Random Walks, a Paradigm to design Distributed Algorithms for Dynamic Networks

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Outline

Plan

Framework study

Context

- Networks
- Distributed Systems (specially Dynamical networks like peer-to-peer or ad-hoc networks)

Constraints

- Management of dynamicity
- Fault tolerance
- Decentralized solutions

Motivation

Conception of solutions to classical problems of distributed algorithmic for dynamic networks

Distributed system

Model of distributed system

- Set of computing ressources that communicates through channels
- $(G = (V, E))$

Communication model :

Messages passing model

Distributed algorithms

Definition

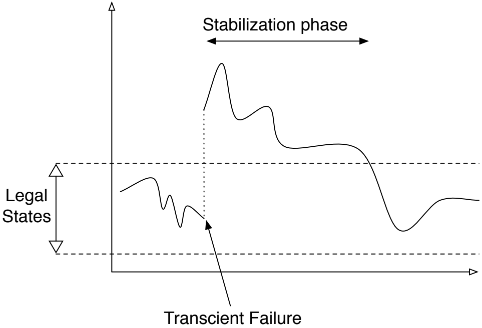
- Set of local algorithms
- Communication primitives (send, receive, ...)
- Local execution on each node

Fault Tolerance

Two approaches

- Robustness
  - Lots of impossibility results
  - Requires less restrictive assumptions
- **Self-stabilization**

Self-stabilization [Dijk74]



Two properties for self-stabilizing alogorithms

- **Convergence :** starting from an illegal state and without failures, the algorithm converges to a legal state
- **Closure :** from a legal state and without failures occurences, the algorithm remains in legal states

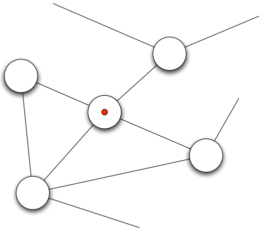
Plan

1. Introduction
   - Main context
   - Tools
2. Random walks caracteristics evaluation
3. Fault tolerant Token Circulation
   - Loss of token
   - Corruption of token
   - Duplication of token
   - Application to ad-hoc networks
4. Network decomposition
   - Motivations
   - Informal description
   - Main mecanisms
5. Futur works

Random walks

- Random moves in a graph
- Algorithmic procedure :

  Reception of the token on the site i
  Choose j uniformly at random among $Neigh(i)$
  Send token to site j



Properties and mains caracteristics

Percussion

On a finite graph, a random walk hits a node **in a finite time**. The mean time starting at a node i to hit node j is $h(i,j)$.

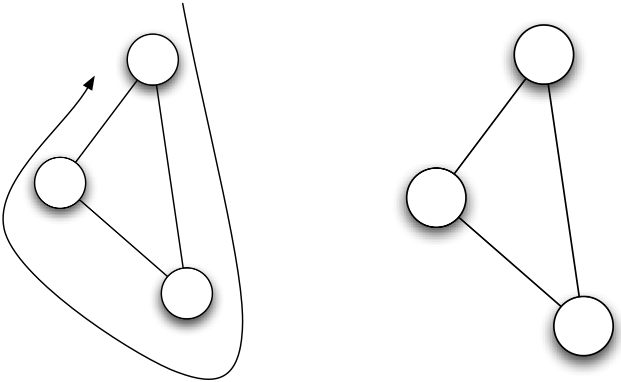
Coverage

On a finite graph, a random walk hits all nodes **in a finite time**. The mean time to visit all nodes is C.

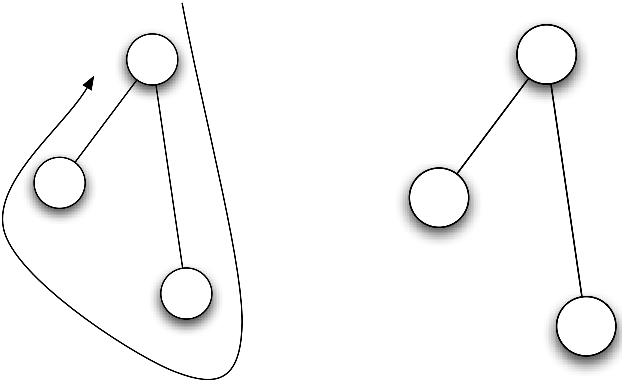
Meeting

On a finite and non-bipartite graph, two random walks meet on the same node **in a finite time**. The mean time before two random walks meet is $Me(x_1, x_2)$.

