Formative and Reflective Measurement Models

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Abstract

We compare formative and reflective measurement models in the context of structural equation models (SEM). The formative model is expressed as part of the structural regression equation. The identification status of the models is different, although they differ only in the direction of some arrows. It is shown, that the reflective model permits the identification of more parameters and requires less restrictions. Formative measurement models are recommended only under a strict theoretical necessity.

Key Words: Structural Equation Models (SEM); Formative and Reflective Measurement Models; Identification; Generalized Least Squares (GLS) Estimation; Maximum Likelihood (ML) Estimation. Pseudo Maximum Likelihood (PML) Estimation.

1 Introduction

The classical structural equation model (SEM) consists of two parts, a measurement model (factor analysis) and a structural (regression) equation between latent variables. Thus, the factors generate the measurements (indicators) which 'reflect' (in some sense) the latent constructs.

On the other hand, one can have the idea, that the indicators generate the latent variables as linear combinations. This approach is called 'formative' in the literature (Bollen and Lennox; 1991; Wilcox et al.; 2008).

In this paper both measurement approaches and a mixture thereof are compared with each other, especially in their ability to identify and estimate latent endogenous regression relationships. Clearly, the classical SEM (e.g. the LISREL model; Jöreskog and Sörbom 2001) is able to accomodate the 'formative' measurement model as part of the structural regression equation.

Although the models look similar (see fig. 2–4) and the parameters are the same (but not the directions), their properties are very different. After establishing identification, all

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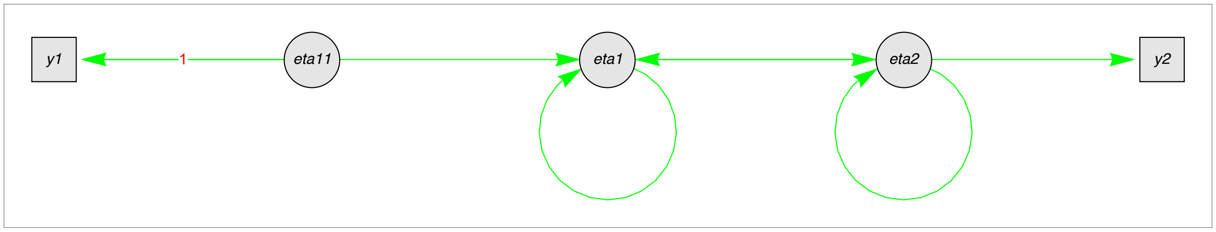

Figure 1: Skeleton diagram for formative and reflective measurement models. Reflective measurement model (right part): The arrow from η_2 to y_2 represents the factor loadings Λ. Left part: the formative measurement model is part of the structural regression (arrows from η_{11} to η_1 with regression matrix B_{111}). Self interaction loops represent the structural matrices B_{11} and B_{22} with zero diagonal. The equation errors ζ and ϵ are not displayed.

models can be estimated with an appropriate number of free parameters using maximum likelihood (ML), or pseudo maximum likelihood (PML) and generalized least squares (GLS), in case of misspecification or unknown distributional properties of the data (cf. Singer; 2016). Therefore, one has a well defined stochastic specification[1], and estimates together with their standard errors can be computed without distributional assumptions.

The paper is structured as follows: In section 2, formative and reflective measurement models are defined in terms of the usual SEM model. The identification problem for three model variants is analyzed in section 3. We also discuss the problems, when unidentified models are estimated. In the conclusion, we state that in a formative model less parameters are identified and that formative measurement models are recommended only under a strict theoretical necessity.

2 Formative and Reflective Measurement Models

In the following the skeleton diagram as shown in fig. 1 is discussed. In the right part of the figure, the conventional reflective measurement model (factor analysis) is displayed. The arrow from η_2 to y_2 represents the factor loadings Λ. Clearly, as seen in the left part of the figure, the 'formative' measurement model is part of the structural regression $\eta = B\eta + \zeta$ (arrows from η_{11} to η_1 with regression matrix B_{111}). The self interaction loops represent the structural matrices B_{11} and B_{22} with zero diagonal and the double arrow stands for the matrices B_{12} and B_{21}. The equation errors ζ and ϵ are not displayed.

Thus the diagram is equivalent to the block form (cf. also appendix)

$$\begin{bmatrix}\eta_{11}\\ \eta_1\\ \eta_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ B_{111} & B_{11} & B_{12}\\ 0 & B_{21} & B_{22}\end{bmatrix}\begin{bmatrix}\eta_{11}\\ \eta_1\\ \eta_2\end{bmatrix} + \begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}I & 0 & 0\\ 0 & 0 & \Lambda\end{bmatrix}\begin{bmatrix}\eta_{11}\\ \eta_1\\ \eta_2\end{bmatrix} + \begin{bmatrix}0\\ \epsilon\end{bmatrix}.$$

In more detail, we want to compare three models (see figs. 2–5) using

[1] We do not discuss the PLS approach here (Sosik et al.; 2009).

