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Talbot interferometer as a time filter

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Dedicated to Prof. Dr. Adolf Lohmann on the occasion of his 75th birthday

Abstract: The Talbot interferometer is analyzed with regards to its temporal characteristics. An expression for its impulse response is derived. The use of the Talbot interferometer as a time filter in optical communications systems is suggested.

Key words: Talbot effect – self-imaging – optical communications – code-division-multiplexing

1. Introduction

Short optical pulses with duration in the ps/fs regime are used for many purposes including optical communications, material processing and time-resolved imaging and spectroscopy, for example. Temporal filtering of optical signals may serve different purposes. One area is pulse shaping of fs/ps-pulses with the purpose to stretch, compress, or flatten a pulse, for example. Pulse compression to achieve very short optical pulses can be achieved, for example, by using the Gires-Tournois interferometer [1], which can be viewed as the reflective version of the Fabry-Perot interferometer, grating pairs [2] and prism pairs [3] making use of the dispersive behaviour of these devices. Froehly et al. [4] described a grating-based pulse shaper based on far-field diffraction which performs a time-to-space mapping and thus allows one to directly modify the frequency spectrum of the optical signal in the spatial domain. A number of authors have demonstrated the use of this device for various purposes [5–7].

All the pulse shaping devices mentioned above work at optical frequencies. However, there is an increasing interest in optical filtering devices that work at communications frequencies, typically in the range from 1 GHz to 1 THz. This interest is motivated by the fact that due to increased demand for bandwidth optical communications systems are currently being developed that operate at 40 Gb/s and 160 Gb/s. At these data rates, electronic filters cannot be used any more. However, for various purposes filtering of the time signals may be of interest. One specific application, that we would like to mention here, is the separation of frequency-multiplexed channels in optical CDMA systems [8].

Hence, it appears to be of interest to consider interferometers that operate in the sub-THz regime and at the same time offer a certain amount of design freedom in order to implement specific filter functions. Here, we suggest the use of the Talbot interferometer for this purpose. A Talbot interferometer consists of two diffraction gratings separated by a multiple or fractional of the Talbot distance. The Talbot interferometer is based on the self-imaging effect [9] using nearfield diffraction. As we will discuss in this paper, the Talbot interferometer can be operated as a time filter suitable for operation in the frequency range between 0.1 THz (or less) up to 10 THz. This is unlike the well-known Fourier-type grating interferometer which is useful only for subpicosecond pulses. In addition, the Talbot interferometer offers a certain amount of design flexibility by using specially designed computer-generated gratings.

This paper is organized as follows: in section 2, we shall give a brief summary of the properties of optical time filters. In section 3, the Talbot interferometer is discussed as a time filter. Section 4 gives a more general derivation of its impulse response and shows an experimental result. Concluding remarks follow in section 5.

2. Interferometers as optical time filters – brief summary

For the purpose of setting up the framework for our later considerations on the Talbot interferometer, it is useful to briefly summarize the general properties of optical interferometers as temporal filters. We consider here linear, time-invariant optical filters. They can be described as delay line filters with a finite impulse response (FIR) or an infinite impulse response (IIR). The Mach-Zehnder interferometer (MZI) and grating interferometers (GIs) are examples of optical FIRs. An example of an optical IIR filter is the Fabry-Perot interferometer (FPI).

Linear filters are characterized in the time domain by their impulse response, $h(t)$, and in the frequency domain by the transfer function, $H(\nu)$, which are Fourier-related:

$$H(\nu) = \int h(t) \exp(2\pi i \nu t)\, dt. \tag{1}$$

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Like many interferometers, the Talbot interferometer generates a discrete impulse response of the general form

$$h(t) = \sum_k h_k \, \delta(t - k\tau). \quad (2)$$

Here Σ_k denotes a sum over k. The characteristic time delay τ results from the different optical paths in the interferometer and the associated time delay. Hence, if we call the path length difference L, then $\tau = L/c$ (c – speed of light). For an FPI, for example, L is twice the cavity length. For a diffraction grating used in the far field (denoted here by FOU-GI) operating in the m-th diffraction order, L is given by the detour between the beamlets emerging from adjacent slits. For the m-th diffraction order, $L = m\lambda$ (λ – wavelength).

