FernUniversität in Hagen
Faculty of Mathematics and Computer Science

Artificial Intelligence Group

Analysis of Challenged Semantics for Abstract Argumentation Frameworks

Bachelor's Thesis

in partial fulfillment of the requirements for
the degree of Bachelor of Science (B.Sc.)
in Informatik

submitted by
Sebastian Wagner

First examiner: Prof. Dr. Matthias Thimm
Artificial Intelligence Group

Advisor: Lars Bengel
Artificial Intelligence Group

Statement

I declare that I have written the bachelor´s thesis independently and without unauthorized use of third parties. I have only used the indicated resources and I have clearly marked the passages taken verbatim or in the sense of these resources as such. The assurance of independent work also applies to any drawings, sketches or graphical representations. The work has not previously been submitted in the same or similar form to the same or another examination authority and has not been published. By submitting the electronic version of the final version of the bachelor´s thesis, I acknowledge that it will be checked by a plagiarism detection service to check for plagiarism and that it will be stored exclusively for examination purposes.

I explicitly agree to have this thesis published on the webpage of the artificial intelligence group and endorse its public availability.

Software created for this work has been made available as open source; a corresponding link to the sources is included in this work. The same applies to any research data.

Kamen, 02.06.2025 .... S. W[illegible]

(Place, Date) (Signature)

Zusammenfassung

Mit Serialisierung können zulässige Extensionen für abstrakte Argumentationsgraphen konstruiert und gleichzeitig die Begründung von Argumenten nachvollziehbar gemacht werden. Darüber hinaus ermöglicht es die Definition neuer Semantiken. In dieser Bachelorarbeit soll die herausgeforderte Semantik, welche mithilfe von Serialisierung unattackierte und herausgeforderte initiale Mengen ausschöpfend selektiert, definiert und untersucht werden. Zur Untersuchung wird die herausgeforderte Semantik im Vergleich zu anderen Semantiken charakterisiert. Weiter wird untersucht, welche gängige Prinzipien für abstrakte Argumentationsgraphen die herausgeforderte Semantik erfüllt und welche Rechenkomplexität gängige Probleme für abstrakte Argumentationsgraphen unter der herausgeforderten Semantik haben.

Abstract

With *serialisability* admissible extensions for abstract argumentation frameworks can be constructed while also making the justification of arguments reasonable. In addition, it allows the definition of new semantics. In this bachelor thesis we want to define and investigate the challenged semantics that exhaustively selects unattacked and challenged initial sets via serialisability. For investigation, the challenged semantics is characterised in comparison to other semantics. We further analyse which common principles for abstract argumentation frameworks the challenged semantics satisfy as well as the computational complexity for common problems for abstract argumentation frameworks under the challenged semantics.

Contents

1 Introduction

In everyday life, people argue about every imaginable topic like politics, sports, weather in many different ways. Argumentation can be seen as a social activity of critical thinking aimed at justifying or challenging an opinion, through positive or negative interconnected arguments and with the purpose of persuading an audience [32]. Due to the fact, that the inferences on arguments of an argumentation can change drastically with every new argument added, a relation can be seen to *non-monotonic reasoning*, a sub area of formal logic, where new information can lead to different conclusions. Consider, for example, following fictive argumentation on the introduction of a new management software:

A_1 We are introducing a new digital administration system.

A_2 The old system is still working and switching is too risky.

A_3 The new system has been tested successfully and offers improved security.

A_4 New systems tend to fail in practice.

Argument A_1 is attacked by argument A_2 that claims to stay at the current administration system. Arguments A_2 and A_3 mutually attack each other due to A_2 questioning the benefits of A_3 and A_3 invalidates A_2, since the system is tested which lower the risks. In a same way argument A_3 attacks A_4. Each new argument changes the viewpoint, whether the introduction of a new management software is a good idea or causes problems, and no final conclusion can be made, if personal factors are left out.

With his paper from 1995, Phan Minh Dung showed, that non-monotonic reasoning is a form of argumentation by introducing a simple mathematical formalism for argumentation, which represents a significant milestone in artificial intelligence [20]. Since then, his formalism of *Abstract Argumentation Frameworks* (AAFs) has been subject of many investigations and scientific papers and has found its way into various areas of artificial intelligence systems such as chatbots [18] or in legal reasoning [28].

Dung's approach is to abstract away from the content of arguments and concentrate on the attacks between them, where an attack indicates that one argument challenges the justification of another [20]. This allows to model an argumentation as a pair consisting of a set of arguments and an attack relation. Besides that, this model of argumentation can be presented as a directed graph, where arguments are the vertices and attacks are the directed edges indicating the source and the destination of an attack. For instance,the directed graph for the previous argumentation is depicted in Figure 1.

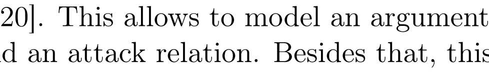


Figure 1: A formalisation of the example argumentation

Given an abstract argumentation framework, possible solutions in form of sets of arguments can be evaluated. Important properties, that these solutions often should comply with, are *conflict-freeness* and *admissibility* [20]. While, for a considered set of arguments, *conflict-freeness* requests no attacks between set members of a considered set of arguments, *admissibility* refers to the ability to defend oneself. In this regard self defence can be understood as follows: if an argument of the considered set is attacked, then this attacker must itself be attacked either due to a mutual attack or through the attack of another argument of the set.