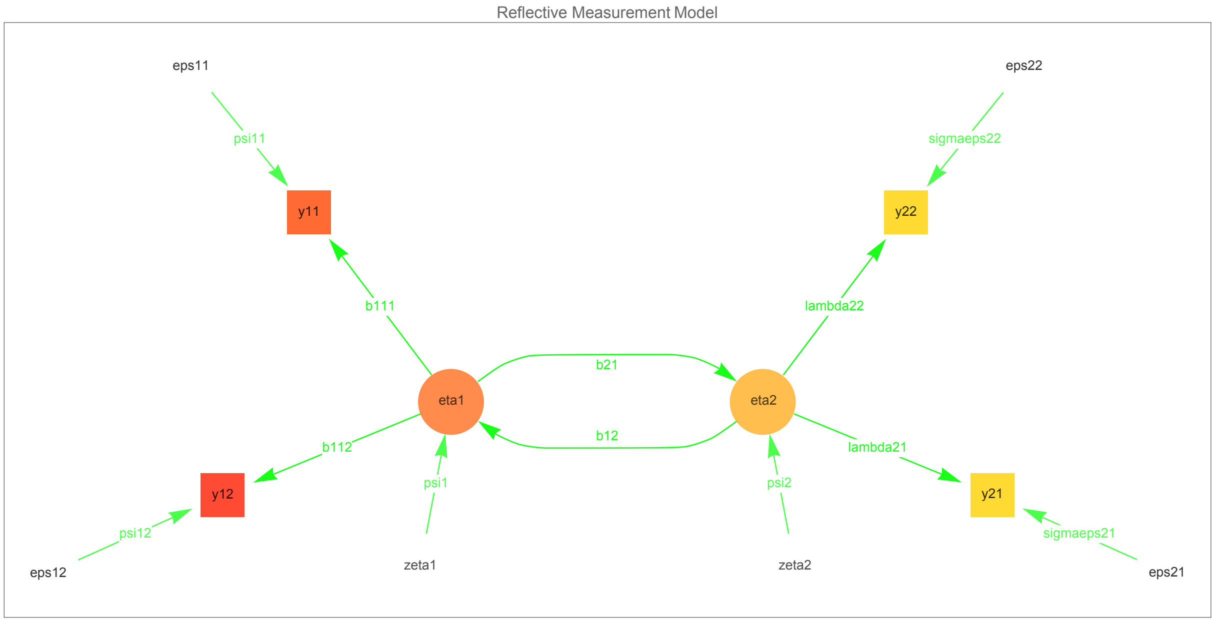

Figure 2: Reflective (1) measurement model.

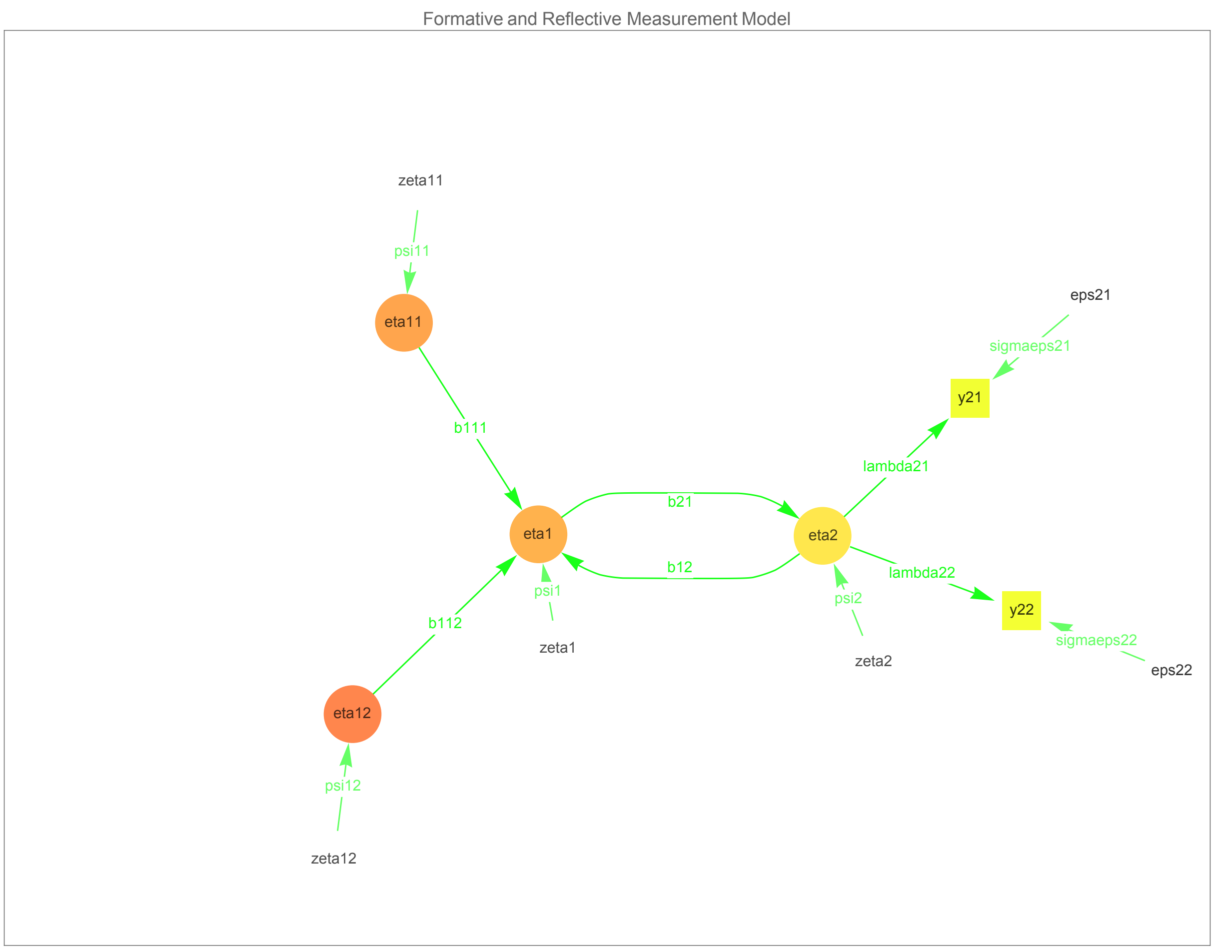

Figure 3: Formative-reflective (2) measurement model.

