Order Sequencing in the Automobile Industry

A Rule Based Approach with Color Change Reduction

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Outline

- A Framework for Order Sequencing
- Rules for Order Sequencing
- A Greedy Approach for Sequence Construction
- Order Clustering to Reduce Color Changes

A Typical Production Plant

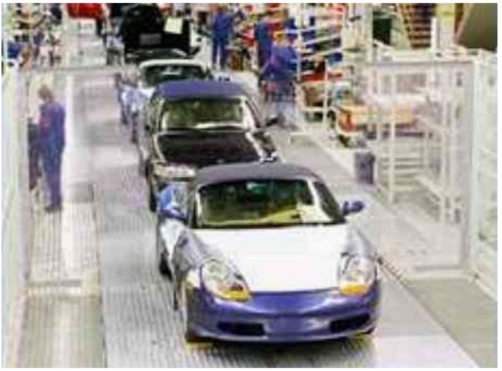
Body Shop

- press, weld, and mount the car body

Paint Shop

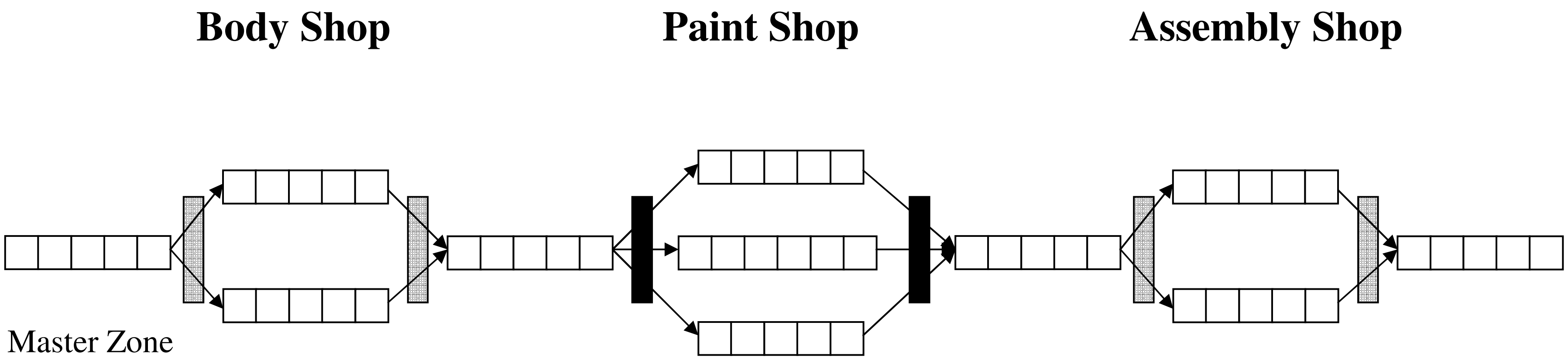
- enamel the body-in-white

Assembly Shop



- implement optional components

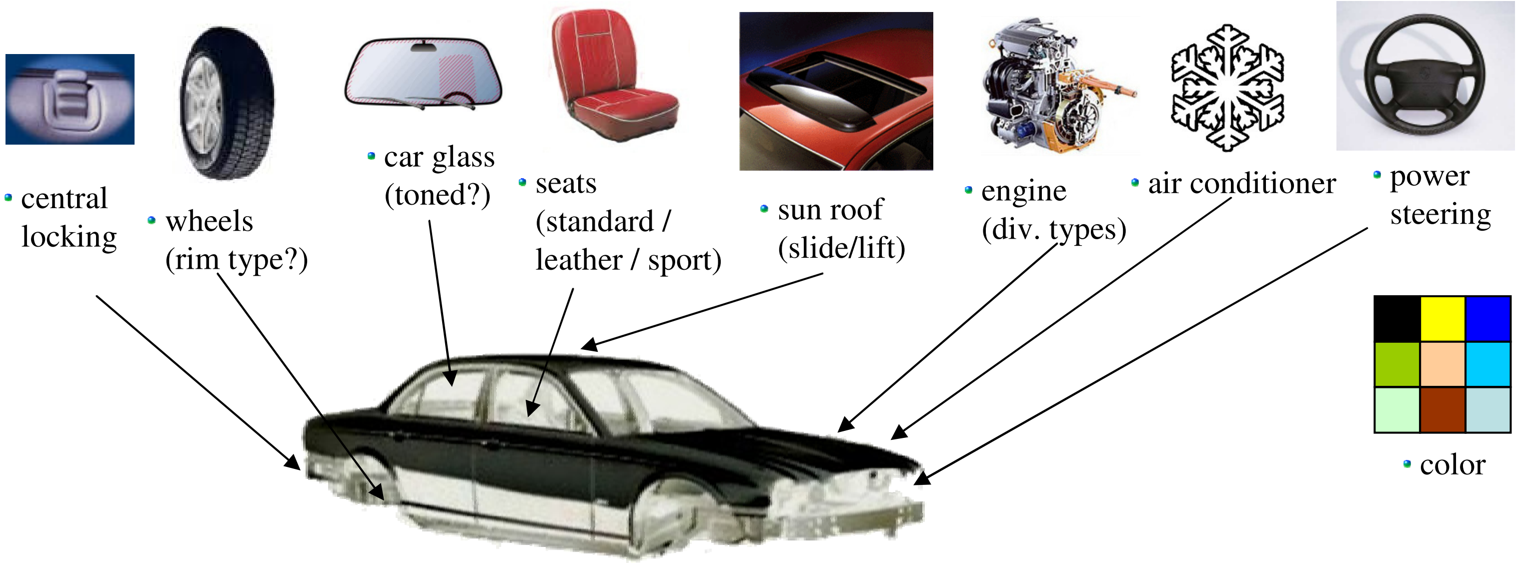
A Typical Production Plant



- For all zones a set of rules must be respected
- Storage systems allow color change reduction by short term interchanges prior to the paint shop