Meanwhile, the idea of abstract argumentation frameworks has been expanded to overcome limitations of the original model resulting from its simplicity. For example, the *Bipolar Argumentation Framework* (BAF) adds a support relation that goes beyond defence [17], the *Weighted Argumentation Framework* (WAF) adds weights to the attacks and introduce an inconsistency budget that allows to neglect weak attacks [22] and the *Probabilistic Argumentation Frameworks* (PrAF) suggests the use of probabilities for arguments or attacks as a quantification of uncertainty regarding their presence [25][24]. Besides the abstract approach, *ABA* [30] and *ASPIC+* [26] are examples for *structured* argumentation, where arguments arise through derivation and attacks arise from rules or conclusions. In this bachelor thesis, however, we only consider the original *Abstract Argumentation Framework* from Dung.

As already mentioned before, the purpose of an argumentation is to convince an audience. In this regards, an audience has its own views and expectations of a solution, that is maybe influenced by their social, cognitive, emotional or contextual situation [32]. An argumentation on therapies for improving medical conditions of an human being is likely to be handled with more skepticism than an argumentation about which film to see in the cinema. In *Abstract Argumentation Frameworks* this can be modelled via *semantics*. These *semantics* are the basis for evaluating *Abstract Argumentation Frameworks* by applying rules and conditions that affect the acceptance of arguments [20]. Generally, there are three styles of semantics definitions, namely *labeling-based* [16], *extension-based* [5] and *ranking-based* [1]. The labeling-based approach defines a set of labels and how they are applied to arguments, while the extension-based characterises which properties a set of arguments must fulfill in order to belong to a semantics [5]. A set that satisfies those properties of a semantics is called an *extension* of that semantics. We will focus on the *extension-based* approach in this bachelor thesis.

For *Abstract Argumentation Frameworks*, various semantics have already been proposed. In his paper, for example, Dung already defined four semantics: *grounded*, *complete*, *preferred* and *stable* [20]. The *grounded* semantics only take into account the minimal acceptable (wrt. set inclusion) set of arguments and has exactly one extension for an arbitrary abstract argumentation framework. For *complete* semantics an extension includes all arguments it defends. An extension is a *preferred* extension if it is admissible and maximal (wrt. set inclusion). Moreover, if an extension is preferred and every argument outside of the extension is attacked by that extension, it is *stable*. There are further semantics from the literature like *ideal* [21], *semi-stable* [15], *strong admissible* [5][14] and many others.

Motivated by the intention to decompose large extensions to better express the reasoning process why arguments may be contained in that extension, a non-deterministic, iterative approach called *serialisability* was formalised [29]. Basic concepts of serialisability are *initial sets* and *reduct*. An initial set is a non-empty, admissible set that is minimal wrt. set inclusion. Initial sets were introduced in [33] and further investigated in [29]. They can be categorised totally into three types: *unattacked* initial sets that are not attacked at all, *unchallenged* initial sets that are not attacked by other initial sets but by other arguments and challenged initial sets that are in conflict with other initial sets [29]. Due to their properties, initial sets can be seen as a solution to a local conflict [29]. A *reduct* can be characterised as a subframework of an abstract argumentation framework AF derived in regards to a set S. It is obtained by removing S and all arguments S attacks from AF [9]. The serialised extension construction itself is quite simple. Iteratively select an initial set of the current considered abstract argumentation framework, then progress to the reduct corresponding to the selection until a certain termination criteria is satisfied. This carries the idea that we first solve a small and local problem and progress to a framework without the nodes already decided. Most of the common admissibility-based semantics can be characterised by serialisation and it is also possible to define new semantics through defining selection and termination on initial sets [29].

One of these semantics is the *unchallenged semantics* analysed in [11]. For the unchallenged semantics only unattacked and unchallenged initial sets are allowed to be selected and it will be iterated until none of these two types of initial sets can be found in the remaining abstract argumentation framework. Since unchallenged initial sets do not contradict any immediately acceptable arguments, it is reasonable in a general consensus to accept them just as unattacked initial sets.

In this bachelor thesis we characterise the *challenged semantics* via serialisability and afterwards analyse it in detail. Instead of unattacked and unchallenged initial sets the challenged semantics exhaustively selects unattacked and challenged initial sets. In contrast to unattacked and unchallenged initial sets, that are by an general consensus acceptable, challenged initial sets describe contradictory positions. You can either accept the one or the other. Challenged semantics can help to understand the effects of selecting the one challenged initial set over the other or to put it another way, what is influenced by the decision for one or the other side. To underline this, we will demonstrate the challenged semantics on the previous argumentation, without clarifying the mathematical prerequisites at this point.

For the previous argumentation depicted in Figure 1, arguments A_2 and A_3 are challenged initial sets. Because we abstracted from the actual content, there is no bias, so arguments A_2 and A_3 can be considered equally acceptable. The choice, which challenged initial set we choose, directly influences the acceptance of arguments A_1 and A_4 If we select A_2 to accept, we have to reject A_1 and A_3 due to attacks of A_2 and can then accept argument A_4, because there is nothing left, that attacks it. If we first select argument A_3, then arguments A_2 and A_4 are rejected and arguments A_1 remains unattacked and can thus be accepted, too.