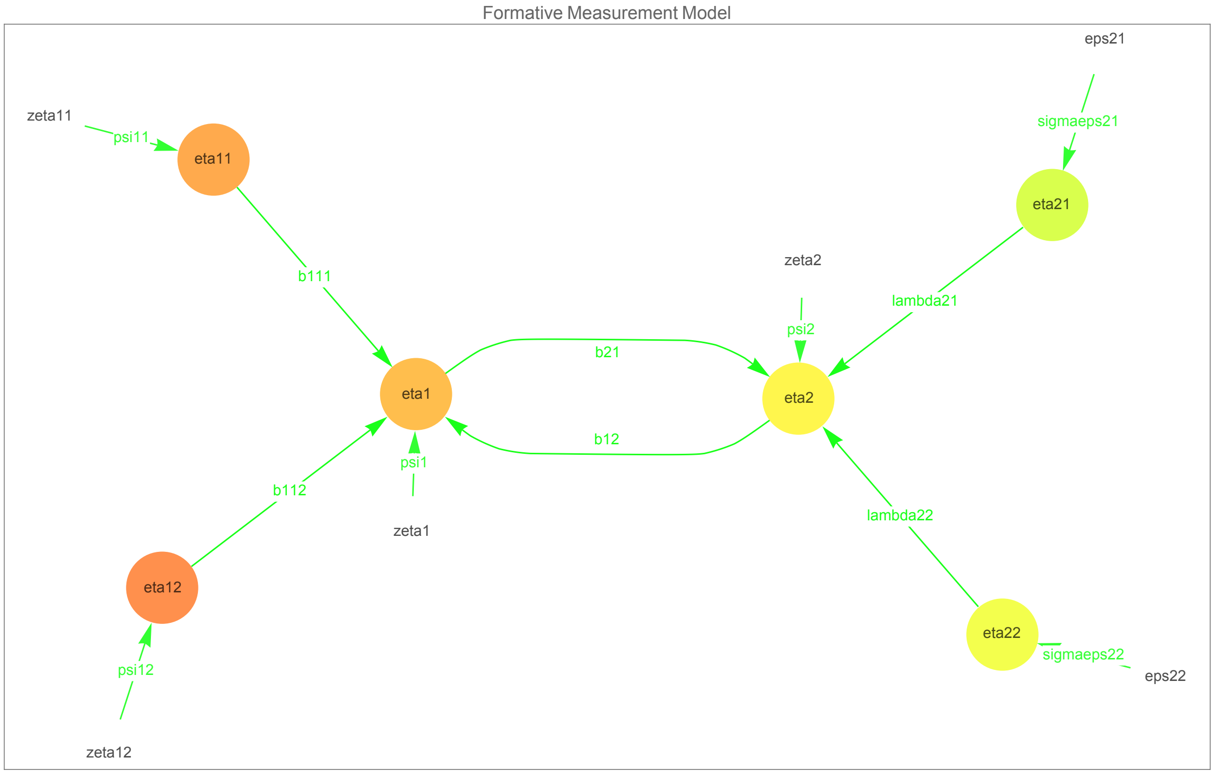

Figure 4: Formative (3) measurement model.