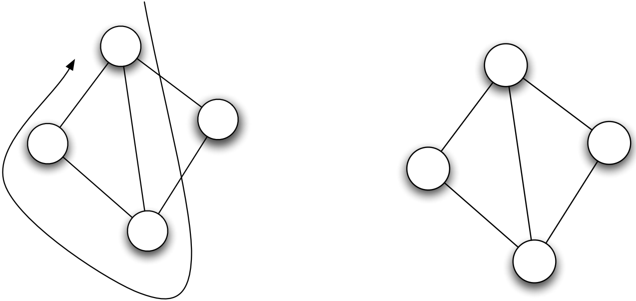
Adaptativity



Adaptativity



Adaptativity



Circulating word

Definition

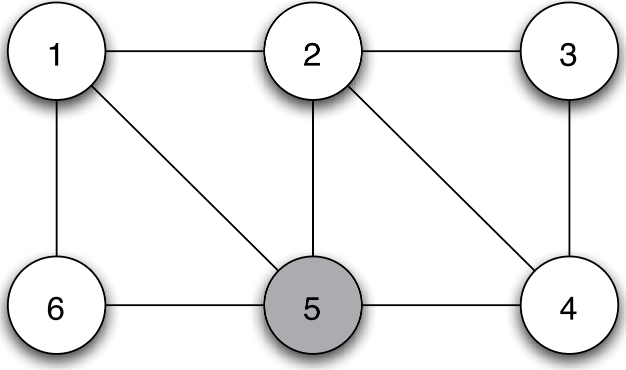
A circulating word is a message that collects information through its moves in the network.

[Lava86]

Circulating word + random walk

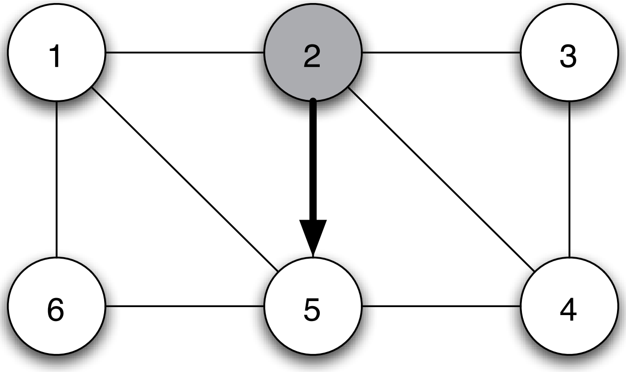
= Tools to collect and broadcast information.

