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Editorial

Fuzzy set theory in medicine

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> Indeed, the complexity of biological systems may force us to alter in radical ways our traditional approaches to the analysis of such systems. Thus, we may have to accept as unavoidable a substantial degree of fuzziness in the description of the behavior of biological systems as well as in their characterization. This fuzziness, distasteful though it may be, is the price we have to pay for the ineffectiveness of precise mathematical techniques in dealing with systems comprising a very large number of interacting elements or involving a large number of variables in their decision trees.
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> Lotfi A. Zadeh, 1969 [14], p. 200

Lotfi A. Zadeh himself anticipated very early that medical diagnosis would be the most likely application domain of his theory [14]. Despite his prominent forecast, work on fuzzy set theory in medicine has largely remained that of individuals and is still considered informal and ad hoc by many. This is the more surprising as fuzzy sets formalize gradation, a natural characteristic of medicine that is incompatible with the discrete nature of classical AI. Interest of the medical AI community in fuzzy set theory should thus be vital.

1. The motivation of fuzzy sets

Nature has provided us with a mind that allows a certain sloppiness in the descriptions of our environment. This sloppiness is sometimes at conflict with the rigor of formal analysis, as the following example demonstrates.

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If the low blood pressure of a patient increases by a small amount, say 1 mmHg, then, because nothing significant has happened, it will still be considered low. A mean arterial blood pressure of 70 mmHg is certainly low. Therefore, by induction, every blood pressure above 70 mmHg should be low, which is, of course, not true.

This old paradox (quoted after [4]) is easily resolved if one accepts that the denotation of the word *low* is not as sharply defined as would be required for induction to be applicable. Clearly, if 70 mmHg means low blood pressure, then 71 mmHg may still rightfully be called low, even though it is not quite as low as 70 mmHg. The higher the blood pressure gets, however, the less adequate the description *low* becomes, until a point is reached where *low* is not an adequate description at all.

AI, in its attempts to draw level with the performance of the human mind, relies to a large extent on symbolic reasoning as a model of human thinking. Symbols, like the words of a language, are names that denote concepts, and concepts are abstractions from the concrete entities that constitute reality. To formally assign the symbols a meaning they are usually associated with sets, collections of entities that represent what the symbol stands for. There are situations, however, in which the meaning of a word or symbol cannot be captured adequately by an ordinary set. The term *cold* for example denotes the range of cold temperatures, which may vary from context to context but, clearly, always lacks a sharp boundary. Analogously, the set of people one might call one's friends is not as sharply defined as would be required if classical sets were to be used. It is Zadeh's contribution to AI that he provided us with a formal framework that allows it to capture the meaning of vague concepts: the theory of fuzzy sets.

2. The definition of fuzzy sets

A *fuzzy subset* $\tilde{A}$ of a (base) set X is specified by its *membership function* $\mu_{\tilde{A}}$,

$$\mu_{\tilde{A}}: X \rightarrow [0,1],$$

assigning to each $x \epsilon X$ the *degree* or *grade* to which x belongs to $\tilde{A}$. Other than ordinary subsets, fuzzy subsets allow the partial membership of their elements, the degree of membership being expressed on a continuous scale from 0 to 1. [0, 1] is called the *valuation set* of $\mu_{\tilde{A}}$. Other valuation sets are also possible; the unit interval is the one introduced by Zadeh [13] and is still the most common.

Clearly, the membership function of a fuzzy subset $\tilde{A}$ of X — called *fuzzy set* $\tilde{A}$ for short—is a generalization of the characteristic function of ordinary subsets, which has a binary valuation set {0,1}. It is therefore legitimate to regard fuzzy sets as generalizations of ordinary sets.

Fuzzy sets come with fuzzy set operators, which are usually specified on the basis of their operands' membership functions. Given two fuzzy sets $\tilde{A}$ and $\tilde{B}$, fuzzy set union, $\cup$, is commonly defined so that

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

and intersection, $\cap$, so that

$$\mu_{\tilde{A}\cap\tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)).$$

The complement, $\neg$, of a fuzzy set $\tilde{A}$ is then defined by

$$\mu_{\neg\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x).$$

Other definitions of these and further set operators have also been proposed, but result in weaker structures (see, for example, [12]). Notice that the above definitions of operators on fuzzy sets include those on ordinary sets as special cases. Fuzzy set theory is a generalization of standard set theory.