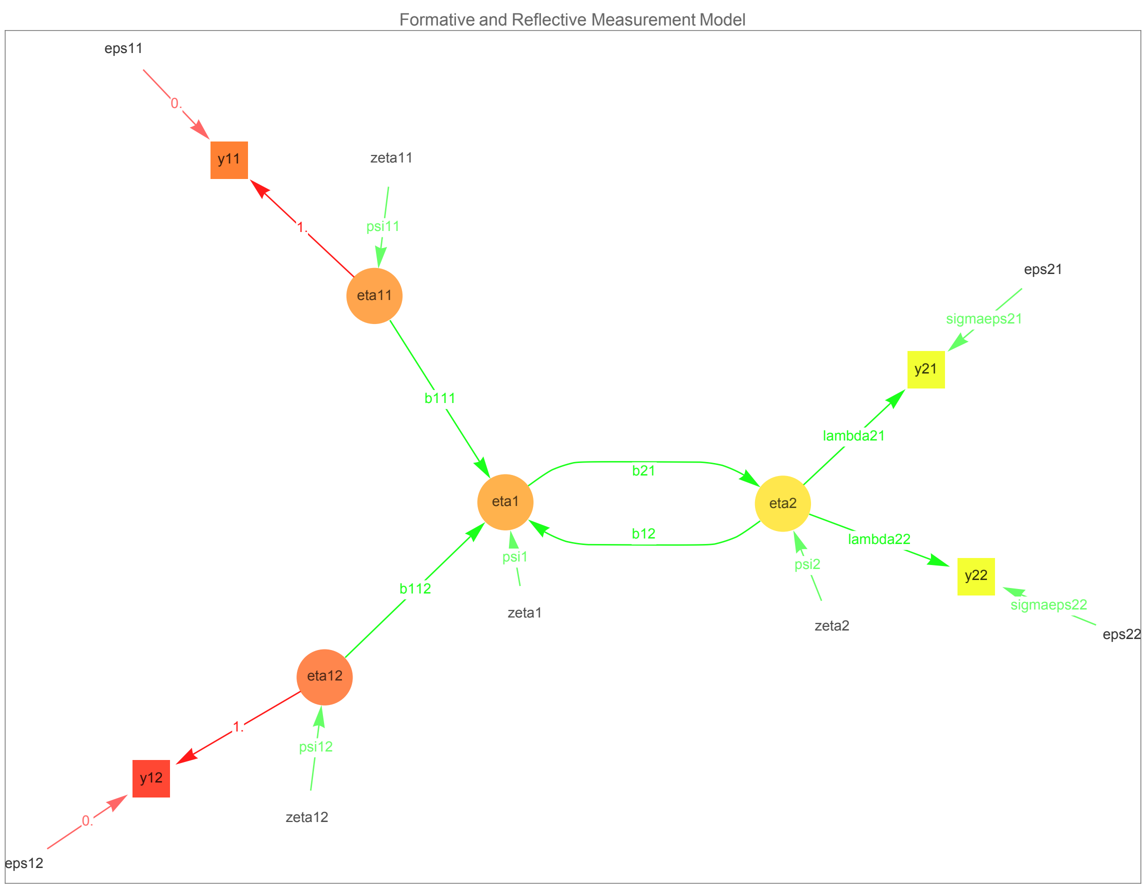

Figure 5: Actually, the formative-reflective model is more complicated. Fixed parameters are red.

1. purely reflective,
2. combined reflective/formative, and
3. purely formative

measurement models. In all models, we have a structural regression part between the 'actual' latent endogenous variables η_1 and η_2. The formative models additionally must accomodate the measurement in the regression equations (cf. fig 5). To facilitate the comparison, we use the same parameter strength.

3 Identification

Before the models can be estimated, it must be analyzed, whether the parameters of the SEM model

$$\begin{aligned} \eta_n &= B\eta_n + \Gamma x_n + \zeta_n \\ y_n &= \Lambda\eta_n + \tau x_n + \epsilon_n, \end{aligned}$$

can be inferred from the measurements. We assume that all structural matrices depend on a parameter vector $\psi : u \times 1$, e.g. $B = B(\psi)$ etc. The mean and variance of the latent states and observations is given as (assuming $x_n = 0$; see appendix)

$$\begin{aligned} \eta_n &= B_1\zeta_n \\ E[\eta_n] &= 0 \\ \mathrm{Var}(\eta_n) &= B_1\Sigma_\zeta B_1' \end{aligned}$$

$$\begin{aligned} E[y_n] &:= 0 \\ \mathrm{Var}(y_n) &:= \Sigma(\psi) = \Lambda\mathrm{Var}(\eta_n)\Lambda' + \Sigma_\epsilon = \Lambda B_1\Sigma_\zeta B_1'\Lambda' + \Sigma_\epsilon, \end{aligned} \tag{1}$$

$n = 1, ..., N$. It is assumed that $B_1 := (I - B)^{-1}$ exists.

Thus we have to solve eqn. (1) for the unknown parameters ψ in B, Σ_ζ (structural part), and Λ, Σ_ϵ (measurement part). If the equation can be solved uniquely, the likelihood function $l(\psi)$ (see eqn. 7) is an injective function of ψ, i.e. $l(\psi) \neq l(\psi')$ for $\psi \neq \psi'$ (for an extensive treatment, see Rothenberg; 1971).

In this case, the model is said to be identified.

3.1 Model 3 (purely formative model)

For model 3 (formative model), we have the structural matrices, dependent on the parameter vector $\psi = \{b_{111}, b_{112}, b_{12}, b_{21}, \psi_{11}, \psi_{12}, \psi_1, \psi_2, \lambda_{21}, \lambda_{22}, \sigma_{\epsilon,21}, \sigma_{\epsilon,22}\}$,

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ b_{111} & b_{112} & 0 & 0 & 0 & b_{12} \\ 0 & 0 & \lambda_{21} & \lambda_{22} & b_{21} & 0 \end{bmatrix},$$

$$\Sigma_\zeta = \begin{bmatrix} \psi_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\epsilon,21} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\epsilon,22} & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_2 \end{bmatrix}.$$

In this case, the identifying equations (1) are very simple, namely

$$\Sigma = \begin{bmatrix} \psi_{11} & 0 & 0 & 0 \\ 0 & \psi_{12} & 0 & 0 \\ 0 & 0 & \sigma_{\epsilon,21} & 0 \\ 0 & 0 & 0 & \sigma_{\epsilon,22} \end{bmatrix}.$$

Thus, only 4 parameters are identified. This stems from the fact, that the factor loading matrix is of the form

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} = [I_4, 0_{4,2}]$$

and thus only the upper left block of $B_1 \Sigma_\zeta B_1'$ is observed. One may say that the latent variables are too strongly defined.