That shows, that the acceptance of argument A_1 or A_4 is directly connected to the conflict between arguments A_2 and A_3 Moreover, the selection of either argument A_2 or A_3 leads to exclusive results (A_1 is only acceptable after A_3 has been accepted and A_4 is only acceptable after A_2 has been accepted). To conclude this in an informal way: You can only have one thing and have to live with the consequences of your choice.

1.1 Related Work

Before presenting the research goals for this bachelor thesis, we discuss related work. Semantics were introduced in various papers. An overview of many semantics is given in [2]. It has to be noted, that we only consider semantics that are based on admissibility. This is due to the fact, that challenged semantics is defined by serialisability, which itself is inherently coupled to admissibility. Thus, the challenged semantics is not compared against semantics like *CF2* [4] or *weak-admissibility* [8]. Moreover, a principle based analysis of semantics has been done, for example, in [5], [19] or [11]. The same holds for studying the computational complexity. An overview is given in [23], while [29] and [11] considers computational problems and their complexity related to serialisability and serialisable semantics. In fact, this bachelor thesis shares similarities to [11] due to the close relationship between unchallenged and challenged initial sets.

As we already know, serialisability is a formalisation that allows to construct extensions of different semantics [29]. This was directly influenced by [33] where an iterative approach is used by continuously applying the characteristic function, a function that returns all defended arguments in regards to a set S, and adding defended arguments to S starting at *initial-like* arguments or their interpretation of initial sets. Another way to express semantics while giving the ability to define new semantics is presented in [6] by *SCC-Recursiveness*. This approach defines a recursive algorithm which passes through all *strongly connected components* (SCC) to construct extensions. Interestingly, serialisability does not imply SCC-recursiveness and vice versa [11] and that although every initial set of an abstract argumentation framework is also an initial set of a single SCC and all conflicts in regards to initial sets are within a single SCC [29].

Other noteworthy work is the transfer of serialisability to ranking-semantics [13], the characterising of serialisation equivalence [10] or the improvement of algorithms that calculates the unchallenged semantics [12].

1.2 Research Questions and Structure of the Thesis

The term "analysis" is broad and unexplanatory. Therefore, we limit the scope of the bachelor's thesis by specifying concrete research questions.

Research Question 1. How can the challenged semantics be defined and how can it be characterised in comparison to other common semantics?

Research Question 2. Which argumentation principles does the challenged semantics satisfy?

Research Question 3. What is the computational complexity of common problems w.r.t. challenged semantics in abstract argumentation frameworks?

Since the research questions already implies a structure, the bachelor thesis is organised as follows: In Section 2 the needed mathematical constructs and ideas such as *Abstract Argumentation Frameworks*, *Initial Sets*, *Reduct* and *Serialisability*, as well as the considered principles are defined. Section 3 starts with the definition of the challenged semantics followed by an example-driven analysis and a comparison to other semantics based on the extensions. Section 4 and 5 analyses the challenged semantics in regards of principle satisfaction and time complexity. In Section 6 we discuss and compare the results of the previous Sections. Section 7 then present ideas for future work and the thesis ends with a conclusion in Section 8.

2 Background

The following section gives a brief overview of definitions, concepts and notation on which this bachelor thesis is based on. First, abstract argumentation frameworks are introduced, along with semantics that allow the selection of admissible sets while considering specific constraints. Then, serialisation is presented, enabling the definition of new semantics, such as the challenged semantics, which will be examined in this bachelor's thesis. Additionally, general principles are explained to facilitate an objective analysis.

2.1 Abstract Argumentation Frameworks

We first define *Abstract Argumentation Frameworks* and *Semantics*, as they are the essential base constructs for this thesis.

Definition 1. An *abstract argumentation framework* is a tuple $AF = (A, R)$, where A is a finite set of arguments and R is a relation $R \subseteq A \times A$. [20]

Each element $(a, b) \in R$, also written as aRb, expresses that argument a *attacks* argument b. This definition enables us to represent an abstract argumentation as a directed graph. Most of the time we consider not just single arguments, but sets of arguments. Therefore, for a set $X \subseteq A$ we introduce $AF|_X = (X, R \cap X \times X)$ as the projection of AF on X. Likewise, for a set $S \subseteq A$, we define

$$S^+_{AF} = \{a \in A \mid \exists b \in S : (b, a) \in R\}$$
$$S^-_{AF} = \{a \in A \mid \exists b \in S : (a, b) \in R\}.$$

The specification of the abstract argumentation framework is omitted when it is clear which argumentation framework the set belongs to.

A set S *attacks* another set S', if $S' \cap S^+ \neq \emptyset$. A set S is *defended* by S' if each attacker $b \in S^-$ of S is attacked by S'. If there are no arguments $a, b \in S$ with $(a, b) \in R$, we say that S is *conflict-free*(cf). In addition, if S defends itself against all attackers, S is *admissible*.

Definition 2. Let $S \subseteq A$ be a set of arguments. S is *admissible*(ad) iff S is conflict-free and S defends every $a \in S$.

