Analytical model for the focusing of pinhole photon sieve
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Recently, Kipp et al. [1] developed a novel diffractive optical element named photon sieve, which consists of a large number of pinholes, for the focusing and imaging of soft x-rays. Their analysis was based on the Fresnel zone plate theory and the numerical calculation of Fresnel-Kirchoff diffraction integral. We present a purely analytical model for the focusing and imaging of pinhole photon sieve (as shown in Fig. 1), by absorbing the idea that the total near-field at the focal plane consists of many individual far-fields of different pinholes [2]. Our model is not limited to an incident spherical wave but applicable to arbitrary paraxial illumination.

We check the validity range of the individual far-field approximation and find that it is highly accurate when the corresponding Fresnel number $N$ is smaller than 0.05. When $N$ increases, the error also increase. However, the sum of the far-field term and the quasi-far-field correction term is still highly accurate even when $N = 0.2$. To understand these results better, we investigate the optical prototype of photon sieve of reference [1], where the parameters $\lambda$ and $q$ are chosen such that $\lambda = 0.6328 \mu m$ and $q = 1 \text{ m}$. We find that, when the diameter $d$ of the pinhole is 200 $\mu m$ ($N$ is about 0.05), the individual far-field approximation is a good one.

The crucial idea of the photon sieve is that all those individual diffracted fields from different pinholes have the same phase value at the target focus point. In this condition, each pinhole has a constructive contribution to the focusing. We derive the generalized condition which are valid for arbitrary paraxial illumination provided that the pinholes are small ($N$ is smaller than 0.05). Of course, when the incident beam is a spherical wave, our condition correspond to the case of Fresnel zones [1]. Especially, for a spherical wave illumination, the individual far-field model predicts that the constructive and the destructive contributions compensate at $d/w$ values of 2.4395, 4.4665, 6.4763, and so on, where $w$ is the width of Fresnel zone. These values are in excellent agreement with those values of about 2.4, 4.4, 6.4 obtained from the numerical calculation of Fresnel-Kirchhoff diffraction integral in reference [1]. Our model can be used for the analysis, fast simulation and optimal design of pinhole photon sieve.