3.2 Model 2 (mixed formative-reflective model)

Here, the dimension of the latent state is lower, since only one formative measurement is involved. We have

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{111} & b_{112} & 0 & b_{12} \\ 0 & 0 & b_{21} & 0 \end{bmatrix}, \; \Sigma_\zeta = \begin{bmatrix} \psi_{11} & 0 & 0 & 0 \\ 0 & \psi_{12} & 0 & 0 \\ 0 & 0 & \psi_1 & 0 \\ 0 & 0 & 0 & \psi_2 \end{bmatrix}.$$

Thus the measurements determined by

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{21} \\ 0 & 0 & 0 & \lambda_{22} \end{bmatrix}, \; \Sigma_\epsilon = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\epsilon,21} & 0 \\ 0 & 0 & 0 & \sigma_{\epsilon,22} \end{bmatrix}$$

have deeper access to the latent states and more parameters are identified, as is shown by the (somewhat complicated) equations

$$\begin{aligned}
\Sigma_{11} &= \psi_{11}, \Sigma_{12} = 0, \Sigma_{13} = \frac{b_{21}b_{111}\lambda_{21}\psi_{11}}{1-b_{12}b_{21}}, \Sigma_{14} = \frac{b_{21}b_{111}\lambda_{22}\psi_{11}}{1-b_{12}b_{21}}, \\
\Sigma_{22} &= \psi_{12}, \Sigma_{23} = \frac{b_{21}b_{112}\lambda_{21}\psi_{12}}{1-b_{12}b_{21}}, \Sigma_{24} = \frac{b_{21}b_{112}\lambda_{22}\psi_{12}}{1-b_{12}b_{21}}, \\
\Sigma_{33} &= \frac{b_{21}^2\left(b_{12}^2\sigma_{\epsilon,21} + \lambda_{21}^2\left(b_{111}^2\psi_{11} + b_{112}^2\psi_{12} + \psi_1\right)\right) - 2b_{12}b_{21}\sigma_{\epsilon,21} + \sigma_{\epsilon,21} + \lambda_{21}^2\psi_2}{(b_{12}b_{21}-1)^2}, \\
\Sigma_{34} &= \frac{\lambda_{21}\lambda_{22}\left(b_{21}^2\left(b_{111}^2\psi_{11} + b_{112}^2\psi_{12} + \psi_1\right) + \psi_2\right)}{(b_{12}b_{21}-1)^2}, \\
\Sigma_{44} &= \frac{b_{21}^2\left(b_{12}^2\sigma_{\epsilon,22} + \lambda_{22}^2\left(b_{111}^2\psi_{11} + b_{112}^2\psi_{12} + \psi_1\right)\right) - 2b_{12}b_{21}\sigma_{\epsilon,22} + \sigma_{\epsilon,22} + \lambda_{22}^2\psi_2}{(b_{12}b_{21}-1)^2}.
\end{aligned}$$

Since we have 9 equations for 12 parameters, some restrictions are needed. First, one should set $\lambda_{21} = 1, b_{111} = 1$ to set a scale for η_1 and η_2. Next, some parameters of the structural regression must be fixed (here to the true values $\psi_1 = 0.3, \psi_2 = 0.4$). The resulting model with 8 free parameters

$$\psi = \{b_{112}, b_{12}, b_{21}, \psi_{11}, \psi_{12}, \lambda_{22}, \sigma_{\epsilon,21}, \sigma_{\epsilon,22}\}$$

and restrictions $b_{111} = 1, \psi_1 = 0.3, \psi_2 = 0.4, \lambda_{21} = 1$ can be shown to be identified, although the explicit solution is quite complicated. Relaxing the restriction $\psi_1 = 0.3$, we run into an identification problem. But see next subsection.

3.3 Model 1 (reflective model)

Here, the dimension of the latent state is still lower, since no formative measurements must be accomodated. We have

$$B = \begin{bmatrix} 0 & b_{12} \\ b_{21} & 0 \end{bmatrix}, \Sigma_\zeta = \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} b_{111} & 0 \\ b_{112} & 0 \\ 0 & \lambda_{21} \\ 0 & \lambda_{22} \end{bmatrix}, \Sigma_\epsilon = \begin{bmatrix} \psi_{11} & 0 & 0 & 0 \\ 0 & \psi_{12} & 0 & 0 \\ 0 & 0 & \sigma_{\epsilon,21} & 0 \\ 0 & 0 & 0 & \sigma_{\epsilon,22} \end{bmatrix}.$$

This model is identified with 9 free parameters

$$\psi = \{b_{112}, b_{12}, b_{21}, \psi_{11}, \psi_{12}, \psi_1, \lambda_{22}, \sigma_{\epsilon,21}, \sigma_{\epsilon,22}\}$$

(see fig. 6). The analogous mixed model with 9 parameters (section 3.2) was not identified, as noted already.

One cannot relax the restriction $\psi_2 = 0.4$ (10 free parameters, 10 equations) without running into a nonidentified model. Thus, from the structural part with parameters $\{b_{12}, b_{21}, \psi_1, \psi_2\}$, only 3 parameters can be estimated.