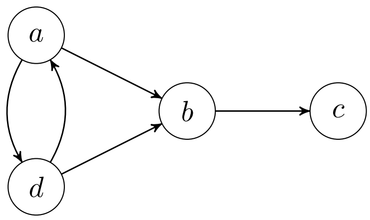


Figure 2: Directed graph for AF_2.

Example 1. Let AF_2 be an abstract argumentation framework with $A = \{a, b, c, d\}$ and $R = \{(a, b), (a, d), (b, c), (d, b)\}$. The corresponding graph is shown in Figure 2.

Let $S = \{a, c\}$ be a set of arguments of AF_2. S is conflict-free, because there is no attack between a and c. S also defends itself against all incoming attacks, since a defends itself against the attack of d, and c by attacking b. Therefore, S is also admissible. The same applies to $S' = \{d, c\}$, where d defends itself against a, and also c by attacking b.

An admissible set is one valid solution in the context of the abstract argumentation framework. The set of all admissible sets of an abstract argumentation framework will be denoted as $\mathsf{adm}(\mathrm{AF})$. Semantics make it possible to select admissible sets, also called extensions, under certain aspects and constraints [2].

Definition 3. An admissible set E is

- a *complete*(co) extension iff for all $a \in A$, if E defends a then $a \in E$,
- a *grounded*(gr) extension iff E is complete and there is no complete extension E' for that $E' \subsetneq E$ holds,
- a *stable*(st) extension iff $E \cup E^+_{AF} = A$,
- a *preferred*(pr) extension iff there is no admissible set E' for that $E \subsetneq E'$ holds,
- an *ideal*(id) extension iff E is the maximal admissible set with $E \subseteq E'$ for each preferred extension E',
- a *semi-stable*(sst) extension iff $E \cup E^+_{AF}$ is maximal,
- a *strongly admissible*(sa) extension iff $E = \emptyset$ or each $a \in E$ is defended by some strongly admissible $E' \subseteq E \setminus a$.

All specifications regarding the size or maximality/minimality of a set are to be understood in terms of set inclusion.

Example 2. Consider AF_2 in Figure 2. The extensions of the given semantics are as follows:

Semantic	Extensions
co	$\{\{a, c\}, \{d, c\}, \emptyset\}$
gr	$\{\emptyset\}$
st	$\{\{a, c\}, \{d, c\}\}$
pr	$\{\{a, c\}, \{d, c\}\}$
id	$\{\emptyset\}$
sst	$\{\{a, c\}, \{d, c\}\}$
sa	$\{\emptyset\}$

Table 1: Extensions of AF_2 for different semantics.

2.2 Serialisability

Recent publications have introduced *serialisability*, a new approach for determining extensions of admissibility-based semantics [29]. The realization that *initial sets* are building blocks of admissibility-based semantics from [33] was a crucial observation.

Definition 4. Let $AF = (A, R)$ be an abstract argumentation framework. A set $S \subseteq A$, $S \neq \emptyset$ is called an *initial set* iff S is admissible and there is no other $S' \neq \emptyset$ for that $S' \subsetneq S$ holds.

The set of all initial sets of an abstract argumentation framework is denoted as $\mathsf{IS}(AF)$. Furthermore, initial sets can be completely divided into three distinct categories [29].

Definition 5. Let $AF = (A, R)$ be an abstract argumentation framework. An initial set $S \in \mathsf{IS}(AF)$ is

1. *unattacked* iff $S^- = \emptyset$
2. *unchallenged* iff $S^- \neq \emptyset$ and there is no other $S' \in \mathsf{IS}(AF)$ that attacks S
3. *challenged* iff there another $S' \in \mathsf{IS}(AF)$ that attacks S.

In the following, unattacked, unchallenged, and challenged initial sets are represented by $\mathsf{IS}^{\not\leftarrow}(AF)$, $\mathsf{IS}^{\not\leftrightarrow}(AF)$, and $\mathsf{IS}^{\leftrightarrow}(AF)$, respectively.

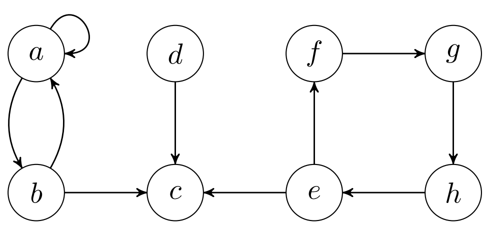


Figure 3: AF_3 with unattacked, unchallenged and challenged initial sets

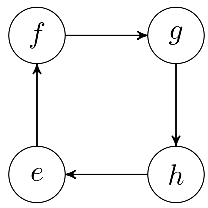


Figure 4: S-reduct $AF_3^{\{b,d\}}$

Example 3. Lets assume we have the argumentation framework AF_3 shown in Figure 3. Then we have $\mathsf{IS}(AF_3) = \{\{b\}, \{d\}\, \{e, g\}, \{f, h\}\}$ with $\mathsf{IS}^{\not\leftarrow}(AF_3) = \{d\}$, $\mathsf{IS}^{\not\leftrightarrow}(AF_3) = \{b\}$ and $\mathsf{IS}^{\leftrightarrow}(AF_3) = \{\{e, g\}, \{f, h\}\}$.