Spanning tree construction



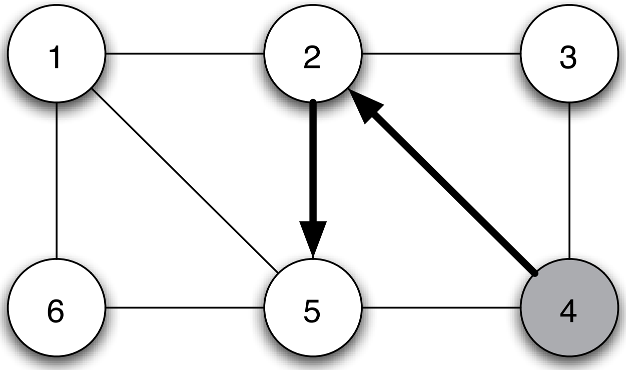
M = {5}

Spanning tree construction



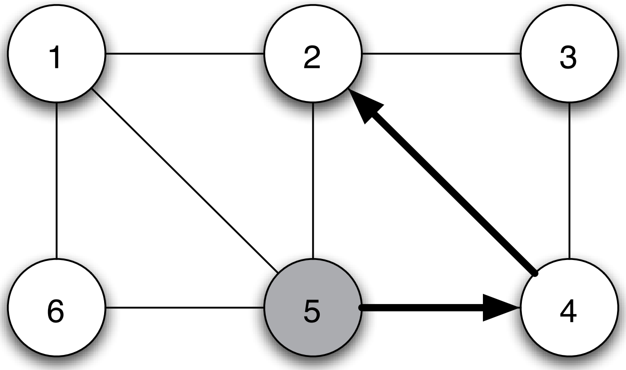
M = {2,5}

Spanning tree construction



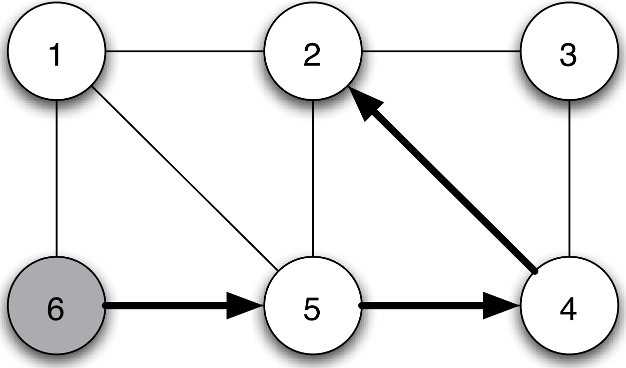
M = {4,2,5}

Spanning tree construction



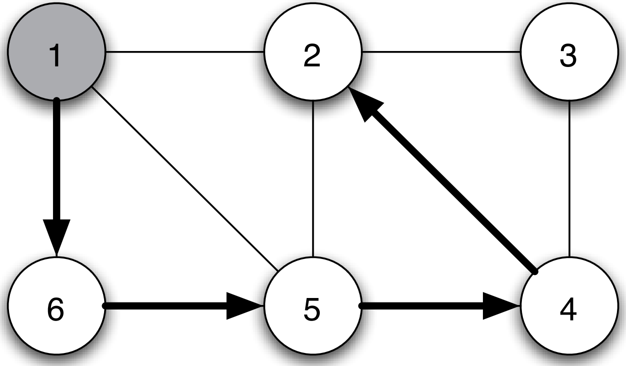
M = {5,4,2,5}

Spanning tree construction



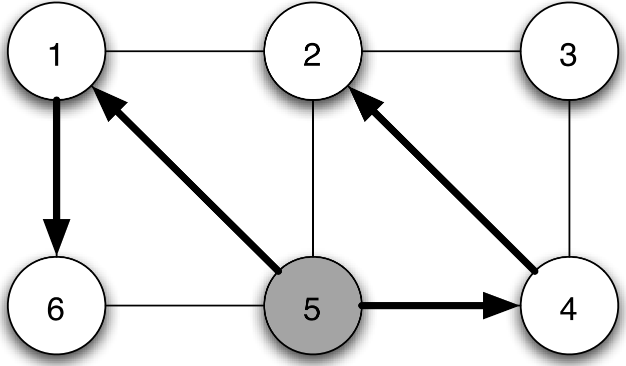
M = {6,5,4,2,5}

Spanning tree construction



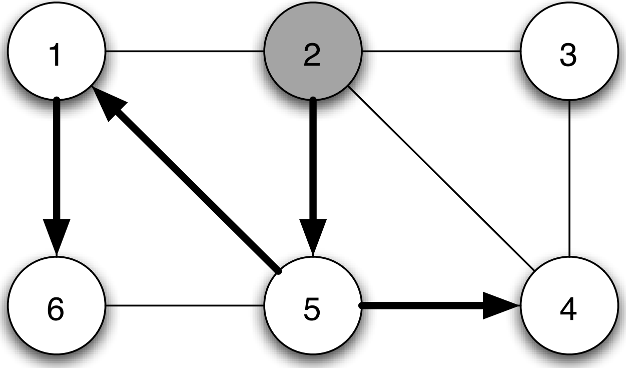
M = {1,6,5,4,2,5}

Spanning tree construction



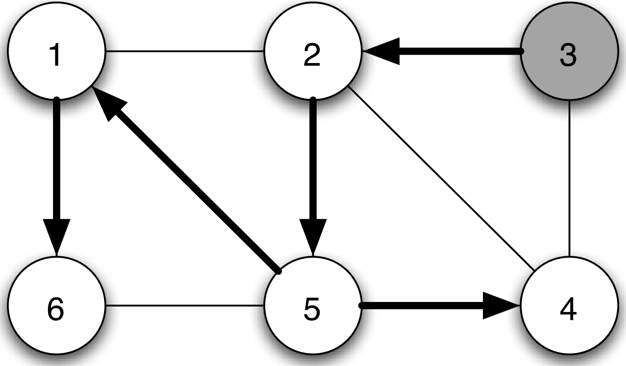
M = {5,1,6,5,4,2,5}

Spanning tree construction



M = {2,5,1,6,5,4,2,5}

Spanning tree construction



M = {3,2,5,1,6,5,4,2,5}

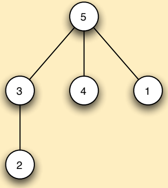
Adaptativity of the spanning tree

Circulating word moves permanently in the network
$\Longrightarrow$ topological information are alway updated

$\Longrightarrow$ Spanning tree is adaptive

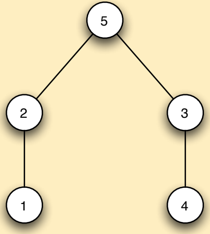
At time T

$$M_T = \{5, 3, 2, 5, 4, 5, 1\}$$



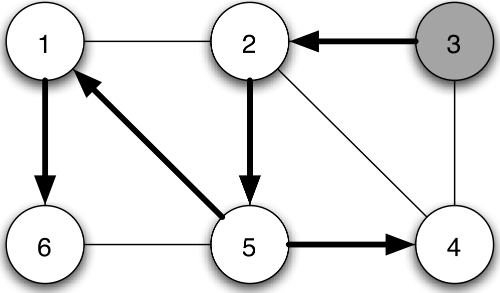
At time $T + \Delta$

$$M_{T+\Delta} = \{5, 2, 1, 5, 3, 4, 5, 3, 2, 5, 4, 5, 1\}$$



Reduction of circulating word size

$$M = \{3, 2, 5, 1, 6, 5, 4, 2, 5\}$$



1	2	3	4	5	6
5	3	3	5	2	1

Circulating word size fixed at n

Plan

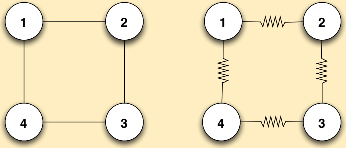
Computation of RW caracteristics

Two approaches

- Markov chains
- **Analogy with electrical networks [DoSn84]**

Results from [CRST+97,Tet91]