The inverse of the time delay τ, $\Delta\nu$, is called the free spectral range (FSR).

$$\Delta\nu = 1/\tau. \quad (3)$$

The FSR may be viewed as the "working range" in which the interferometer operates. In the case of a diffraction grating, for example, $\Delta\nu$ represents the separation of different diffraction orders in the time frequency domain. The spectral resolution, $\delta\nu$, is determined by the inverse of the maximum time delay, $\tau_{\max}$, generated by the filter:

$$\delta\nu = 1/\tau_{\max}. \quad (4)$$

Considering the case of a diffraction grating again, $\tau_{\max} = N\tau$ where N is the number of periods in the grating. $\delta\nu$ is the width of a single diffraction peak in the frequency domain.

Depending on the specific implementation of an interferometer, the path length differences can vary over many orders of magnitude. FPIs can be implemented, for example, with discrete mirrors or as very thin layers made of dielectric or semiconductor materials. Hence, L may take on values between a few wavelengths up to several meters which corresponds to values for the FSR between the order of 100 MHz to 100 THz. For a FOU-GI with $L = m\lambda$, the FSR $\Delta\nu$ is very large. For example, for $\lambda = 1.5$ μm and $m = 1$ the FSR is $\Delta\nu = 200$ THz.

FPI and MZI are filters which offer little "design flexibility". In terms of filter theory, the design flexibility can be expressed by the degree of the filter. For a non-recursive filter, the degree is given by the number of delta peaks in the impulse response. For the MZI, for example, the impulse response consist of two delta peaks where only the splitting ratio may be varied. Hence the transfer function is always sinusoidal. The same argument is valid for the FPI. This means, although these two interferometer types can be operated over a large frequency range, depending on their implementation, their design flexibility is limited. Differently for the FOU-GI: it offers a large amount of design freedom. By using computer-generated holograms, amplitude and phase of each beamlet can be modified individually [6]. However, due to its large FSR it is not useful for applications in the frequency range defined above, namely between 1 GHz and 1 THz.

3. Talbot interferometer as a delay line filter

Here, we consider the use of a Talbot interferometer as a temporal filter. Since it is based on near-field (or Fresnel) diffraction, we denote it by FRS-GI. The idea is to use the paths of the different diffraction orders between two gratings as the paths of a delay line filter. This is different from the FOU-GI [4] or the Treacy interferometer [2] which work within a single diffraction order. The output of the Treacy interferometer represents the combination of the +1st and −1st diffraction orders generated by the grating pair. The Talbot interferometer, on the other hand, uses the first grating as a $1 \times N$ beamsplitter and the second grating as a $N \times 1$ beam combiner.

The principle of the Talbot interferometer is shown in fig. 1. The first grating, G_1, with a period p and assumed to be located in $z = 0$, is illuminated by a plane wave. The wavefield $u(x, z)$ behind the grating is periodic in z-direction with a longitudinal periodicity $z_T = 2p^2/\lambda$: $u(x, z) = u(x, z + z_T)$. If G_1 is a phase grating, $u(x, z_T)$ will be constant in amplitude and only vary in its phase. Therefore, a second phase grating, G_2, located in $z = z_T$ with a phase-conjugate profile can be used by compensate the phase modulation of the wavefield. In this case, the wavefield behind the second grating, $u_2(x)$, is a plane wave traveling in direction of the zeroth order. In this configuration, one may view the Talbot interferometer as a "Talbot cavity" which was described for coherent beam combination earlier [10, 11].

Different design options exist: 1) The distance between the gratings may be an integer of the Talbot distance or a fractal thereof. 2) G_1 may be a phase grating or an amplitude grating. In the latter case, a wavefield with phase-only modulation occurs in certain fractal Talbot planes. This situation is reciprocal to the case of the Talbot array illuminators [12, 13]. 3) The gratings may be specially designed to generate a desired intensity distribution among the diffraction orders using well-known iterative design techniques. 4) The output may be anyone of the orders behind G_2.