Orders and Commodities

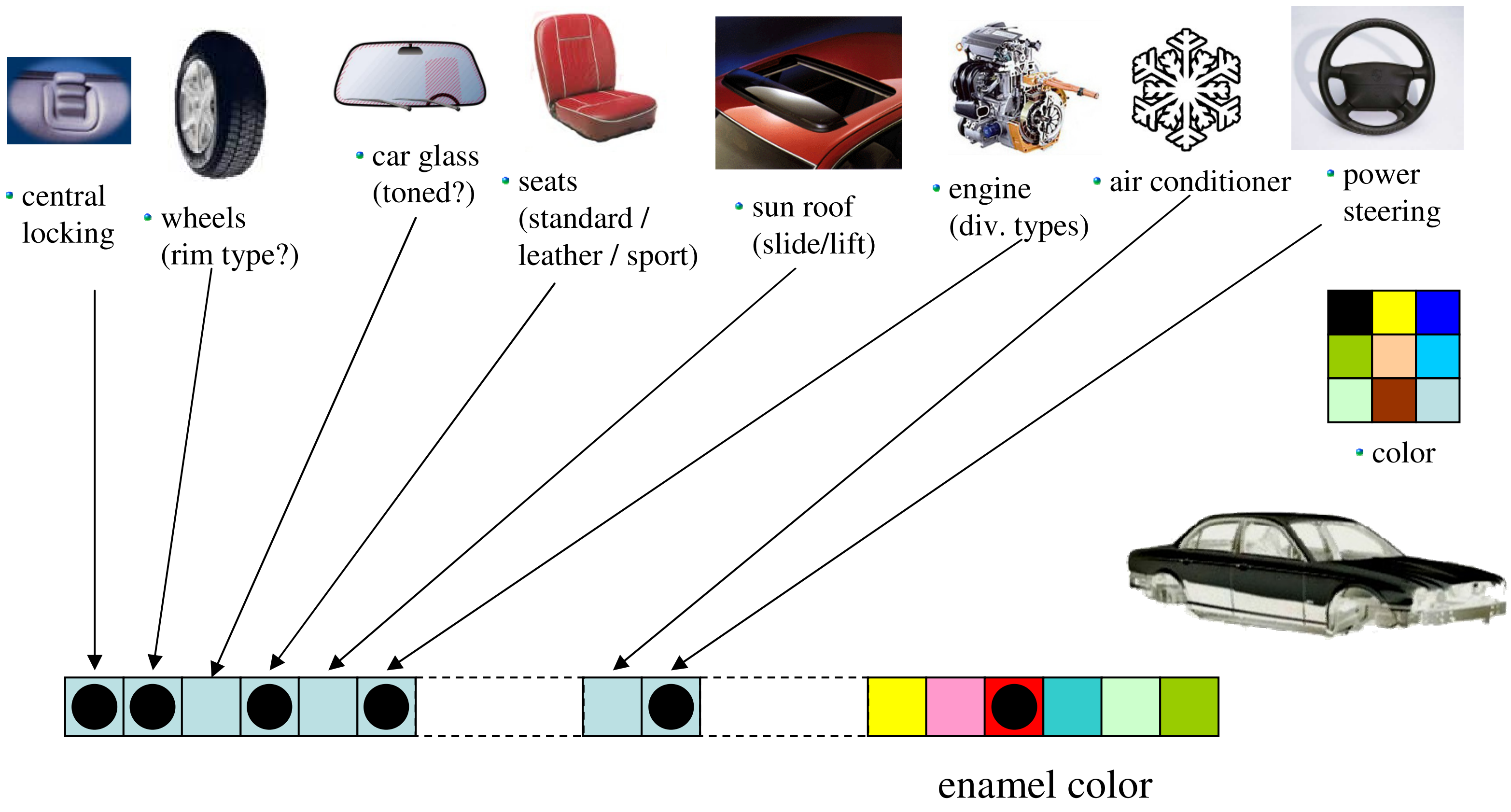
Commodities:



- central locking
- wheels (rim type?)
- car glass (toned?)
- seats (standard / leather / sport)
- sun roof (slide/lift)
- engine (div. types)
- air conditioner
- power steering
- color

Order:

Commodities:



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- color

Order:

As Bit Vector:

enamel color

Manufacturing Errors

- Production process is essentially probabilistic
- Commodities are delayed due to manufacturing errors

