes (bottom).
is calculated by the Doppler frequencies and the local fringe spacing known from the tracer position:

\[ v(z) = f_1(v, z)d_1(z) = f_2(v, z)d_2(z). \] (2)

A necessary prerequisite for the determination of the position is two different fringe spacing functions \( d_{1,2}(z) \) with a monotonically changing quotient function \( q(z) \). The uncertainty of the position measurement depends on the slope of the calibration curve and on the signal-to-noise ratio (SNR) of the measurement signals: The steeper the calibration curve, the more precise will be the determination of the position. For good performance, one fringe spacing curve has to be chosen to be increasing and the other to be decreasing to maximize the slope of the quotient function, \( q(z) \). This configuration can be achieved if the beam waist of one wavelength is place before the central crossing plane such that the wave fronts in the intersection volume are convex and a fringe system with divergent fringes results, (Fig. 1, top). The beam waist of the other wavelength is to be placed behind the crossing plane, such that the wave fronts in the intersection volume are concave and a convergent fringe system results (Fig. 1, bottom). Previously this positioning was achieved by individual adjustment of two individual collimation optics.6 The different beams were combined by a dichroic mirror and were supplied with adjustment devices to superpose the beams collinearly, which restricts the possibility for miniaturization.

3. Achievement of a Profile Sensor with Fiber-Optic Beam Delivery and a Diffractive Lens

For a good performance of the velocity profile sensor, the bichromatic measurement volume has to fulfill the following requirements:

(A) Because the sensor is based on coincidence measurements, the two interference fringe systems should be of comparable size and superpose perfectly.

(B) For the generation of convergent and divergent fringes, the beam waist of one wavelength is to be placed before the crossing plane of the partial beams, and the beam waist of the other wavelength is to be placed behind the crossing plane. In other words, a spatial separation of the beam waists along the optical axis is required.

For the purpose of miniaturization, fiber-optic beam delivery can be employed, so both wavelengths are delivered by one single fiber. The emitted beams are superposed \textit{a priori} without additional effort. Consequently, fine adjustment devices are not necessary and the setup can be built significantly simpler and smaller. The beam waists of both wavelengths are located at the fiber end face, which does not fulfill requirement (A) for spatial separation of the beams waists needed for the profile sensor. To separate the beam waists along the optical axis, the physical effect of dispersion can used. It has to be ensured that the individual interference fringe systems overlap (requirement (B)), which is possible only with dispersion-free imaging optics. By defined dispersion management it is possible to fulfill both requirements; see the sensor concept depicted in Fig. 2.

The spatial separation of the beam waists has to be made by the optics before the beam splitting element, which in this case is a grating, whose +1st and −1st diffraction orders are used. The grating is then placed in the center of the beam waists. To collimate and the partial beams and cause them to intersect behind the grating, an optic with zero dispersion is needed to place the crossing planes of both wavelengths at the same position such that the measurement volumes coincide. The optics images the beam waists and the source point of the partial beams to the same beam waists and crossing point of the partial beams.

For the choice of the lenses it is useful to take a closer look at imaging elements. Optical elements can generally be subdivided into three classes:

(A) \textit{Reflective optics} do not show dispersion in any way. However, they change the direction of the optical axis and are sensitive to alignment. They are suitable for folded beam paths.

(B) \textit{Refractive optics} exhibit moderate dispersion, which is caused only by material dispersion of the glass used. By combining lenses of different materials, one can design achromats with significantly reduced chromatic aberration.

(C) \textit{Diffractive optical elements (DOEs)} have a structure that is of the same order of magnitude as the wavelength employed, so diffraction becomes the dominating effect. A DOE alters the local intensity and phase of an incident laser beam and is described by complex transmission function \( T(x, y) \). To derive the intensity distribution behind the DOE, one has to apply the scalar diffraction theory. For details we refer the reader to Ref. 13. In the near-infrared wavelength region DOEs have a dispersion approximately 30 times higher than those of refractive optics. See Fig. 3 for a comparison of the focal lengths.