Another important concept is the *S-reduct*, or simply reduct [7]. It enables us to remove a set S of arguments from an abstract argumentation framework as well as the arguments attacked by S.

Definition 6. Let $AF = (A, R)$ be an abstract argumentation framework and $S \subseteq A$ a set of arguments. The S-Reduct AF^S is defined by $AF^S = (A', R')$ with $A' = A \backslash (S \cup S^+)$ and $R' = R \cap A' \times A'$.

Example 4. Consider AF_3 and the set $S = \{b, d\}$. Since S attacks some arguments, there is $S^+ = \{a, c\}$. If we remove S and S^+, then we are left with $AF_3^{\{b,d\}} = \{e, f, g, h\}$ shown in Figure 4.

Serialisability combines initial sets with reducts in an iterative manner. Sequentially, an initial set is selected from the graph under consideration and is then removed by building the reduct, until no more initial sets remain. The selected initial sets represent a *serialisation sequence* [13].

Definition 7. A *serialisation sequence* for an abstract argumentation framework AF is a sequence $\mathscr{S} = (S_1, \dots, S_n)$ for which $S_1 \in \mathsf{IS}(AF)$ holds and $S_i \in \mathsf{IS}(AF^{S_1 \cup \dots \cup S_{i-1}})$ holds for all $2 \leq i \leq n$.

Let $\mathscr{S} = (S_1, \dots, S_n)$ be a serialisation sequence. We denote the set $S = S_1 \cup \dots \cup S_n$ as the *$\mathscr{S}$-induced* set. Since we only select initial sets that are, by themselves, always admissible, the set induced by $\mathscr{S}$ is also admissible [29]. Furthermore, every admissible, non empty set can be expressed by at least one serialisation sequence while a serialisation sequence induces exactly one admissible set [29].

Example 5. Lets assume we have AF_3. $\mathsf{IS}(AF_3)$ is shown in Example 3. We might pick the unchallenged argument d first and add it to the serialisation sequence $\mathscr{S}_1 = (\{d\})$. Now we reduce the graph. $AF_3^{\{d\}}$ is shown in Figure 5. For $AF_3^{\{d\}}$ the initial sets are $\mathsf{IS}(AF_3^{\{d\}}) = \{\{b\}, \{e, g\}, \{f, h\}\}$. We might now choose b as the next initial set: $\mathscr{S}_1 = (\{d\}, \{b\})$. The reduct $AF_3^{\{b,d\}}$ is already shown in Figure 4 and has the intial sets $\mathsf{IS}(AF_3^{\{b,d\}}) = \{\{e, g\}, \{f, h\}\}$. We might choose $\{f, h\}$. The serialisation sequence then is $\mathscr{S}_1 = (\{d\}, \{b\}, \{f, h\})$ and the reduct $AF_3^{\{b,d,f,h\}}$ has no more arguments and therefore no more initial sets. The $\mathscr{S}_1$-induced set would be $S_1 = \{b, d, f, h\}$. We would get the same induced set S_1 with the serialisation sequence $\mathscr{S}_2 = (\{b\}, \{d\}, \{f, h\})$.

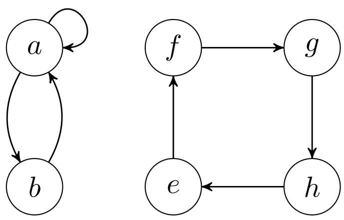


Figure 5: Reduct $AF_3^{\{d\}}$

It is important to notice that the concept of serialisation does not make any suggestions which initial set to choose. Therefore serialisability is a non-determinstic algorithm. This is why Example 5 is in some kind a simplification, because it just look at one possible path. Due to the fact, that building a reduct may influence the initial sets we might need to look for every possible serialisation sequence to have a complete view.

Most of the shown semantics can also be characterised by serialisability.

Theorem 1. *Let $AF = (A, R)$ be an abstract argumentation framework and $E \subseteq A$. Then E is*

- *a complete extension iff there is a serialisation sequence $\mathscr{S} = (S_1, \ldots, S_n)$ for $E = S_1 \cup \cdots \cup S_n$ and its hold that $\mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \cdots \cup S_n})$,*
- *a grounded extension iff there is a serialisation sequence $\mathscr{S} = (S_1, \ldots, S_n)$ for $E = S_1 \cup \cdots \cup S_n$ and for all S_i, $i = 1, \ldots, n$ it holds that $S_i \in \mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \cdots \cup S_{i-1}})$ and additionally $\mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \cdots \cup S_n}) = \emptyset$ holds,*
- *a preferred extension iff there is a serialisation sequence $\mathscr{S} = (S_1, \ldots, S_n)$ for $E = S_1 \cup \cdots \cup S_n$ and it holds that $\mathsf{IS}(AF^{S_1 \cup \cdots \cup S_n}) = \emptyset$,*
- *a stable extension iff there is a serialisation sequence $\mathscr{S} = (S_1, \ldots, S_n)$ for $E = S_1 \cup \cdots \cup S_n$ and it holds that $AF^{S_1 \cup \cdots \cup S_n} = (\emptyset, \emptyset)$.*

