The Flow Lattice of Oriented Matroids

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Recently Hochstättler and Nešetřil introduced the flow lattice of an oriented matroid as generalization of the lattice of all integer flows of a digraph or more general a regular matroid. This lattice is defined as the integer hull of the characteristic vectors of signed circuits.

We characterize the flow lattice of an oriented matroid $\mathcal{O}$ for the cases that $\mathcal{O}$ is uniform or has rank 3. For both cases we determine the dimension of the lattice and give a complete characterization including the construction of a basis of signed circuits. We prove that in the uniform case the dimension is $n - 1$ iff the matroid is reorientation equivalent to a neighborly matroid polytope of odd rank. Hereby we partially solve a problem stated in the famous book of Björner, Las Vergnas, Sturmfels, White and Ziegler about oriented matroids.