For the required separation of the beam waists the chromatic aberration of refractive lenses turned out not to be sufficient. To achieve a significantly higher separation we employ a diffractive lens that exhibits strong chromatic aberration. This aberration is usually a disadvantage, but here it is the intended effect. A diffractive lens consists of an amplitude or a phase
structure of concentric rings. Its focal length is given by

\[ f(\lambda) = \frac{R_0^2}{2\lambda}, \quad (3) \]

where \( R_0 \) denotes the radius of the central ring and \( \lambda \) is the laser wavelength. Obviously the focal length depends strongly on the wavelength, which is a consequence of the diffraction.

The diffractive lens used for the experiments was a special design with design parameters as listed in Table 1.

To maximize the optical power in the measurement volume, high efficiency of the diffractive lens is needed. Low efficiency will result in a poor SNR of the burst signals and therefore in an increased error of the Doppler frequency estimation. Hence to get the desired efficiency we designed the lens across the central area with four phase levels. The outermost zones are structured with only two levels. To explain this reduction, we treat the working principles of a diffractive lens. To produce discrete multiples of the wavelength for a delay in propagation to the desired focus, the lens consist of zones of decreasing width from the origin to the margin. This width cannot drop below the minimum feature size of the fabrication process used, which is \( \lambda / 1.5 \) m. If the four-level structure reaches this limit there must be a change to a two-level structure. Thus the lens consists of a four-level structure for a radius of less than 3.3 mm; the last distance until the margin of the lens is fabricated with two levels only. The total diffraction efficiency is then given by the weight of two areas, \( A_I \) and \( A_{II} \), and their corresponding efficiencies:

\[ \eta_{ges} = \frac{\eta_I A_I + \eta_{II} A_{II}}{A_{ges}} = 69\%, \quad (4) \]

where the efficiency of a two-level structure is 0.405 and that of a four-level structure is 0.811. The two surfaces of the glass substrate cause additional losses:

\[ \eta_T = T^2 = \left( \frac{4\eta_{Air}\eta_{SiO_2}}{\eta_{Air} + \eta_{SiO_2}} \right)^2 = 93\%. \quad (5) \]

So one can expect a total efficiency of 64%. The measured value was \( \sim 62\% \) and fits the calculation quite well. This result indicates that there are no significant fabrication errors.

4. Experimental Setup and Results

The experimental setup is shown schematically in Fig. 4. The radiation of two fiber-coupled laser diodes with 660- and 825-nm wavelength was combined by a single-mode fiber coupler and guided via a single-mode fiber to the sensor head, which was arranged on a 30-cm-long rail. The wavelength and fiber parameters were chosen such that propagation of light occurred in single-mode operation for both wavelengths. For this reason the wavelength difference of the lasers should be minimized. The difference should be large enough that the spatial separation of the beam waists is sufficient and that the wavelengths can be clearly separated by a dichroic mirror without ambiguity. The wavelength difference of 165 nm is a good compromise between these requirements. In the sensor head an aspheric refractive lens was used to collimate the light and was followed by a diffractive lens to focus the beams. A transmission phase grating with a grating constant of 10 \( \mu \)m was placed in the center of the separated beam waists. The +1st and \(-1\)st diffraction orders were used as

![Fig. 4. Experimental schematic of the velocity profile sensor with fiber optics and a diffractive lens.](image)
the LDA partial beams; other diffraction orders were blocked by beam stops. A Keplerian telescope consisting of achromatic lenses of 50- and 60-mm focal length and designed for the near-infrared region was used to collimate the partial beams and make them intersect. In the sensor head, only the collimating lens behind the fiber was provided with x, y, and z adjustments; all other components were attached rigidly to the rail and needed no further adjustment. Scattered light from the measurement volume can be detected in either a backward [Fig. (4)] or a forward scattering arrangement. (To achieve directional discrimination, one can additionally introduce acousto-optic modulators into one partial beam path or allow them to act as the beam splitter to introduce a carrier frequency into the Doppler signal. This will, however, restrict the possibility for miniaturization.)