It is a common phenomenon that generalizations of formal constructs have weaker properties and thus form weaker structures than their specializations. In the case of fuzzy set theory some laws of Boolean algebra that hold for ordinary set theory do not hold for its fuzzy extension. In particular, if union, intersection, and complement are defined as above,

$$\tilde{A} \cap \neg\tilde{A} \neq \phi$$

and

$$\tilde{A} \cup \neg\tilde{A} \neq X,$$

i.e. the laws of noncontradiction and excluded middle do not apply. This is not, as is sometimes concluded, a flaw of fuzzy set theory, but merely the price to pay for greater expressiveness.

That there is indeed an advance in expressive power of fuzzy sets over ordinary sets is illustrated by the following fundamental relationship. To every fuzzy set $\tilde{A}$ corresponds a family of ordinary sets

$$(A_\alpha)_{\alpha\in[0,1]}$$

called the α-*level-sets* or α-*cuts* of $\tilde{A}$. Each A_α is defined as

$$\{x \in X \mid \mu_{\tilde{A}}(X) \geq \alpha\},$$

i.e. as the set of elements of X whose degree of membership in $\tilde{A}$ is greater than or equal to the threshold α. The definition implies that the A_α are ordered by α so that

$$A_\alpha \supseteq A_{\alpha'}$$

for all

$$0 < \alpha \leq \alpha' \leq 1.$$

α can be interpreted as assigning some rank to each set of the family, for example a degree of specificity or the degree to which the set is an extension of a certain concept. A fuzzy set may therefore be viewed as comprising the information captured in a possibly infinite number of nested ordinary sets, each having a rank assigned to it.

With the dissemination of fuzzy set theory it has become common to speak of *fuzzy logic* whenever fuzzy sets and some computation based thereon are involved.

As a matter of fact, *fuzzy logic* has become *the* keyword of the whole field (it is, for example, included in the U.S. National Library of Medicine's Medical Subject Headings, whereas *fuzzy set theory* is not), although its definition and its relation to standard logic is by no means as agreed upon as that of fuzzy set theory to standard set theory ([3], for Zadeh's most recent definition of fuzzy logic – see [15]). For most work based on fuzzy logic, however, it is sufficient to understand the fundamentals of fuzzy set theory.

3. The controversy over fuzzy sets

From the perspective of other sciences fuzzy set theory has been perceived as perfectly natural: 'The notion of a fuzzy set... is an entirely uncontroversial extension of respectable mathematical concepts.' [9]. Yet, the mathematical community itself has indulged in lengthy discussions concerning the necessity, soundness and adequacy of fuzzy set theory [2,6,11].

It seems that although fuzzy set theory has been contested on purely theoretical grounds, most of the debate is due to its supposed rivalry with probability theory, the predominant uncertainty calculus of the past and present. The irritation among the followers of probability theory is not without reason: even though Zadeh made it quite clear right from the beginning that 'the notion of a fuzzy set is completely nonstatistical in nature' [13], fuzzy set theory and its relatives, in particular possibility theory, have been recognized as accessible means of treating uncertainty, thereby entering the territory of probability theory and challenging its prescriptive sovereignty.

The struggle is, of course, not one of displacement, but one of finding the innate positions. To illustrate this, consider the following situation. We see a bottle half filled with water, and we know of another bottle we cannot see that it is either full or empty, the chances being even. There are at least two different aspects to this situation: one can focus on the truth value of a statement saying that a particular bottle is filled to a certain extent, or one can ask for the chance of such a statement being true. In case of the first bottle, the degree of (partial) truth of the statement 'the bottle is full' may be set at 0.5, whereas the chance that it is totally true is 0. In case of the second bottle, the degree of truth of the same statement is unknown (as is the actual state of the bottle; yet we know that it is either 1 or 0), but the chance that the statement is again totally true is 0.5.

Both fuzzy set theory and probability theory can, in principle, be used to model and reason about either aspect of this situation: both are mathematical constructs that are independent of any particular use. However, one will agree that the first aspect, pertaining to the partial match of a proposition and a perfectly known state of affairs, is more naturally modelled in terms of fuzzy set theory, while the second, relating to the partial ignorance of what is actually present, is the classical domain of probability theory.

4. The power of fuzzy sets

Classical AI is discrete in nature. Its models of reality are built from enumerable sets of symbols. Because the complexity of these models invariably increases with the number of symbols employed, resolution[1] and simplicity of AI systems are almost mutually exclusive properties. As a result, medical AI systems often lack the gradation that would be required to render the continuity of the addressed medical problem. For example, would it not seem natural that different patients presenting with comparable symptoms be given comparable diagnoses? Yet many diagnostic systems lack this fundamental property: due to system-immanent thresholds, similar cases may be separated during the qualitative abstraction of continuous parameters and are subsequently treated differently, possibly resulting in significantly differing diagnoses. Analogously, would it not seem likely that a slight alteration over time in the parameters observed of one patient changes the diagnosis only slightly? Instead, however, symbolic dynamic systems usually respond to a continuous change in the patient's condition with an abrupt change in state.