Example 6. Consider again AF_2 in Figure 2. The abstract argumentation framework has $\mathsf{IS}(AF_1) = \{\{a\}, \{d\}\}$. We first take a look on the serialisation sequences for the complete semantics(co). Since co does not give restrictions on which initial sets to choose from, we start with $\{a\}$: $\mathscr{S}_{co_1} = (\{a\})$. From $AF_2^{\{a\}}$ we can then choose $\mathsf{IS}^{\not\leftarrow}(\{c\})$ and get $\mathscr{S}_{co_1} = (\{a\}, \{c\})$ and no more arguments in the reduct. For the second sequence, we instead start with $\{d\}$: $\mathscr{S}_{co_2} = (\{d\})$. In $AF_2^{\{d\}}$ the unattacked initial set $\mathsf{IS}^{\not\leftarrow}(\{c\})$ remains: $\mathscr{S}_{co_2} = (\{d\}, \{c\})$. With $\mathscr{S}_{co_3} = (\emptyset)$ there is a third serialisation sequence that fulfill all requirements set in theorem 1.

For the preferred semantics (pr) we also have $\mathscr{S}_{pr_1} = (\{a\}, \{c\})$ and $\mathscr{S}_{pr_2} = (\{d\}, \{c\})$, but $\emptyset$ is not a valid solution for pr due to $\mathsf{IS}(AF_2) \neq \emptyset$.

With serialisablity we can also characterize new semantics like the unchallenged semantics [11].

Definition 8. Let $AF = (A, R)$ be an abstract argumentation framework. We say that *uc(AF)* is the set containing all subsets $S \subseteq A$ for which there exists a serialisation sequence $\mathscr{S}_{uc} = (S_1, \ldots, S_n)$ such that $S = S_1 \cup \cdots \cup S_n$ and it holds that $S_1 \in \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\not\leftrightarrow}(AF)$ and for all S_i, $i = 2, \ldots, n$ that $S_i \in \mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \cdots \cup S_{i-1}}) \cup \mathsf{IS}^{\not\leftrightarrow}(AF^{S_1 \cup \cdots \cup S_{i-1}})$ and $\mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \cdots \cup S_n}) \cup \mathsf{IS}^{\not\leftrightarrow}(AF^{S_1 \cup \cdots \cup S_n}) = \emptyset$.

In the unchallenged semantics the selection of initial sets is restricted to unattacked and unchallenged inital sets, until none of these can be further selected in the corresponding reduct.

Example 7. We look again at AF_3 in Figure 3. As in example 5 for $\mathscr{S}_{uc_1}$ we can first select $\{d\}$ and then $\{b\}$. From $AF_3^{\{d,b\}}$ no more initial sets can be selected under the constrains of the unchallenged semantics. Therefore, $\mathscr{S}_{uc_1} = (\{d\}, \{b\})$ The same initial sets can be selected in other order leading to $\mathscr{S}_{uc_2} = (\{b\}, \{d\})$. Both serialisation sequences induce the same set $S_1 = \{b, d\}$.

2.3 Principles

Principles allow an objective analysis of semantics and help to make them comparable. This chapter introduces principles from the literature that are considered in the investigation of the challenged semantics. In particular, we define unattacked sets for the introduction of *directionality*, give some additional definitions for the introduction of *SCC-Recursiveness* [6] and specify *Strong Admissibility* below.

Definition 9. Let $AF = (A, R)$ be an abstract argumentation framework. A set $U \subseteq A$ is called *unattacked* iff there is no $a \in (A \setminus U)$ that attacks $b \in U$.

The set of unattacked sets of AF will be written as $\mathfrak{US}(AF)$.

Definition 10. Let $AF = (A, R)$ be an abstract argumentation framework. A set $S \subseteq A$ is a *strongly connected component* of AF, if there is a directed path between any pair $a, b \in S$ in AF and there is no $S' \supset S$ with that property.

The set of strongly connected components for AF will be denoted as SSCs_{AF}.

Example 8. Let us investigate AF_3 from Figure 6 in terms of strongly connected components. $\{a, b\}$ is a strongly connected component, because there is a path from a to b and vice versa. Another strongly connected component is $\{e, f, g, h\}$. Due to its circle shape each argument has a path to each other. The argument d has only outgoing edges so that it is not possible to reach d in any way, therefore it is a strongly connected component on its own. Nearly the same holds for c, which does not have any outgoing edges so that there is no path to other arguments. Hence, $\mathsf{SSCs}_{AF_3} = \{\{a, b\}, \{c\}, \{d\}, \{e, f, g, h\}\}$.

For the following concepts we need a slightly modified perspective for the set of attackers of an given set S.

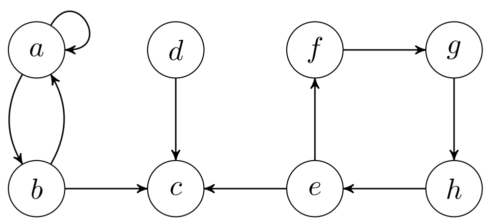


Figure 6: AF_3 from Figure 3 reproduced for ease of reference.

Definition 11. Let $AF = (A, R)$ be an abstract argumentation framework and $S \subseteq A$ be a set of arguments. We define: $\mathsf{op}_{AF}(S) = \{a \in A \mid a \notin S \wedge aRS\}$

Therefore $\mathsf{op}_{AF}(S)$ contains all attacked arguments of S that are not in S themselves.