➡ An exact optimization model is not sensible

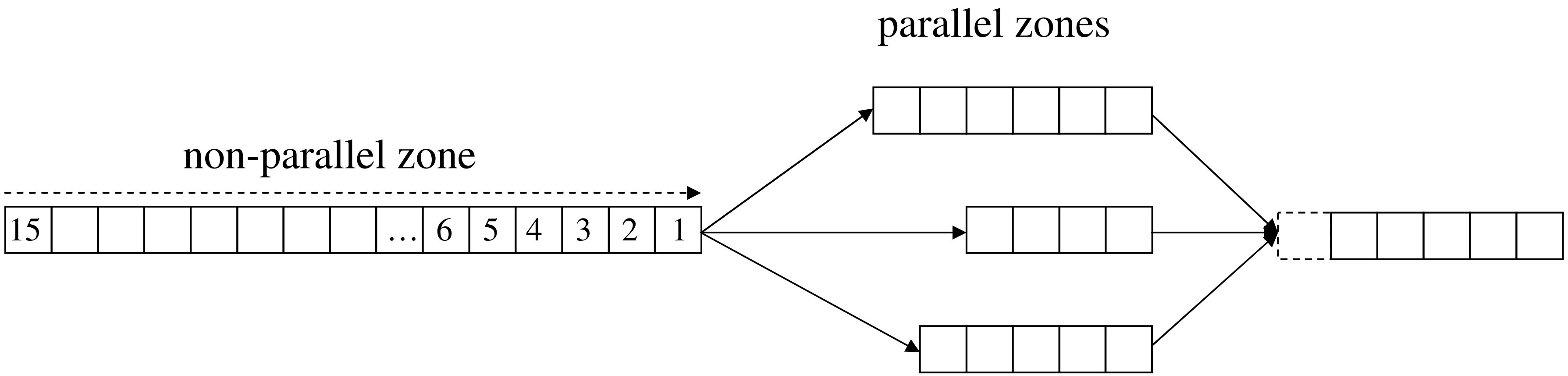
- Three major goals:
  - Robustness
  - Transparency
  - Performance

➡ Instead of objectives we use rules with priority to evaluate the result

➡ We model the production process in deterministic way

Deterministic Routing of Order Positions

- What happens at the split point of zones?
- Zones have different velocity / capacity
- The time an order needs to pass a zone is equal for parallel zones



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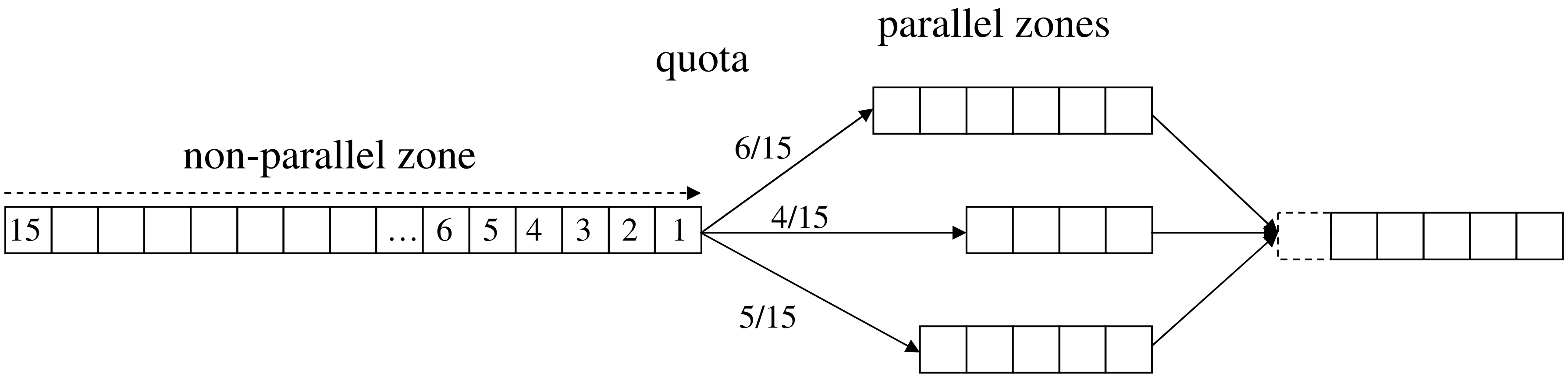