- $h(i,j) = m \times R(i,j) + \frac{1}{2} \sum_{k \in V} \deg(k) \times (R(j,k) - R(i,k))$
- $m \times R \leq C \leq O(m \times R \times \log n)$



Computation of RW caracteristics $\Longrightarrow$ Computation of $R(i,j)$

Electricity laws

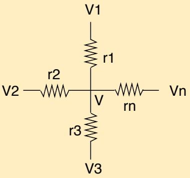
Ohm's law

$$U_{AB} = R(A, B) \times i_{AB}$$

Kirchhoff's law

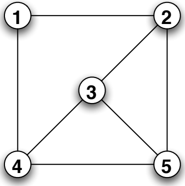
The sum of ingoing currents equals the sum of outgoing currents.

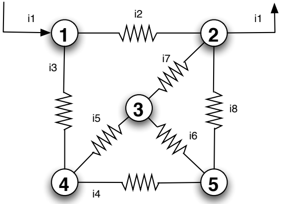
Millman's theorem



$$\frac{\frac{V - V_1}{r_1} + \frac{V - V_2}{r_2} + \cdots + \frac{V - V_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}} = 0$$

Computation of $R(1,2)$



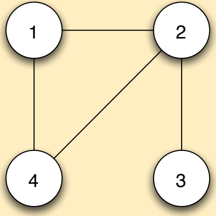


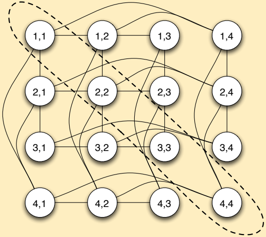
$$\left\{\begin{array}{lllll} V_1 & & & & = 1 \\ & V_2 & & & = 0 \\ & -V_2 & +3V_3 & -V_4 & -V_5 = 0 \\ -V_1 & & -V_3 & +3V_4 & -V_5 = 0 \\ & -V_2 & -V_3 & -V_4 & +3V_5 = 0 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} V_1 = 1 \\ V_2 = 0 \\ V_3 = \frac{1}{4} \\ V_4 = \frac{1}{2} \\ V_5 = \frac{1}{4} \end{array}\right.$$

$$\Longrightarrow \left\{\begin{array}{lll} i2 = & V_1 - V_2 & = 1 \\ i3 = & V_1 - V_4 & = \frac{1}{2} \\ i1 = & i2 + i3 & = \frac{3}{2} \end{array}\right. \Longrightarrow R(1,2) = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Computation of meeting time

Case of 2 RW





Plan

Token circulation

Primitive for :

- Election
- Efficient broadcast information
- Ressources allocation
- Structures Maintenance
- ...

Existing solutions

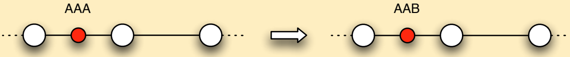
Based on the construction and maintenance of a virtual spanning structure [ChWe02]

Communication failures

Self-stabilization in messages passing model

- loss of token

- Corruption of token



- Duplication of token

Plan

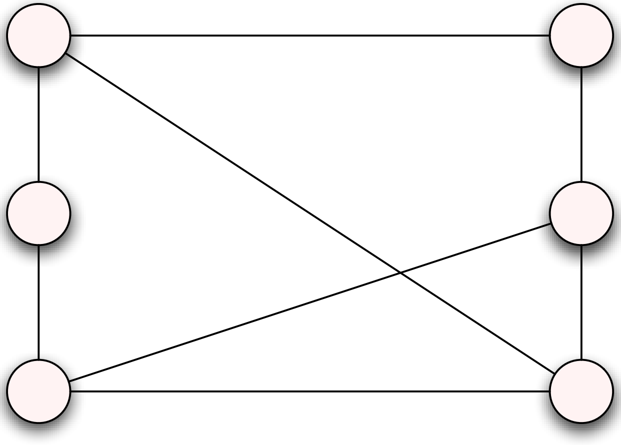
Loss of token

Communication deadlock

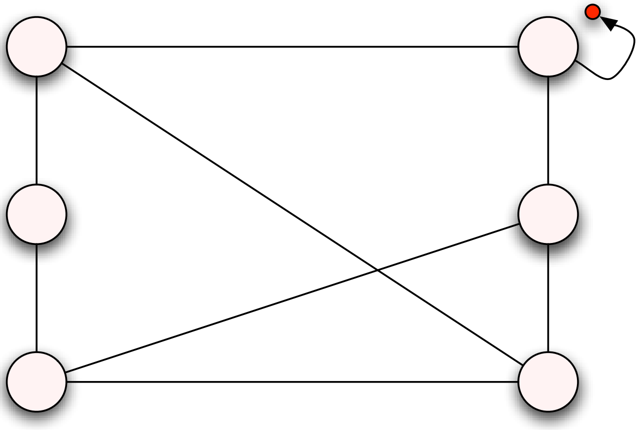
Solution : Timeout

- At the reception of the token, a node reset its timeout
- If the timeout trigger, its node produce a new token