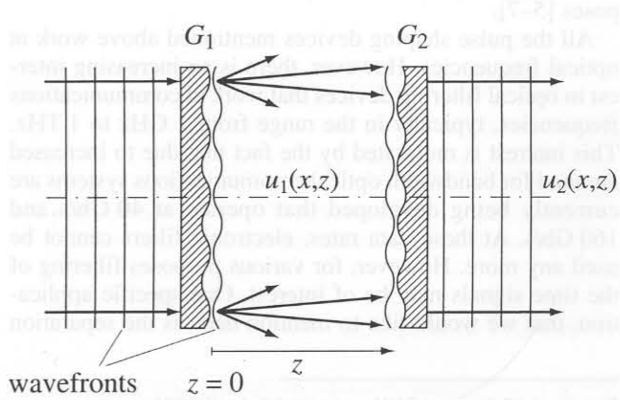

Fig. 1. **Talbot interferometer consisting of two gratings, G_1 and G_2. In the "resonant" case, the second grating combines all diffraction order generated by G_1 in the k-th output order (here: $k = 0$).**

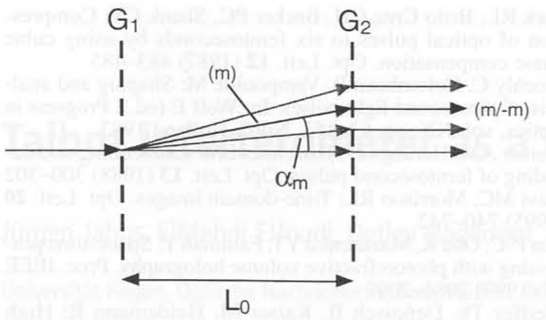

Fig. 2. Visualization of the paths of the diffraction orders. L_0 is the path of the zeroth order between the two gratings.

We now want to derive an expression for the impulse response of the Talbot interferometer for the case that G_1 is illuminated under normal incidence and that the output is given by the zeroth order behind grating G_2. First, we calculate the time delays τ_k which occur in eq. (2). The m-th order generated by G_1 and the m-th order generated by G_2 combine to contribute to that zeroth order (see fig. 2). The time delay τ_m between the 0-th and the m-th diffraction order can be calculated from the path length difference $\Delta L_m = L_m - L_0$ by a simple Pythagoras calculation and a Taylor series approximation. It is

$$L_m = L_0\,(1 + \tan^2\alpha_m)^{1/2} \approx L_0\,(1 + \sin^2\alpha_m)^{1/2}$$
$$= L_0\,[1 + (m\lambda/p)^2]^{1/2} \approx L_0\,[1 + (1/2)\,(m\lambda/p)^2]. \quad (5)$$

Now, we consider the specific case $L_0 = z_T = 2\,p^2/\lambda$ for which one obtains:

$$\Delta L_m = m^2\,\lambda \quad (6)$$

and hence

$$\tau_m \approx m^2\,\tau \quad \text{with} \quad \tau = \lambda/c. \quad (7)$$

Here, the quadratic dependence of τ_m on m should be noted. Representing the two gratings G_1 and G_2 by Fourier series:

$$g_1(x) = \Sigma_m\, g_m^{(1)} \exp(2\,\pi\, i\, m x/p) \quad (8)$$

and

$$g_2(x) = \Sigma_n\, g_n^{(2)} \exp(2\,\pi\, i\, n x/p) \quad (9)$$

we can write for the impulse response function:

$$h^{(0)}(t) = \Sigma_k\, g_k^{(1)}\, g_{-k}^{(2)}\, \delta(t - k^2\,\lambda/c). \quad (10)$$

We have added the superscript '0' to indicate that the output represents the 0-th order behind G_2.

4. General description

For a more general description the impulse response, we have to consider the wavefield behind grating G_2. A distance z behind G_1 the wavefield $u_1(x, z)$ is

$$u_1(x, z) = \Sigma_m\, g_m \exp(-2\,\pi\, i\, m^2\, z/z_T) \exp(2\,\pi\, i\, m x/p). \quad (11)$$

Behind G_2 the wavefield is given as:

$$\begin{aligned} u_2(x) &= u_1(x, z)\, g_2(x) \\ &= \Sigma_m\, \Sigma_n\, g_m^{(1)}\, g_n^{(2)} \exp(-2\,\pi\, i\, m^2\, z/z_T) \\ &\quad \cdot \exp(2\pi\, i\,(m-n)\, x/p] \\ &= \Sigma_k\, h_k \exp(2\,\pi\, i\, n x/p) \end{aligned} \quad (12)$$

with

$$h_k = \Sigma_m\, g_m^{(1)}\, g_{m-k}^{(2)} \exp(-2\,\pi\, i\, m^2\, z/z_T). \quad (13)$$

Here, we have introduced a new variable $k = m - n$. h_k describes the amplitudes of the k-th diffraction order behind G_2. Eq. (13) represents a discrete convolution of $g_m^{(1)'} = g_m^{(1)} \exp(-2\,\pi\, i\, m^2\, z/z_T)$ and $g_m^{(2)}$.