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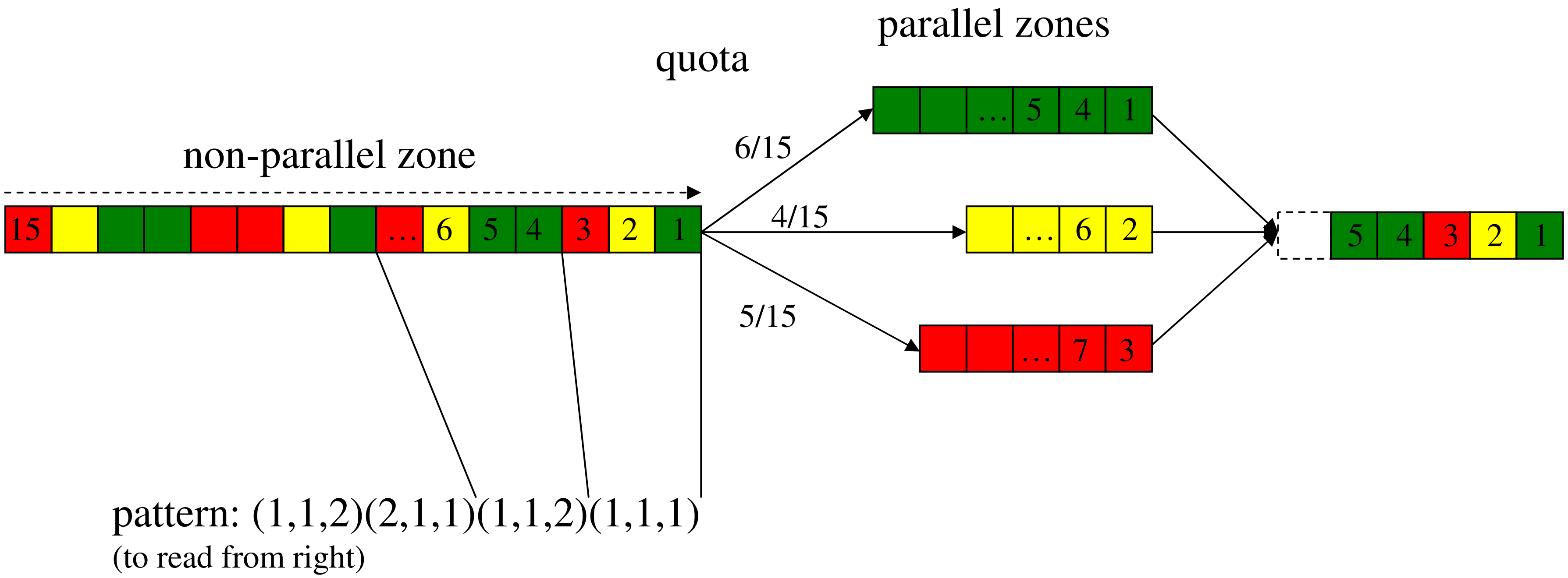
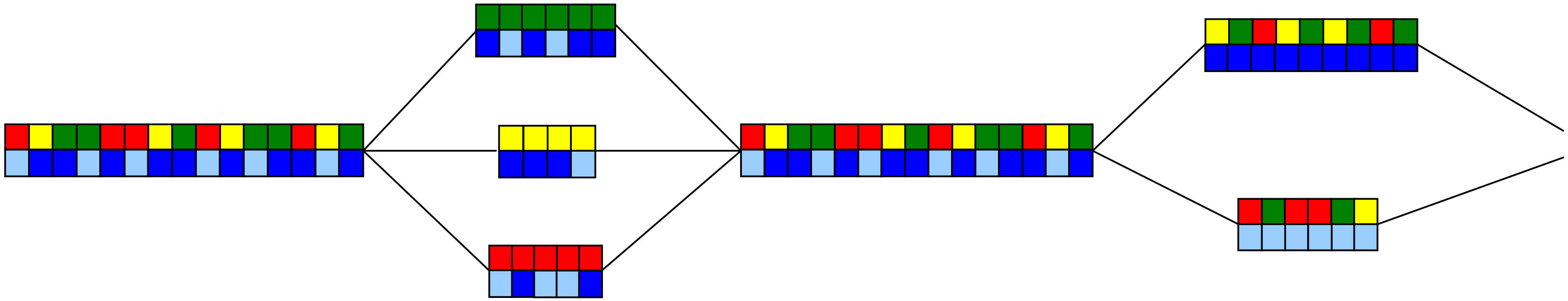


Deterministic Routing of Order Positions

- There is more than one split point

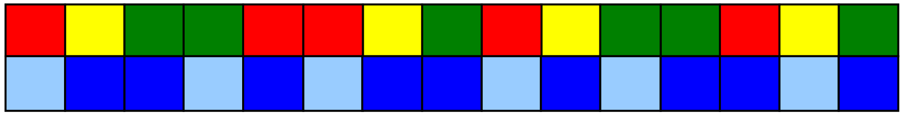


(1,1,2)(2,1,1)(1,1,2)(1,1,1)

(1,2)(1,1)(1,2)(1,1)(1,2)(1,1)

Deterministic Routing of Order Positions

It suffices to consider the master sequence and a set of patterns



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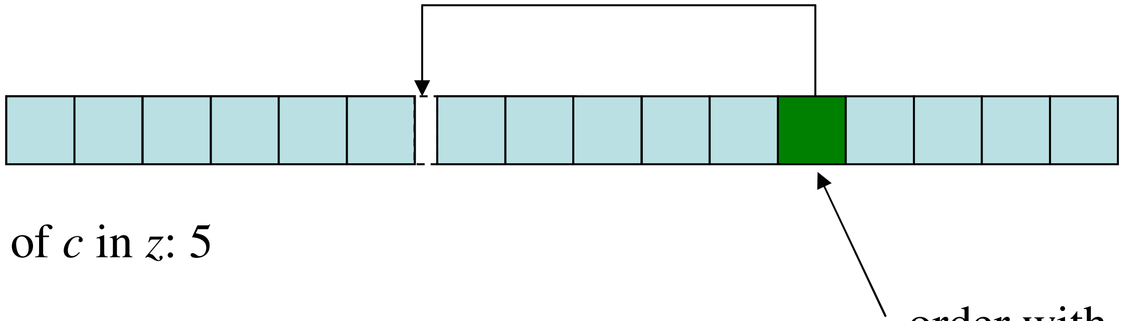
(1,2)(1,1)(1,2)(1,1)(1,2)(1,1)

Manufacturing Errors

- Manufacturing errors could occur during the production process
- Use statistical data to determine the expected delay of a commodity on a zone

zone z:



expected delay of c in z: 5

order with commodity c

- c will appear later in zones succeeding z (respecting quotas)

➡ This enables to compute a more realistic order sequence

The Rule Set

- Rules can be applied to each pair (c,z) of commodity and zone
- Any order containing c routed through zone z must respect this rule
- A priority p=1,…,10 is assigned to each rule
- Achieve a sequence with lexicographically minimal number of rule breaches