Definition 12. Given an abstract argumentation framework $AF = (A, R)$, a set $E \subseteq A$ of arguments and a strongly connected component $S \in \mathsf{SSCs}_{AF}$. We define:

- $D_{AF}(S, E) = \{a \in S \mid (E \cap \mathsf{op}_{AF}(S))Ra\}$,
- $P_{AF}(S, E) = \{a \in S \mid (E \cap \mathsf{op}_{AF}(S)) \not{R}a \wedge \exists b \in (\mathsf{op}_{AF}(S) \cap a^{-}_{AF}) : E \not{R}b\}$,
- $U_{AF}(S, E) = S \setminus (D_{AF}(S, E) \cup P_{AF}(S, E))$.

The function abbreviations D, P and U stand for *defeated*, *provisionally defeated* and *undefeated* with respect to the set E composed of arguments we want to accept [6]. Defeated are therefore those arguments of the strongly connected component, that are directly attacked by E from outside of S. Arguments of S are provisionally defeated if they are attacked from outsite of S, but not by arguments of E. All remaining arguments are undefeated, which means that they are not attacked by but instead defended by E.

With these functions it is possible to define a recursive algorithm to examine abstract argumentation frameworks based on their strongly connected components [6].

Definition 13. Given an abstract argumentation framework $AF = (A, R)$ and a set of arguments $C \subseteq A$.

1. A function $\mathscr{BF}(AF, C)$ is called *base function*, if, given an argumentation framework $AF = (A, R)$ such that $|\mathsf{SSCs}_{AF}| = 1$ and a set $C \subseteq A$, we have that $\mathscr{BF}(AF, C) \subseteq 2^A$,
2. Given a base function $\mathscr{BF}(AF, C)$, we define the function $\mathscr{GF}_{\mathscr{BF}}(AF, C) \subseteq 2^A$ as follows: for any $E \subseteq A, E \in \mathscr{GF}(AF, C)$ if and only if
   - in case $|\mathsf{SSCs}_{AF}| = 1, E \in \mathscr{BF}(AF, C)$,
   - otherwise, $\forall S \in \mathsf{SSCs}_{AF} : E \cap S \in \mathscr{GF}_{\mathscr{BF}}(AF|_{S \setminus D_{AF}(S,E)}, U_{AF}(S, E) \cap C)$.

The generic selection function $\mathscr{GF}$ allows us to select every possible strongly connected component recursively under consideration of previously accepted arguments in E . This is done with the help of graph projections on strongly connected components without their defeated arguments. If the projection only contains a single strongly connected component, the acceptable arguments of this component are evaluated with the base function $\mathscr{BF}$.

The next definition introduces strong defence which is essential for the *Strong Admissibility* Principle.

Definition 14. Let $AF = (A, R)$ be an argumentation framework and $E \subseteq A$ is a set of arguments. We say that an argument $a \in A$ is *strongly defended* by E, written as $sd(a, E)$, iff $\forall b \in A : bRa \Rightarrow \exists c \in E \setminus \{a\} : cRb$ and $sd(c, E \setminus \{a\})$

Now, we are in a position to specify the principles considered in this bachelor's thesis for analysing the challenged semantics.

We start by showing the *Conflict-Freeness* Principle that has a general purpose and that every good semantics should satisfy.

Principle 1. A semantics σ satisfies the principle of *Conflict-Freeness* [5] iff for every abstract argumentation framework AF, every $E \in \sigma(AF)$ is conflict-free with respect to the attack relation.

This means that there are not attacks between arguments of any extension for an arbitrary abstract argumentation framework AF.

The next principles deal with defence. For example, the following *Admissibility* Principle deals with whether every extension of a semantics is always admissible.

Principle 2. A semantics σ satisfies the principle of *Admissibility* [5] iff for every abstract argumentation framework AF, every $E \in \sigma(AF)$ is conflict-free and defends itself in AF.

In other words every extension of a semantics is an acceptable solution.

The *Strong Admissibility* Principle requires the stricter interpretation of strong defence.

Principle 3. A semantics σ satisfies the principle of *Strong Admissibility* [5] iff for every abstract argumentation framework AF, for every $E \in \sigma(AF)$ and every $a \in E$ it holds that $a \in E \Rightarrow sd(a, E)$.

Stated differently, every argument included in an extension of a semantics is at least defended by another argument of the extension. Hence, an argument is justified not only by itself, when it is attacked.

We also consider two attenuations of admissibility. The first one is the *Reduct-Admissibility* Principle.

Principle 4. A semantics σ satisfies the principle of *Reduct-Admissibility* [19] iff for every abstract argumentation framework AF and $E \in \sigma(AF)$ we have that $\forall a \in E$: if b attacks a then $b \notin \bigcup \sigma(AF^E)$.

The intuition behind the *Reduct-Admissibility* Principle is, that a set of arguments have to defend itself only against arguments, that have a chance to be accepted [8]. While the *Reduct-Admissibility* Principle states that the attacker of the extension may not be accepted in the reduct, the upcoming *Semi-Qualified Admissibility* Principle states that an extension only has to defend itself against arguments that are contained in at least one extension of the semantics [19].