Now we consider the case of a fractional Talbot distance between G_1 and G_2, i.e. $z = (M/N)\, z_T$, and furthermore assume the output of the interferometer to be the k-th diffraction order. The impulse response then assumes the form:

$$\begin{aligned} h^{(k)}(t) = \Sigma_m\, g_m^{(1)}\, g_{m-k}^{(2)} \exp(-2\,\pi\, i\, m^2\, M/N) \\ \cdot\, \delta(t - (m-k)^2\, \tau). \end{aligned} \quad (14)$$

The shape of the impulse response is determined by the amplitudes of the diffraction orders. Here, it is of particular interest that one can design beam splitter gratings with a large variety of diffraction patterns. This has been exploited for the implementation of so-called Fourier-type array illuminators, see, for example, ref. [15].

We consider one specific example: $z = z_T$, $g_2(x) = u_1^*(x, z)$ and $k = 0$. In this specific case, it is $h^{(0)}(t) = \Sigma_k\, |g_k|^2\, \delta(t - k^2\,\lambda/c)$. This situation is demonstrated experimentally in fig. 3. Two binary phase gratings were used with an etch depth to cause a phase shift of π at the illumi-

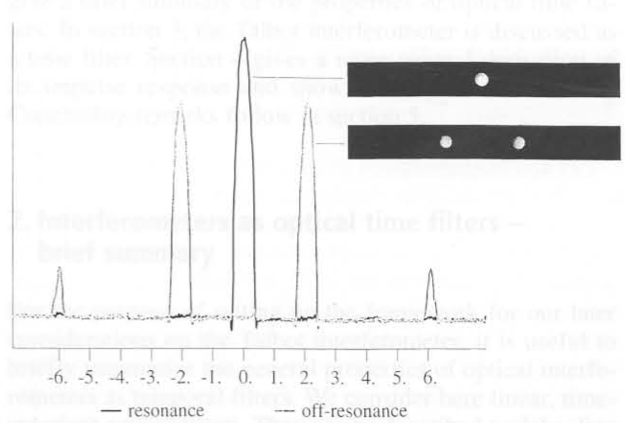

Fig. 3. Experimental results for Talbot cavity in resonance (solid line) and off-resonance (shaded line). The figure shows the diffraction patterns of the wavefield transmitted by G_2 as intensity plots. In the resonance case, the second grating matches exactly the wavefront generated by the first, so that in the ideal case, only the zeroth order is present behind G_2. For the experimental result shown here, the grating distance was chosen to be equal to the Talbot distance z_T. Furthermore, G_2 was had to be aligned in the lateral direction. In the off-resonance case, G_2 does not match the phase of the incoming wavefront and hence a variety of different diffraction orders occur. In the experiment, G_2 was moved out of the first Talbot plane.

nating wavelength $\lambda = 632.8$ nm. By properly adjusting both gratings ("resonance case"), one obtains a strong zeroth order behind G_2 as visualized by the figure. By moving either grating, the cavity goes "off resonance" and the light is distributed from the zeroth to higher orders.

5. Conclusion

We suggested and analyzed the use of the Talbot interferometer as a temporal filter. As compared to the widely intestigated FOU-GI, it does not require signals consisting of subpicosecond pulses and may therefore be interesting for communications systems in the sub-THz range. A disadvantage of the Talbot interferometer is the fact that the transfer function is not as straightforward to derive as for the Fourier-setup. However, using proper design it may be useful for address en- and decoding in optical CDMA systems. It may be implemented as a free-space optical and as waveguide device. A waveguide implementation could be based on the so-called "multimode-interference devices" [16] to yield very compact modules.

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