The Rule Set

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- Any order containing *c* routed through zone *z* must respect this rule
- A priority *p*=1,…,10 is assigned to each rule
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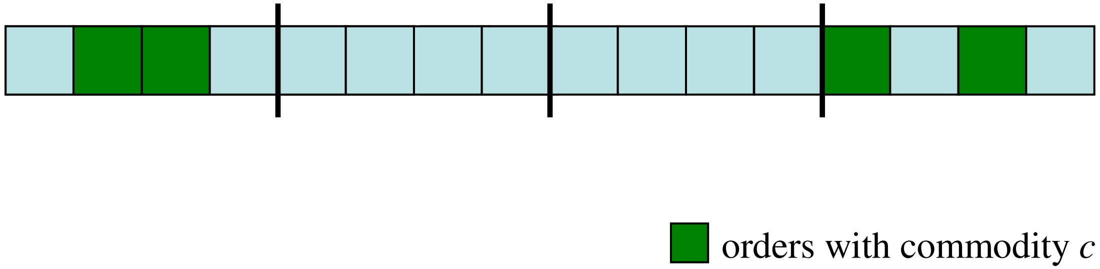
- **Clustering Rule**: A commodity is allowed to occur in *s* out of *S* slots of the master sequence

cluster *c* in 2 out of 4 slots

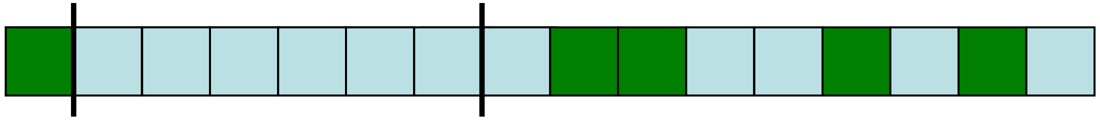


The Rule Set

- **Clustering Rule**: A commodity is allowed to occur in *s* out of *S* slots of the master sequence

- **Banning Rule**: A commodity is banned from an interval of a zone

ban *c* from 2,…,7



🟩 orders with commodity *c*

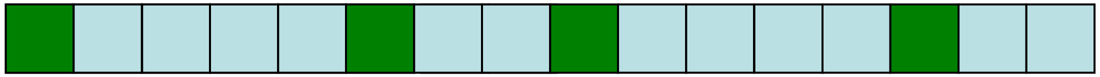
The Rule Set

- **Clustering Rule**: A commodity is allowed to occur in *s* out of *S* slots of the master sequence

- **Banning Rule**: A commodity is banned from an interval of a zone

- **Ratio Rule**: A commodity is allowed in at most *x* out of *y* consecutive sequence positions of a zone

allow *c* in 2 out of 8 positions



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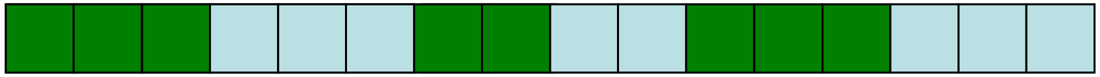
- **Grouping Rule**: Orders that hold a commodity *c* should occur in groups with minimal and maximal size, where groups must keep a minimal distance

size of groups: 2,…,3
minimal distance: 2



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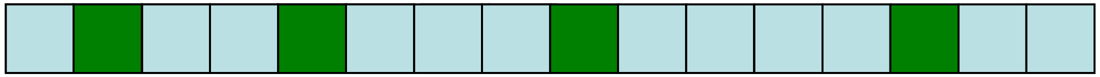
minimal distance: 2



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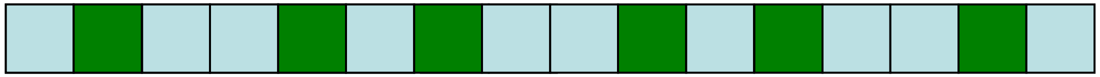
- **Spreading Rule**: Orders with a commodity should be evenly spread

average distance: 1.5



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- **Spreading Rule**: Orders with a commodity should be evenly spread

- Two concurrent strategies for rule evaluation
  - Count the number of rule breaches for each priority (quantity of rule breaches)
  - Increase a penalty value for a broken rule (quality of rule breaches)

- Clustering rule is considered separately

Greedy Master Sequence Construction

Simplification:

- Clustering rules are zone independent
- There is at most one clustering commodity per order (suitable for color clustering)

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- Compute each slot of the master sequence separately (Divide & Conquer)

Greedy Master Sequence Construction

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- Clustering rules are zone independent
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➡ Orders are assigned to slots of the master sequence

➡ Compute each slot of the master sequence separately (Divide & Conquer)

- Known approach: Goal Chasing (Monden 1983)
  - Choose step by step an order which assures an even resource consumption
  - Only suitable for spreading

$$\left|\text{desired resource consumption} - \text{current resource consumption}\right|^2 \rightarrow \min$$

Greedy Master Sequence Construction

Simplification:

- Clustering rules are zone independent
- There is at most one clustering commodity per order (suitable for color clustering)

- Orders are assigned to slots of the master sequence
- Compute each slot of the master sequence separately (Divide & Conquer)