Fuzzy sets, on the other hand, have become known for their 'ability to introduce notions of continuity into deductive thinking' [10]. Transferred to practice this means that the use of fuzzy sets allows a conventional symbolic system (specified in the form of rules, tables, or whatever) to adopt continuous behaviour. This should indeed be of considerable interest to the medical AI community, because, as indicated above, medicine is essentially a continuous domain.

How can the use of fuzzy sets resolve the mismatch between the discreteness of symbolic systems and the continuity of medical reality? For clear-cut cases (cases whose parameters are typical representatives of the symbols employed) a system built on fuzzy sets—a fuzzy system—produces the same results as its underlying symbolic skeleton. For borderline[2] cases, however, it brings to bear the gradation that is implicit in the meaning of the symbols and explicated by the fuzzy sets: it determines the degrees of fit of what is actually present and its internal descriptions and propagates these degrees through the system to its output, where they serve to qualify the results of the reasoning process. For example, as the actual blood pressure of a patient increases, the degree of fit of the description 'low blood pressure' (or the degree of truth of an equivalent statement) decreases, and the degrees of truth of all conclusions derived therefrom change in the direction determined by the involved logical connectives.

There is a special subset of fuzzy systems that is designed to operate in a completely continuous, ie essentially real-valued, environment. If carefully designed, the output of such a system is a continuous function of its input and, if present, of its internal memory. This function is usually a better approximation of the modelled medical relationship than its underlying discrete specification; at the same time, it is much easier defined in fuzzy terms than would be possible analytically.

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[1] The degree to which the model can reproduce the nuances of reality.

[2] Note that in a fuzzy system 'borderline' is not an additional category, but a smooth transition zone that is defined by the overlapping of the fuzzy sets partitioning the problem domain.

Many practical applications of fuzzy set theory in medicine make use of this principle: fuzzy scores, continuous versions of conventional scoring schemes, and fuzzy alarms are just examples. Best developed is the approach for fuzzy control, the most successful application of fuzzy set theory to date in which a tabular or rule-based mapping from input to output variables effectively implements a continuous control law. Fuzzy qualitative simulation and, more generally, fuzzy model-based diagnosis are promising candidates for future research.

Fuzzy set theory is not an alternative to, but an enhancement of classical AI approaches. By virtue of fuzzy sets, symbolic systems may exhibit continuous behaviour and thus address medical problems more adequately. Although the theory of fuzzy sets may not be the only formalism to interface the symbolism of AI with the continuity and gradation of reality, it requires minimum remodelling and is highly effective. This makes it a powerful tool.

5. This issue

The articles compiled in this special issue address a wide variety of medical and technical problems. Interestingly, most of the work is practically oriented.

The first two articles make use of fuzzy systems as function approximators. In the first one a model-based controller for the closed-loop delivery of a muscle relaxant is presented. Unlike many other fuzzy controllers, this work relies on a deep model of the process under control, with a small fuzzy controller stepping in to adapt unknown process variables. The resulting system has been shown to perform well under simulated and real conditions [7].

The second article applies the same principle, the approximation of functions through fuzzy systems, to the field of clinical alarming. For this purpose the domain knowledge acquired from thirteen cardiac anaesthesiologist is compiled into 188 rules mapping input to output variables. The approach has been evaluated under both off-line and on-line conditions [1].

The third article describes a formalized approach aiming to facilitate the transformation of fuzzy processing systems that, despite their demonstrated usefulness, have remained in a prototypical stage of development to compact pieces of software that meet the requirements of industrial production systems [5]. With the aid of such semiautomatic transformation, fuzzy systems may eventually become as naturally integrated into clinical devices as they already are in many consumer products.

The final contribution in the form of a research note presents an ongoing research project that deals with the formalization of terms from the American College of Radiology (ACR) Breast Imaging Lexicon using fuzzy sets [8]. Formalization is crucial to the standardization of medical diagnosis and for all attempts to make it more objective; despite its undeniable aptness, fuzzy set theory has not yet played a role in such endeavours.

Although its application in medicine is still far from being mainstream, there is a noticeable tendency towards making use of fuzzy set theory whenever it appears practical to do so. However, many remain sceptical. To them, the work compiled in this special issue may serve as a case in point that fuzzy sets have their rightful place in medical AI. To all others, it should be an encouragement to consider the fuzzy aspects of their own work, and a source of inspiration.

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