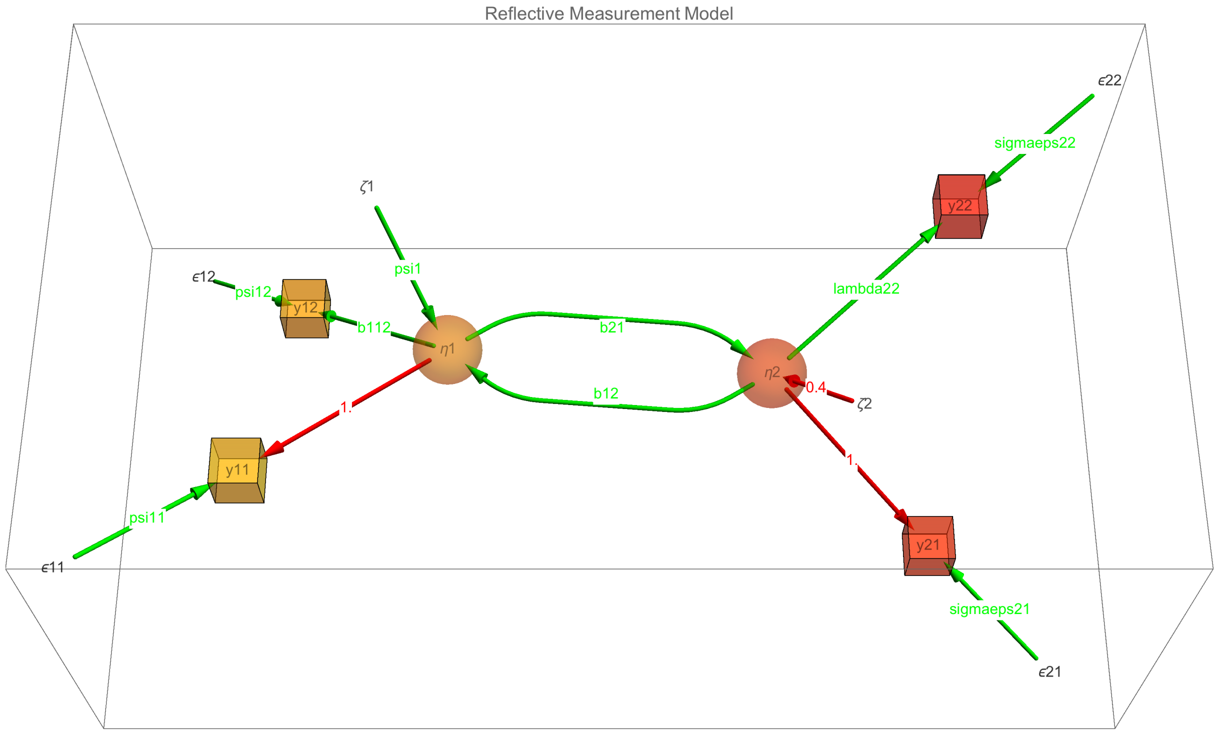

Figure 6: Reflective measurement model with 9 free parameters. Fixed parameters are red.

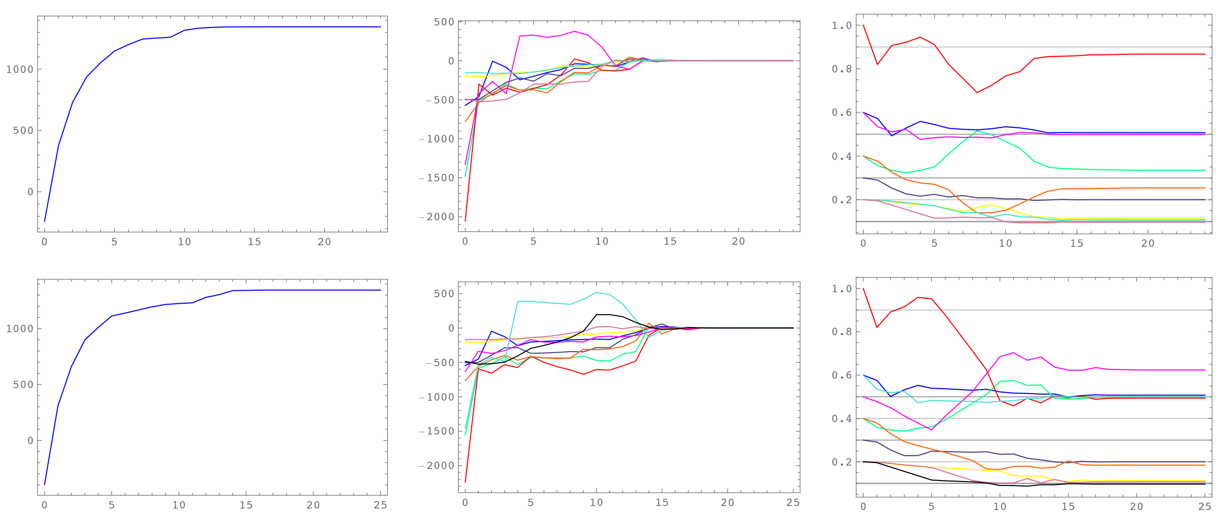

Figure 7: Reflective measurement model: Top: Convergence of 9 ML estimates (right) and score function (middle). The likelihood function is displayed in the left picture. $N = 5000$ observations. Bottom: nonidentified model (10 parameters)

In case of nonidentification, the Fisher information matrix

$$F_{ij} = E[s_i s_j],$$

with score function $s_i := \partial l/\partial\psi_i, i, j = 1, ..., u$ is singular and the negative Hessian $J_{ij} = -\partial^2 l/\partial\psi_i\partial\psi_j$ (observed Fisher information) can even have negative eigenvalues (Rothenberg; 1971).