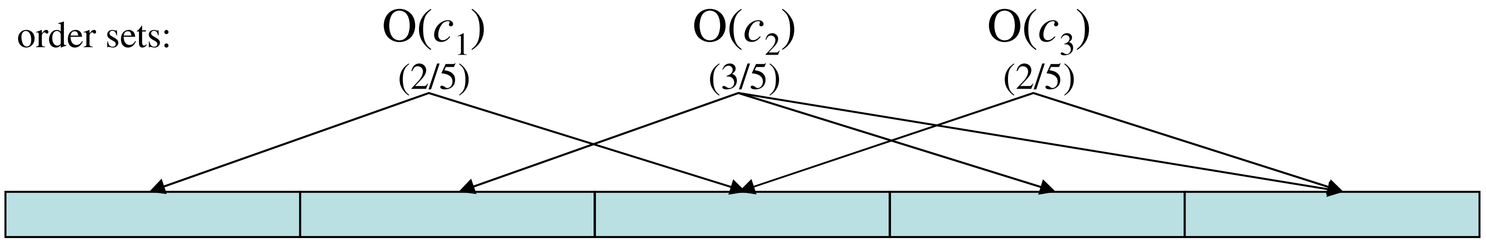
- Modified Goal Chasing approach:
  - Choose in every step a “best” order
    - with lexicographically minimized rule breach vector b=(b_1,…, b_{10})
    - and minimized penalty (priority-weighted penalties)

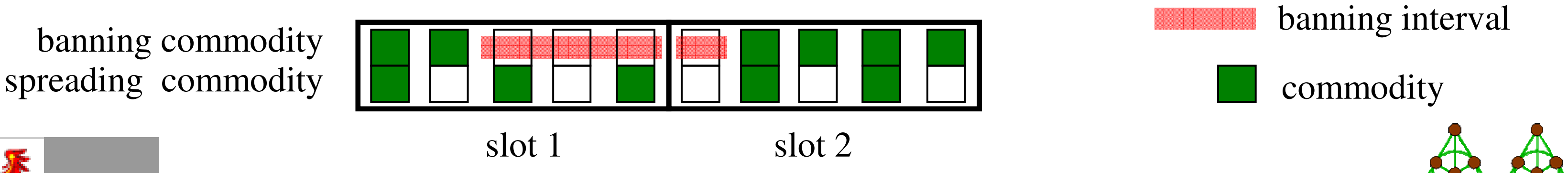
- Fully configurable by the choice of priorities

Order Clustering Algorithm

- Let $c_1,\ldots,c_u$ denote all clustering commodities and O(c) the set of all orders containing commodity c.

1. Assign to each clustering commodity its preferred slots
   - Other banning rules could imply varying ratio of a commodity
   - Spreading commodities must be evenly distributed over the slots





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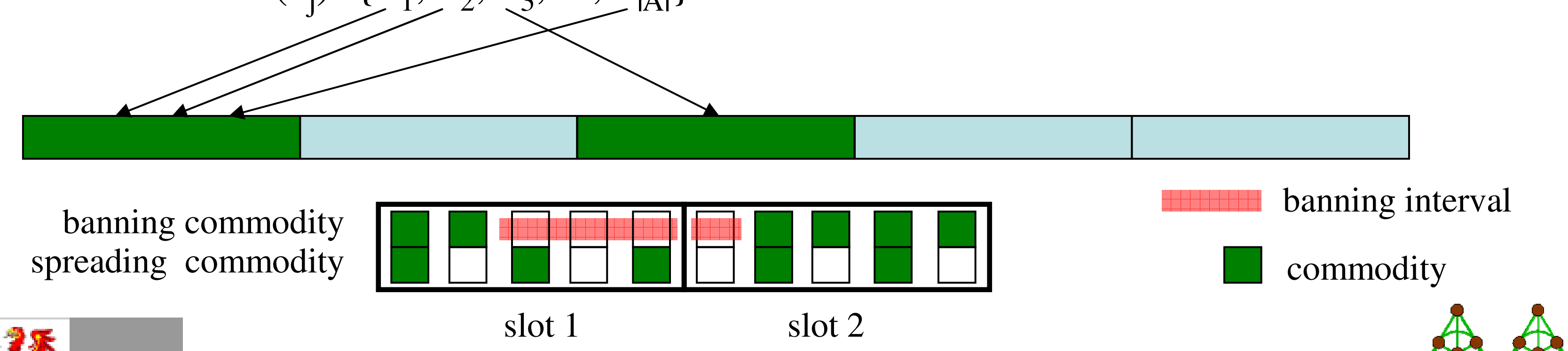
   - Other banning rules could imply varying ratio of a commodity
   - Spreading commodities must be evenly distributed over the slots
2. Assign each order to a preferred slot
   - Consider banning and spreading rules

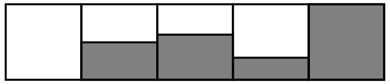
$O(c_j)$: $\{o_1, o_2, o_3,\ldots, o_{|A|}\}$



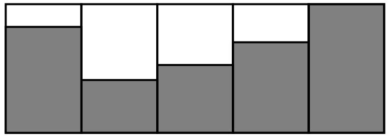
Best Vector Packing into Bins Problem

- Let $e_1,\ldots,e_r$ denote all spreading commodities
- For each set O(c_i) introduce a vector w_i (U is the set of non-clustering orders)

$$w_{i,j} \quad := \begin{cases} |O(c_i) \cap O(e_j)| & j = 1,\ldots,r \\ |O(c_i)| & j = r+1 \end{cases}$$



$$w_{u+1,j} := \begin{cases} |U \cap O(e_j)| & j = 1,\ldots,r \\ |U| & j = r+1 \end{cases}$$



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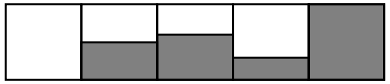