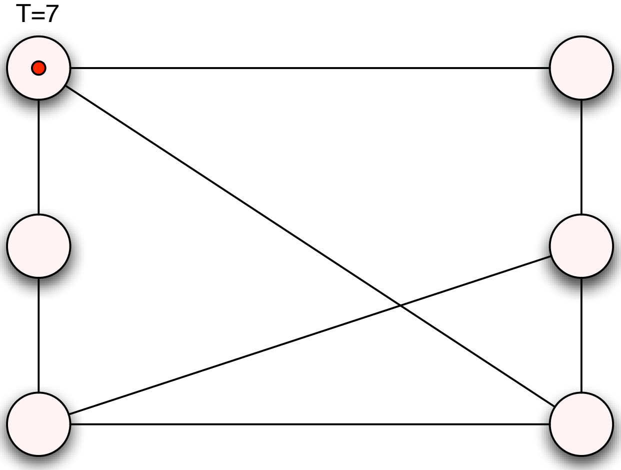
Timeout



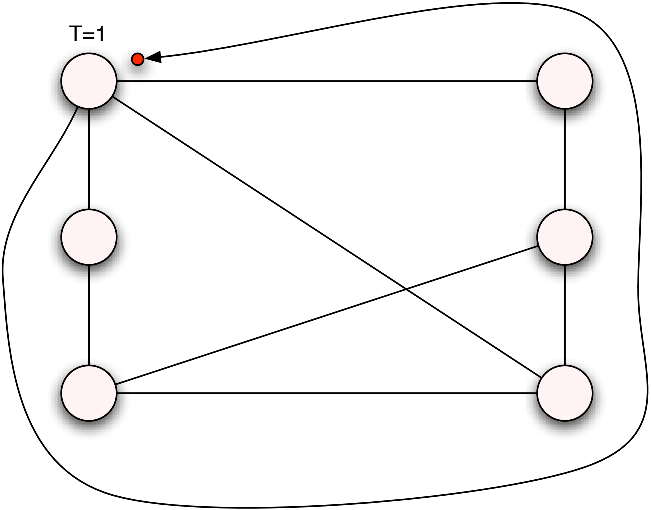
Timeout



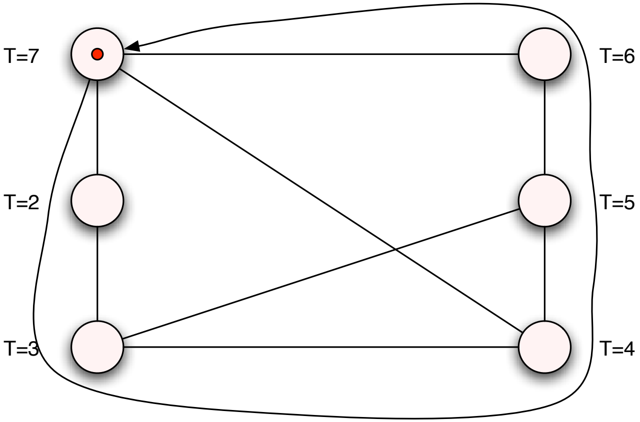
Timeout



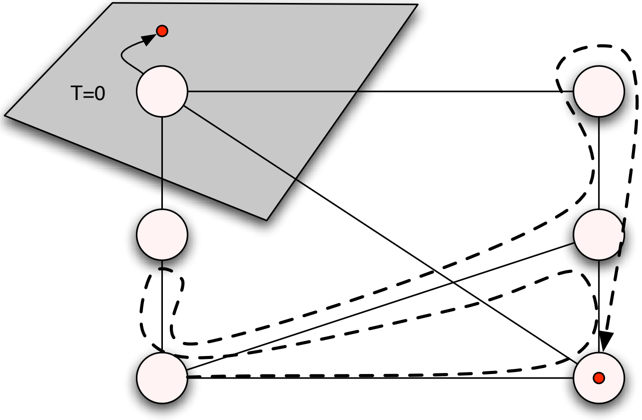
Timeout



Timeout



Timeout



Timeout

Random walk policy as token's moves

[DoSW02] : $T = C \times i$

$\Longrightarrow$ Token creation still possible

There is no bound on the visiting time for a given node.