In case of identification, the system can be estimated. Here the ML method is used. The convergence of the 9 parameter estimates is displayed in fig. 7 (top). In case of non-identification (fig. 7, bottom and table 2), the estimates of the nonidentified parameters converge to wrong values (depending on the starting values of the optimization algorithm) and the squared standard errors are partly negative. Clearly one can see in tables 1–2 that the estimates (of the identified parameters) are close to the true values (a large sample $N = 5000$ was used.)

	true	$\hat{\psi}_{ML}$	std
b_{112}	0.5	0.507457	0.0107924
b_{12}	0.3	0.334935	0.0346588
b_{21}	0.9	0.867258	0.0376121
ψ_{11}	0.1	0.113758	0.00875246
ψ_{12}	0.2	0.20007	0.00455517
ψ_1	0.3	0.254154	0.0281956
λ_{22}	0.5	0.498855	0.00588345
$\sigma_{\epsilon,21}$	0.1	0.107366	0.00938148
$\sigma_{\epsilon,22}$	0.1	0.0970145	0.0029884

Table 1: Reflective measurement model with 9 free parameters. ML estimates and asymptotic standard errors (std $= \sqrt{\text{diag}(J^{-1})}$).

	true	$\hat{\psi}_{ML}$	std
b_{112}	0.5	0.507457	0.0107937
b_{12}	0.3	0.496665	$0. + 0.806537i$
b_{21}	0.9	0.494121	$0. + 2.72801i$
ψ_{11}	0.1	0.113758	0.0087553
ψ_{12}	0.2	0.20007	0.00455538
ψ_1	0.3	0.184385	$0. + 0.194547i$
ψ_2	0.4	0.623614	$0. + 2.23954i$
λ_{22}	0.5	0.498855	0.00588389
$\sigma_{\epsilon,21}$	0.1	0.107366	0.00938341
$\sigma_{\epsilon,22}$	0.1	0.0970145	0.00298854

Table 2: Reflective measurement model with 10 free parameters (nonidentified). Partly wrong ML estimates and imaginary asymptotic standard errors (std $= \sqrt{\text{diag}(J^{-1})}; i := \sqrt{-1}$).

4 Conclusion

Although the models 1–3 look very similar and have the same parameter values, their identification status is different. The purely formative measurement model requires a higher dimensional regression specification in order to accomodate the formation of the latent states (the formative measurement model is actually part of the structural regression). In consequence, some parameters of the structural regression cannot be identified and consequently not estimated. In contrast, the classical factor analytic model gives 'more freedom' for the latent states and permits more possibilities in the identification and estimation of structural regressions. Mixed formative/reflective models are somewhat intermediate.

In conclusion, formative measurement models should be used only under a strict theoretical necessity.

Appendix

SEM modeling

We use the SEM model (for details, see Singer 2016)

$$\eta_n = B\eta_n + \Gamma x_n + \zeta_n \tag{2}$$

$$y_n = \Lambda\eta_n + \tau x_n + \epsilon_n, \tag{3}$$

$n = 1, \ldots, N$. The structural matrices have dimensions $B : P \times P,\ \Gamma : P \times Q,\ \Lambda : K \times P,\ \tau : K \times Q$ and $\zeta_n \sim N(0, \Sigma_\zeta)$, $\epsilon_n \sim N(0, \Sigma_\epsilon)$ are mutually independent normally distributed error terms $\Sigma_\zeta : P \times P,\ \Sigma_\epsilon : K \times K$. We assume that all structural matrices depend on a parameter vector $\psi : u \times 1$, i.e. $\Sigma_\zeta(\psi)$ etc. For example one can specify $\Sigma_\zeta(\psi) = G_\zeta(\psi)G'_\zeta(\psi)$ to obtain a positive semidefinite matrix. The true parameter vector will be denoted as ψ_0.

In the structural and the measurement model, the variables x_n are *deterministic* control variables. They can be used to model intercepts and for dummy coding. Stochastic exogenous variables ξ_n are already included by extending the latent variables $\eta_n \to \{\eta_n, \xi_n\}$. For example, the LISREL model with intercepts is obtained as

$$\begin{aligned}
\begin{bmatrix}\eta_n \\ \xi_n\end{bmatrix} &= \begin{bmatrix}B & \Gamma \\ 0 & 0\end{bmatrix}\begin{bmatrix}\eta_n \\ \xi_n\end{bmatrix} + \begin{bmatrix}\alpha \\ \kappa\end{bmatrix}1 + \begin{bmatrix}\zeta_n \\ \zeta_n^*\end{bmatrix} \\
\begin{bmatrix}y_n \\ x_n\end{bmatrix} &= \begin{bmatrix}\Lambda_y & 0 \\ 0 & \Lambda_x\end{bmatrix}\begin{bmatrix}\eta_n \\ \xi_n\end{bmatrix} + \begin{bmatrix}\tau_y \\ \tau_x\end{bmatrix}1 + \begin{bmatrix}\epsilon_n \\ \delta_n\end{bmatrix} \\
\text{Var}(\begin{bmatrix}\zeta_n \\ \zeta_n^*\end{bmatrix}) &= \begin{bmatrix}\Psi & 0 \\ 0 & \Phi\end{bmatrix} \\
\text{Var}(\begin{bmatrix}\epsilon_n \\ \delta_n\end{bmatrix}) &= \begin{bmatrix}\Sigma_\epsilon & 0 \\ 0 & \Sigma_\delta\end{bmatrix}.
\end{aligned}$$

