Implicit Factorization of Structured Matrices

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Abstract

To compute the eigenvalues of a symmetric or skew–symmetric matrix A, we can use a one–sided Jacobi–like algorithm. This algorithm begins by a suitable factorization of A. In some applications, A is given implicitly by its factors but the factors may be unsuitable for the Jacobi process. To avoid explicit computation of A, the factors have to be preprocessed.

For example, matrix $A = B^\tau C \pm C^\tau B$ is a such matrix with implicitly given factors, i.e.

$$A = \begin{bmatrix} B^\tau & C^\tau \end{bmatrix} \begin{bmatrix} 0 & I \\ \pm I & 0 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} := G^\tau J G.$$

In case of a plus sign, the solution is the diagonalization of J, followed by the indefinite QR factorization of G. In the other case (a minus sign), the matrix A is, obviously, skew–symmetric and can be implicitly factorized using symplectic and block–symplectic matrices. We describe and analyze the corresponding factorization algorithm.

The error bound of this factorization fits well in the relative perturbation theory for eigenvalue computation of skew–symmetric matrices.

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