Reloading Wave

Our approach : a solution decided by the token

Broadcast a reset timeout order to all nodes

Principle

- A nodes receiving this wave, reset its timeout and continue the propagation.
- The wave is propagated periodically through an adaptive spanning tree stored in a circulating word inside the token

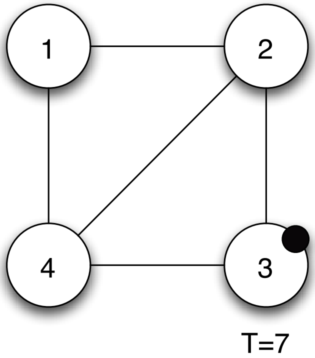
When propagate the wave ?

Remarks :

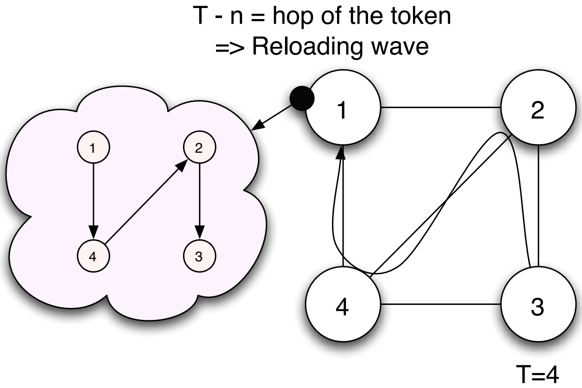
- After a visit, a node produces a new token in T time units
- The propagation of the wave takes in the worst case n time units

The token should propagate the reloading wave each $T - n$ time units

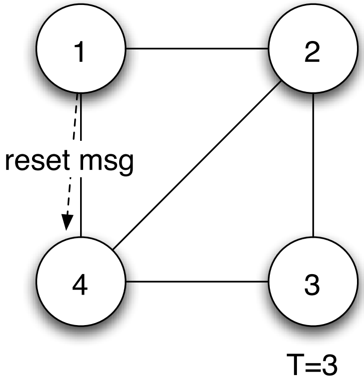
Example



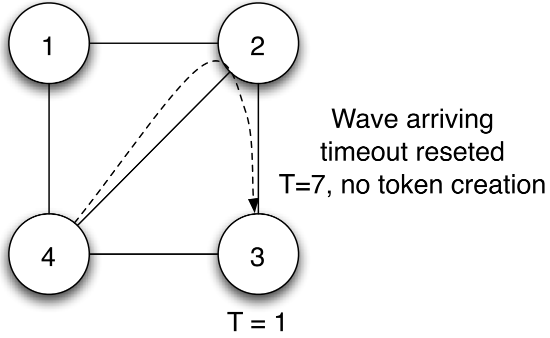
Example



Example



Example

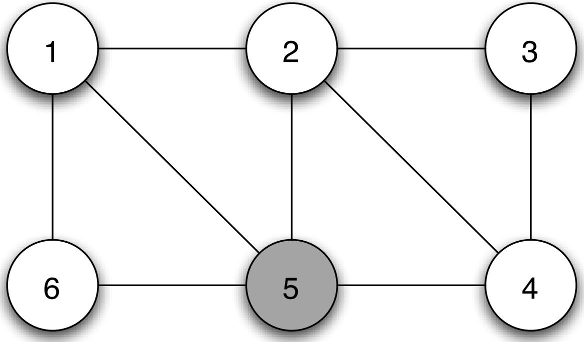


Plan

Corruption of token

Solution

Test realized locally on each visited nodes.



M = {5,1,6,5,3,2,5}

Here $3 \notin \mathit{Neigh}(5)$, node 5 corrects the word : $M = \{5, 1, 6, 5\}$.

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   - **Duplication of token**
   - Application to ad-hoc networks
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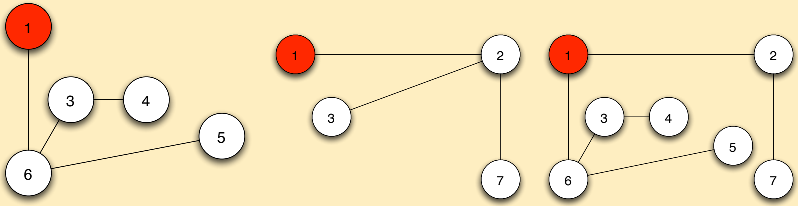
Duplication of token

A solution [IsJa90] (state model)

Meeting property

Mergure of topological information



Scheme of proof

Legal state $\mathcal{EL}$

- One complete and consitent token $\mathcal{JUCC}$
- All nodes have been visited $\mathcal{TSV}$