Since the error vectors are normally distributed, the indicators y_n in the measurement model (3) are distributed as $N(\mu_n, \Sigma)$, where

$$\begin{aligned}
\eta_n &= B_1(\Gamma x_n + \zeta_n) \\
E[\eta_n] &= B_1\Gamma x_n \\
\text{Var}(\eta_n) &= B_1\Sigma_\zeta B_1'
\end{aligned}$$

$$E[y_n] := \mu_n(\psi) = \Lambda E[\eta_n] + \tau x_n = [\Lambda B_1\Gamma + \tau]x_n := C(\psi)x_n \tag{4}$$

$$\text{Var}(y_n) := \Sigma(\psi) = \Lambda\text{Var}(\eta_n)\Lambda' + \Sigma_\epsilon = \Lambda B_1\Sigma_\zeta B_1'\Lambda' + \Sigma_\epsilon. \tag{5}$$

In the equations above, it is assumed that $B_1 := (I - B)^{-1}$ exists. In short form one can write the SEM as a regression equation

$$\begin{aligned} y_n &= \mu_n(\psi) + \nu_n = C(\psi)x_n + \nu_n \\ \nu_n &\sim N(0, \Sigma(\psi)). \end{aligned}$$

Thus, the log likelihood function for the N observations $\{y_n, x_n\}$ is

$$l(\psi) = -\tfrac{N}{2} \left(\log|\Sigma| + \operatorname{tr}\left[\Sigma^{-1} \tfrac{1}{N} \sum_n (y_n - \mu_n)(y_n - \mu_n)' \right] \right). \tag{6}$$

One can insert the sample covariance matrix $S = \frac{1}{N}\sum_n (y_n - \bar{y})(y_n - \bar{y})'$ in (6) which yields the form (for the case $\mu_n = \mu$)

$$l = -\tfrac{N}{2} \left(\log|\Sigma| + \operatorname{tr}\left\{ \Sigma^{-1} \left[S + (\bar{y} - \mu)(\bar{y} - \mu)' \right] \right\} \right). \tag{7}$$

In contrast to ML estimation, in least squares estimation no probability distribution of the data is assumed. Thus one may define the equation errors as $\zeta_n \sim (0, \Sigma_\zeta)$, $\epsilon_n \sim (0, \Sigma_\epsilon)$ without normality assumption but retains the correct specification of the first and second moments μ_n and Σ. The GLS fit function for the model without intercepts is given in the usual form as

$$F = \tfrac{N}{2} \operatorname{tr}\left[(\Sigma - S)V \right]^2, \tag{8}$$

where the weight matrix $V = S^{-1}$ is the inverse sample covariance matrix of y_n. The so defined GLS fitting function requires the positive definiteness (and thus nonsingularity) of S. In cases of singular (or nearly singular) S, one can use the variable matrix $V = \Sigma^{-1}(\psi)$ or other nonsingular constant matrices as weight function.

In contrast, the likelihood function (7) is well defined for singular S ($N \leq K$), since no log determinants of the sample moment matrices are involved, as is suggested by the ML fitting function of LISREL (cf. LISREL 8 reference guide, p. 21, eqns. 1.14, 1.15, p. 298, eqn. 10.8; Jöreskog and Sörbom 2001). In Browne (1974), this is called a Wishart likelihood function. The covariance matrix $\Sigma(\psi)$ (eqn. 5) of the indicators y_n must be nonsingular, however.[2]

If the error terms are not normally distributed, the likelihood (6) can be considered as a pseudo likelihood (cf. Gourieroux et al.; 1984; Arminger and Schoenberg; 1989) with correct first and second moments. It yields consistent estimates, but requires corrections in the asymptotic standard errors.

References

Arminger, G. and Schoenberg, R. J. (1989). Pseudo maximum likelihood estimation and a test for misspecification in mean and covariance structure models, *Psychometrika* **54**(3): 409–425.

Bollen, K. and Lennox, R. (1991). Conventional wisdom on measurement: A structural equation perspective, *Psychological Bulletin* **110**(2): 305.

Browne, M. W. (1974). Generalized least squares estimators in the analysis of covariances structures, *South African Statistical Journal* **8**: 1–24.

Gourieroux, C., Monfort, A. and Trognon, A. (1984). Pseudo maximum likelihood methods: Theory, *Econometrica* **52, 3**: 681–700.

[2] Otherwise the singular normal distribution can be used (Mardia et al.; 1979, p. 41). This case occurs in the presence of restrictions between the components of y_n.

Jöreskog, K. and Sörbom, D. (2001). *LISREL 8. User Reference Guide*, Scientific Software International, Lincolnwood, IL.

Mardia, K., Kent, J. and Bibby, J. (1979). *Multivariate Analysis*, Academic Press, London.

Rothenberg, T. (1971). Identification in parametric models, *Econometrica* **39,3**: 577–591.

Singer, H. (2016). SEM modeling with singular moment matrices. Part III: GLS Estimation., *Journal of Mathematical Sociology* **40**(3): 167–184.

Sosik, J. J., Kahai, S. S. and Piovoso, M. J. (2009). Silver bullet or voodoo statistics? a primer for using the partial least squares data analytic technique in group and organization research, *Group & Organization Management* **34**(1): 5–36.

Wilcox, J. B., Howell, R. D. and Breivik, E. (2008). Questions about formative measurement, *Journal of Business Research* **61**(12): 1219–1228.

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