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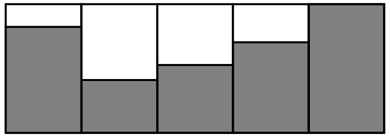
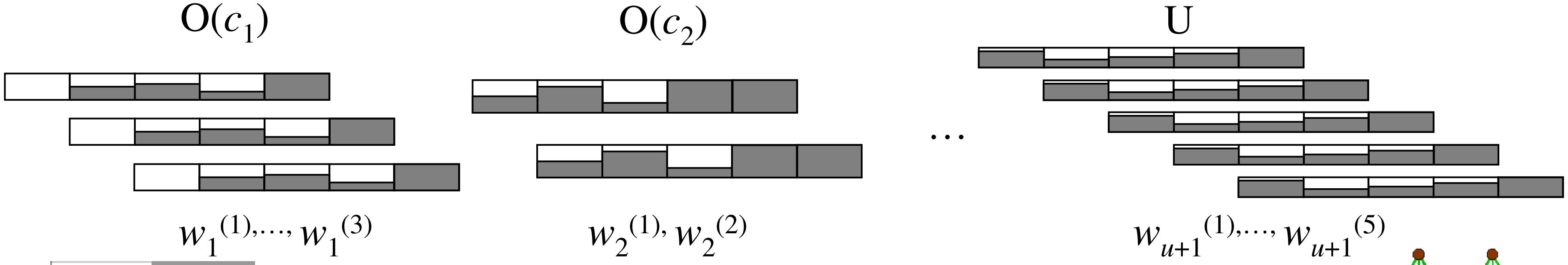


- Split these vectors according to the desired number of slots


O(c_1)
O(c_2)
U
…
$w_1^{(1)},\ldots, w_1^{(3)}$
$w_2^{(1)}, w_2^{(2)}$
$w_{u+1}^{(1)},\ldots, w_{u+1}^{(5)}$

Best Vector Packing into Bins Problem

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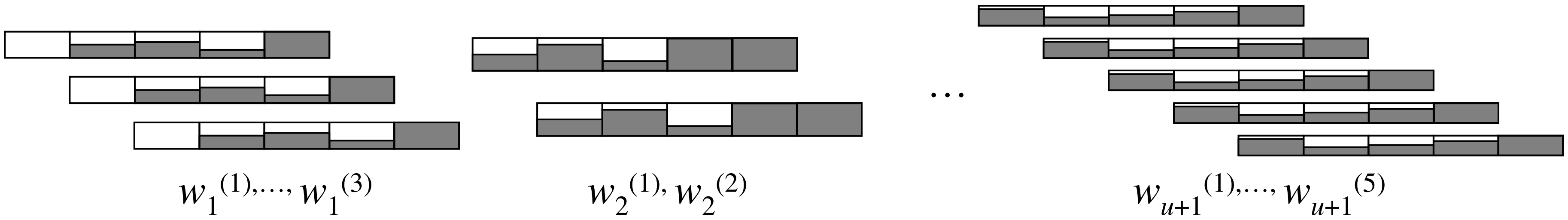
$w_1^{(1)},\ldots, w_1^{(3)}$
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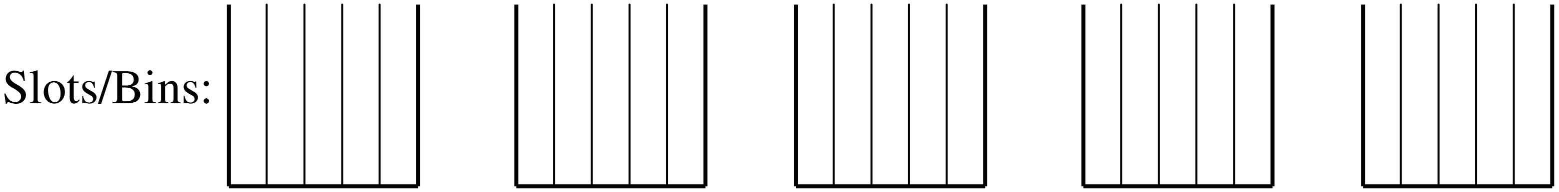
…
$w_{u+1}^{(1)},\ldots, w_{u+1}^{(5)}$

- Slots contain vectors indexed from 1 to r+1
- The last component is reserved to ensure that a slot is filled completely



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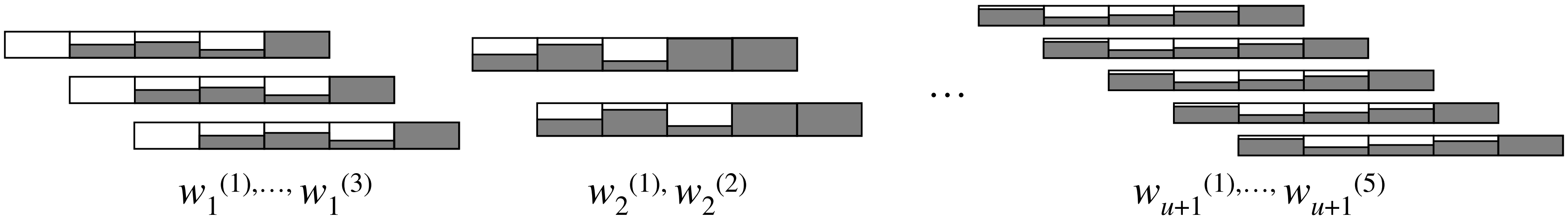

…
$w_1^{(1)},\dots, w_1^{(3)}$
$w_2^{(1)}, w_2^{(2)}$
$w_{u+1}^{(1)},\dots, w_{u+1}^{(5)}$