Lemmas

- A visited node can not produce new tokens
- The number of visited nodes increases
- All nodes become visited nodes : $\mathcal{C} \longrightarrow \mathcal{TSV}$
- All tokens become consistent
- There exist at least one complete token
- For all configurations satisfying $\mathcal{C} \in \mathcal{TSV}, \mathcal{C} \longrightarrow \mathcal{JUCC}$

Theorem

$$\forall \mathcal{C}, \mathcal{C} \longrightarrow \mathcal{EL}$$

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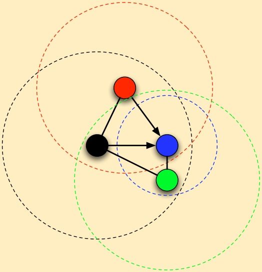
Model

Communication model

- Nodes communicate through radio waves of different range
- Oriented graph



Mobility

- Dynamic graph
- Nodes moves are a natural behavior
- Alternative solutions to flooding

Adaptation of circulating word management

Permanent algorithm $\Longrightarrow$ size growths infinitely

Our reduction technique

- based on usefull information.
- Allows to renew topological information used.

Adaptation of circulating word management

Remarks

The word $M =< 5, 4, 1, 3, 1, 2, 3, 4, 2, 5, 1 >$ allows the construction of spanning tree enrooted on an arbitrary node

$$\underbrace{5, 4, 1, 3, 1, 2, 3, 4, 2, 5}_{Constructor\ Cycle}, 1$$

$\Longrightarrow$ our algorithm maintains a constructor cycle

Definition

A cycle $\mathcal{C}(i, j)$ in the word M is called *constructor* if :

$$\forall k \in identities(M), \exists l \in \{i, \ldots, j\} | M[l] = k$$

Results

Our algorithm

- maintains an adaptive image of the network
- manages connexions and disconnexions
- bounds the size of the circulating word

Lemma

The size of the circulating word is bounded by $\frac{n^2+8n}{4}$

Simulations results

Size of the network	10	100	500
Averrage size of the word	17	225	1342
Deviation	1,9	20	192

k exclusion for ad-hoc network

k exclusion

- Extension of mutual exclusion
- At most k nodes can get critical section at a given time

k distinct tokens will circulate (Set of colors $\mathcal{K}$)

Convergence

- Preliminary phase : deletion of corrupted tokens
- Production phase : produce at least one token by colors of $\mathcal{K}$
- Mergure phase : deletion of duplicated tokens

Mobility assumption

Self-stabilizing if :

An edge chosen to belong to the spanning tree should permit the propagation of the reloading wave during the time it belongs to this tree.

Plan

Motivation

Limitation of Random Walks

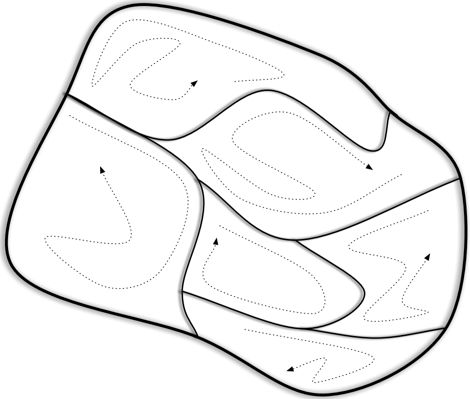
- In the most general case, Bounds on hitting, cover and meeting time are in $O(n^3)$
- Acceptable in theory but not for practical application.

Differents kinds of solutions (depending of the task to achieve)

- Increasing the number of random walks
- Build and maintain a hierarchical structure over the network

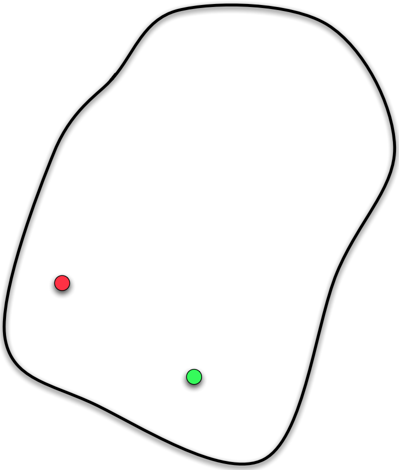
Plan

Goal

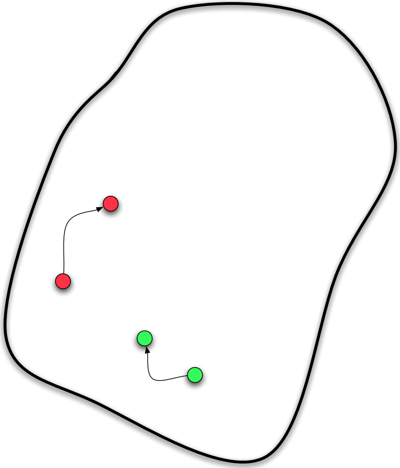


Get a decomposition of the network into partition. Each of them should have a number of node comprised between m_{min} and m_{max}.