A calibration setup was used for signal generation. A 4-μm tungsten wire, acting as the scattering object, was fixed on an optical chopper and rotated with well-defined velocity and position through the measurement volume. As the distance from the wire to the chopper axis was significantly larger (several centimeters) than the extent of the interference fringe system, the movement of the wire can to a good approximation be considered linear. The receiving optics consisted of a single lens that coupled the light into a multimode fiber. After reflection from a dichroic mirror for wavelength separation, the light was guided onto two photodetectors. Signals were recorded by a PC with an analog-to-digital converter card. A LabView software program calculated a fast Fourier transform and determined the center frequency by fitting a Gaussian curve to the Doppler signal. This will, however, restrict the possibility for miniaturization.

We characterized the sensor by measuring the caustic curves about the measurement volume with a beam scanner; see Fig. 5. The results for the beam waist position, beam waist radius, and beam quality factor were $z_0 = 13.299 \text{ mm}$, $w_0 = 17.8 \mu\text{m}$, and $M^2 = 1.2$, respectively, for the red wavelength and $z_0 = 16.469 \text{ mm}$, $w_0 = 35.0 \mu\text{m}$, and $M^2 = 2.3$ for the infrared wavelength. Because of high dispersion of the diffractive lens, the beam waists of the two wavelengths are well separated by 3.2 mm. The long wavelength causes a smaller focal length, so the beam waist is located closer to the sensor (higher $z$ values). The diffractive lens has a focal length of 20 mm at 660 nm, so for 825 nm a focal length of 16 mm results, as determined by Eq. (3). This yields a spatial separation of the beam waists of 4.0 mm and is in good agreement with the measured value of 3.2 mm. The deviation to the theoretical value might be explained by the fact that, because of nonlinear imaging of the Gaussian beam, the beam waist position behind the second Keplerian telescope and, therefore, the separation depends also on the beam waist's position and diameter before the telescope. The measurement volumes have lateral diameters of $\sim 60 \mu\text{m}$ and $\sim 80 \mu\text{m}$, as can be seen from Fig. 5. With a fringe spacing of 6 μm the measurement volumes exhibit approximately 10 and 13 fringes for the 660-nm and 825-nm wavelengths, respectively. These numbers are high enough to ensure a precise Doppler frequency estimation by the fast Fourier transformation.

![Fig. 5](image1.png)  
**Fig. 5.** Caustic curves about the measurement volume. The beam waists of the 825-nm beams are located before the measurement volume, generating divergent fringes, whereas the beam waists of the 658-nm beams are located behind, generating convergent fringes.

![Fig. 6](image2.png)  
**Fig. 6.** Normalized amplitudes of the fast Fourier transform Doppler peak. A satisfactory superposition of the individual interference fringe system occurs with an effective length of the measurement volume of 700 μm.
opposite signs of the slope result for the fringe spacing functions, indicating convergent and divergent fringes. In the central plane of the measurement volume at \( z = 9.26 \) mm the curves cross. At this position the fringe spacing is achromatic and determined only by geometrical optics.\(^6,14\) Its value is 
\[
\frac{d}{H} = \frac{\beta g}{2}
\]
where \( g \) is the period of the diffraction grating and \( \beta = 60/50 \) is the magnification factor of the Keplerian telescope. The expected value of \( d = 60 \) \( \mu \)m is in excellent agreement with the measurement; see Fig. 7, top. Outside the central crossing plane the fringe spacing is affected by the wave-front curvature of the Gaussian beam. Calibration function \( q(z) = \frac{d_1(z)}{d_2(z)} \) is monotonically increasing, with a mean slope of 0.69 \( \text{mm}^{-1} \). This is the highest slope achieved to our knowledge, and therefore better spatial resolution can be expected. In a previous publication\(^6\) a spatial resolution of 1.6 \( \mu \)m at mean slope of the calibration curve of 0.23 \( \text{mm}^{-1} \) was reported. Consequently, a spatial resolution of 0.5 \( \mu \)m can be expected here if the same SNR is assumed.