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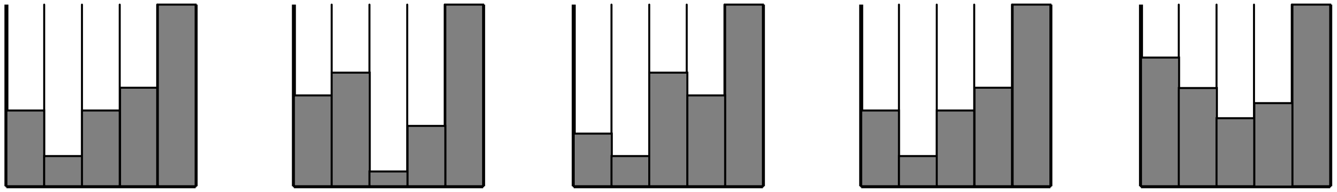
Desired slot contents: $$w^*_{s,j} := \begin{cases} \frac{|O(e_j)|}{S} \cdot b(e_j, s) & j = 1,\dots,r \\ \frac{n}{S} & j = r+1 \end{cases}$$

Slots/Bins:



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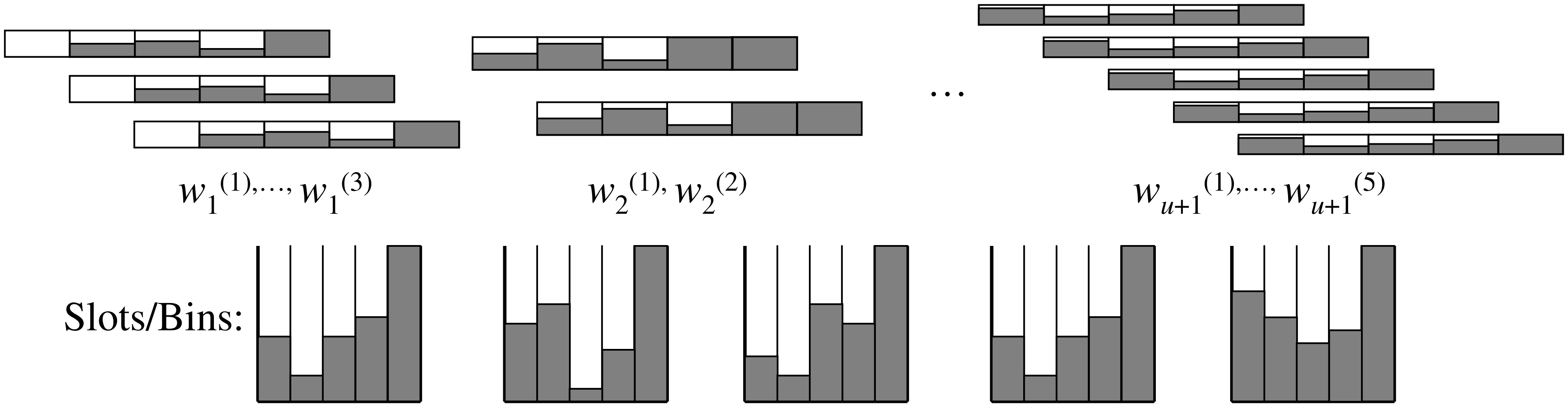

$w_1^{(1)},\ldots, w_1^{(3)}$
$w_2^{(1)}, w_2^{(2)}$
…
$w_{u+1}^{(1)},\ldots, w_{u+1}^{(5)}$
Slots/Bins:

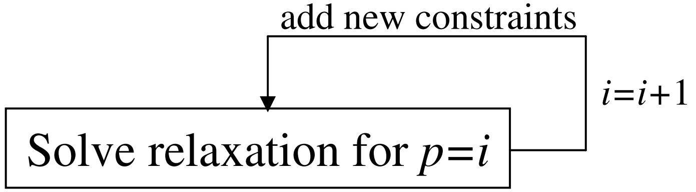
- Greedy approach: In each step choose a pair of vector and bin, so that the vector improves on the bins contents best possible

- This yields an assignment of an order o to its set of preferred slots $S^*(o)$.

Assignment of Orders to Slots

- Given for each order o a set $S^*(o)$ of preferred slots
- Assign each order to a preferred slot (respect other banning / spreading rules)

- Binary variable $x_{o,s}$ to indicate that o is assigned to slot s.
- Spreading objective: $\sum_{o:c\in o} x_{o,s} = O(c)/S\cdot b(c,s), \quad \forall s=1,\dots,S,\ c\in C_{Spr}$
- Clustering objective: $\sum_{o:c\in o,\ s\notin S^*(o)} x_{o,s} = 0, \quad \forall s=1,\dots,S,\ c\in C_{Clu}$
- Slot size constraint: $\sum_{o} x_{o,s} = n/S, \quad \forall s=1,\dots,S$
- Banning constraint: $\sum_{o:c\in o} x_{o,s} \le n/S-|B(c,s)|, \quad \forall s=1,\dots,S,\ c\in C_{Ban}$



add new constraints

i=i+1

Solve relaxation for p=i

- Number of variables can be reduced significantly

Results

- Easily configurable
  - simple rule definitions
  - transparent parameter tuning with priorities

- High performance
  - Operates with greedy subroutines and on small linear programs only
  - About 3 minutes on 1500 orders and realistic rules (Sun 450 MHz, 1GB)

- Color batch size increased by 50%

- The algorithm is currently used in all automobile plants of the Ford Motor Company across Europe

BTU

Thanks for your attention.