Goal



Goal



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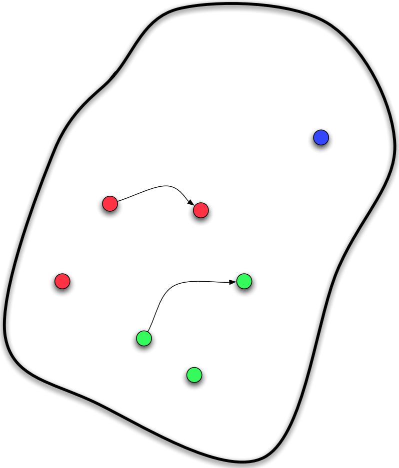

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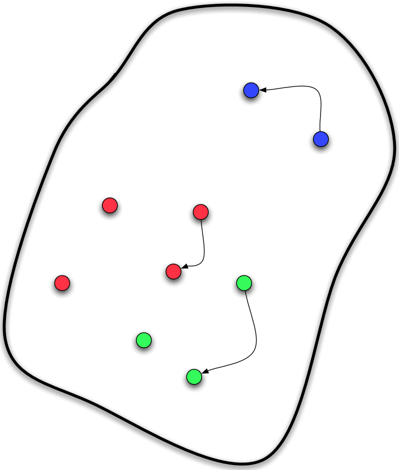

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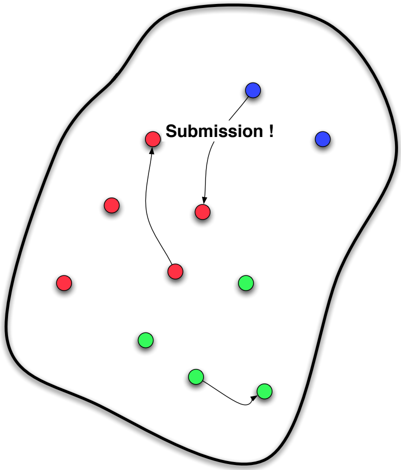
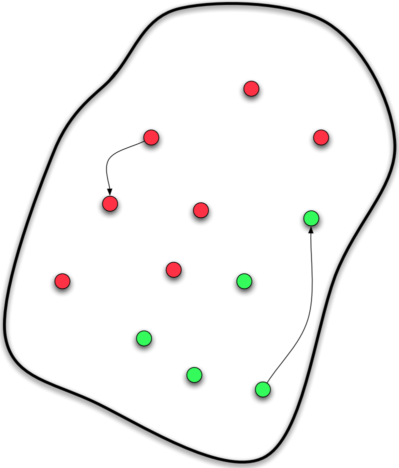


Goal



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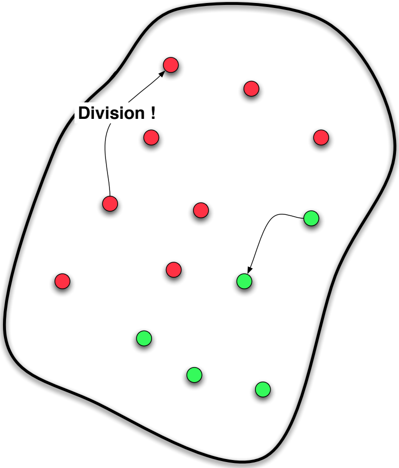
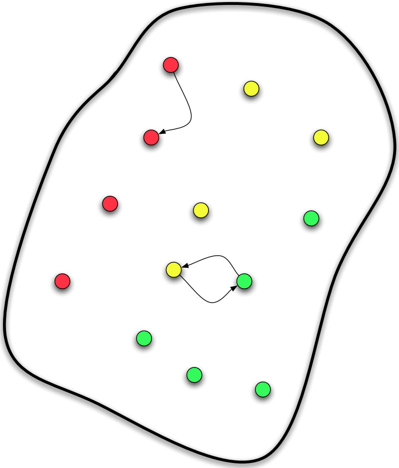


Goal



Plan

Annexion

Condition

The local random walk annexes a new node and

$$Nb\_nodes \leq m_{max}$$

Operations

- Mark the new node with the LRW color.
- Udpate LRW information

Division

Condition

The local random walk annexes a new node and

$$Nb\_nodes > m_{max}$$

Operations

- Udpate LRW information
- Send a wave on the spanning structure to split it into two parts

Submission

Condition

The local random walk visits another partition and

$$Nb\_nodes < m_{min}$$

Operations

- Kill the LRW
- Propagate a wave through the partition to change the color partition

Plan

Futur works

Adaptive hierarchization

- Formal proof of the algorithm
- Self-stabilizing behaviour ?

Experimentations

- Topology of the partitions created
- Stability of the partition regarding the mobility of the network

Applications

Hybridation of random walks and *local flooding* for ad-hoc networks

Random Walks, a Paradigm to design Distributed Algorithms for Dynamic Networks

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