The spatial resolution was tested with the same setup used for calibration. The wire rotated with fixed velocity and position through the measurement volume. A PC calculated the Doppler frequencies and the velocity and position by means of Eqs. (1) and (2). The measurement was repeated 20 times at each position, and the average and a 1σ standard deviation were calculated. The standard deviation defines the spatial resolution of the sensor.

Figure 8 shows the dependence of the standard deviation on position along the optical axis. In the center of the measurement volume the spatial resolution falls below 1 \( \mu \)m, with 650 \( \text{nm} \) as the lowest value. Toward the margins of the measurement volume the spatial resolution gets worse owing to the decreasing SNR of the burst signals but is below 5 \( \mu \)m throughout the entire measurement volume. The spatial resolution might be a little less in flow measurements with pointwise scattering particles; in that case an averaging along the \( y \) axis will be made, where a fringe spacing variation also exists. However, this effect is \( \sim 2 \) orders of magnitude smaller\(^11\) and can usually be neglected. A problem for the experimental determination is the mechanical stability of the chopper, which might not be significantly better than \( \sim 1 \) \( \mu \)m. Better submicrometer spatial resolution should be possible with a more stable calibration setup.

5. Summary
We have reported an extended laser-Doppler anemometer setup that allows the position of a tracer particle inside the measurement volume to be determined with submicrometer accuracy. The sensor uses two superposed interference fringe systems of different wavelength, one with convergent and the other with divergent fringes. The ratio of the Doppler frequencies from the individual fringe systems corresponds to the position at which a tracer particle passed the measurement volume.

In this paper we have demonstrated, for the first time to our knowledge, that a laser-Doppler velocity profile sensor can be achieved with fiber-optic beam delivery and a compact sensor head suitable for miniaturization. Both wavelengths (660- and 825-nm laser diode radiation) are delivered via one single-mode fiber to the sensor head. Because the beams of both wavelengths originate from the same fiber, the beams are \( a \) \( \text{priori} \) propagating collinearly through the sensor head, so only few adjustment devices are needed and the setup can be kept simple and compact.

Generation of superposed convergent and divergent fringe systems is achieved by a special arrangement of lenses with low and high dispersion. First, a highly dispersive diffractive lens is used to separate the beam waists of the two wavelengths along the
optical axis. Thereafter, a diffraction grating is placed for beam splitting. It is followed by an imaging optics with low dispersion (i.e., chromatic aberration) to make all partial beams intersect at one position and to form the measurement volume. Here the beam waist of the infrared beam is located before the crossing plane and an interference fringe system with divergent fringes is generated, whereas the beam waist of the red beam is located behind the crossing plane, generating convergent fringes. The slope of the calibration function is $0.69 \text{ mm}/11002$ and is therefore significantly higher than reported previously $0.23 \text{ mm}/11002$.

The theoretical spatial resolution can be estimated to be $500 \text{ nm}$. In the experiment a spatial resolution of $650 \text{ nm}$ could be demonstrated in the center of the measurement volume. For a miniaturization, all components can be integrated into a tube of $1\text{-cm}$ diameter and a few centimeters’ length. The dimensions of the sensor head will scale with the working distance required.

In conclusion, we have demonstrated that a laser-Doppler velocity profile sensor can be achieved by means of fiber-optic beam delivery with the two wavelengths guided through a single fiber to the sensor head. The sensor head can be made compact and is suitable for further miniaturization. The sensor is capable of measuring velocity profiles with high spatial resolution and therefore offers great potential for many applications to microfluidics and nanofluidics as well as for the investigation of turbulent flows.

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