Principle 5. A semantics σ satisfies the principle of *Semi-Qualified Admissibility* [19] iff for every abstract argumentation framework AF and $E \in \sigma(AF)$, we have that $\forall a \in E$, if b attacks a and $b \in \bigcup \sigma(AF)$ then $\exists c \in E$ s.t. c attacks b.

The last defence-based principle is the *Reinstatement* Principle.

Principle 6. A semantics σ satisfies the principle of *Reinstatement* [5] iff for every abstract argumentation framework $AF = (A, R)$ and $E \in \sigma(AF)$ it is true that $a \in E$ if E defends some $a \in A$.
The idea behind this principle is, that every argument that is defended, should also be accepted.

Extensions that contain as many arguments as possible are often desired. This is expressed by the next principle.

Principle 7. A semantics σ satisfies the principle of *I-Maximality* [5] iff for every abstract argumentation framework AF and $E_1, E_2 \in \sigma(AF)$, it is true that $E_1 \subseteq E_2 \Rightarrow E_1 = E_2$.
A semantics that satisfies this principle focuses only on solutions that are as comprehensive as possible.

The upcoming *Directionality* Principle describes, that the justification of an argument should only depend on the justification of their ancestors and not on the justification of their successors.

Principle 8. A semantics σ satisfies the principle of *Directionality* [5] iff for every abstract argumentation framework $AF = (A, R)$ and $\forall U \in \mathfrak{US}(AF)$ we have $\sigma(AF, U) = \sigma(AF|_U)$ with $\sigma(AF, U) = \{E \cap U \mid E \in \sigma(AF)\}$

Sometimes, it may be desirable to refrain from making a definite decision about an argument, especially when no clear justification can be found. This is formalised by the next principle.

Principle 9. A semantics σ satisfies the principle of *Allowing Abstention* [3] iff for every abstract argumentation framework AF and for every $a \in A$, if there exist two extensions $E_1, E_2 \in \sigma(AF)$ such that $a \in E_1$ and $a \in E_2^+$ then there exists an extension $E_3 \in \sigma(AF)$ such that $a \notin (E_3 \cup E_3^+)$.

The idea behind the next principle is that the point of view of an abstract argumentation framework can be merged with the point of view of the resulting reduct.

Principle 10. A semantics σ satisfies the principle of *Modularization* [7] iff for every abstract argumentation framework AF it is true that $E_1 \in \sigma(AF) \wedge E_2 \in \sigma(AF^{E_1}) \Rightarrow E_1 \cup E_2 \in \sigma(AF)$.

Another principle we consider is the *SCC-Recursiveness* Principle. Extensions were calculated on the basis of strongly connected components and their ancestors. *SCC-Recursiveness* also allows to characterize new semantics via the definition of a base function [6].

Principle 11. A semantics σ satisfies the principle of *SCC-Recursiveness* [6] iff there is a base function $\mathscr{BF}_\sigma$ such that for every abstract argumentation framework $AF = (A, R)$ we have that $\sigma(AF) = \mathscr{GF}_{\mathscr{BF}_\sigma}(AF, A)$.

A *SCC-Recursive* semantics can be calculated by considering only the SCC itself and the acceptance of arguments of ancestor SCCs.

To conclude the principles, we give a definition for the *Naivety* Principle.

Principle 12. A semantics σ satisfies the principle of *Naivety* [5] iff for every abstract argumentation framework $AF = (A, R)$, every $E \in \sigma(AF)$ is conflict-free and maximal with respect to $\mathsf{cf}(AF)$.

Simply put, a semantics that satisfies the *Naivety* principle neglects the idea of defence and only considers maximal sets without mutually attacking arguments.

3 Characterising Challenged Semantics

In the following section the challenged semantics will be defined and analysed in comparison to other semantics.

3.1 Challenged semantics

With the knowledge from the background section it is possible to give a definition for the challenged semantics.

Definition 15. Let $AF = (A, R)$ be an abstract argumentation framework.
Let $\mathsf{c(AF)}$ be the set containing all subsets $S \subseteq A$ for which there exists a serialisation sequence $\mathscr{S}_c = (S_1, \dots, S_n)$ with $S = S_1 \cup \dots \cup S_n$ and it holds that $S_1 \in \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ and for all S_i, $i = 2, \dots, n$ that $S_i \in \mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \dots \cup S_{i-1}}) \cup \mathsf{IS}^{\leftrightarrow}(AF^{S_1 \cup \dots \cup S_{i-1}})$ and $\mathsf{IS}^{\not\leftarrow}(AF^{S_1 \cup \dots \cup S_n}) \cup \mathsf{IS}^{\leftrightarrow}(AF^{S_1 \cup \dots \cup S_n}) = \emptyset$.

In simple terms, the challenged semantics is defined by putting restrictions on serialisation sequences. On the one hand only challenged initial sets as well as unattacked initial sets are selected. On the other hand a serialisation sequence ends when no more unattacked or challenged initial sets can be selected in the corresponding reduct. The following examples illustrate the application of the challenged semantics.

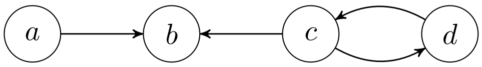


Figure 7: AF_4 with unattacked and challenged initial sets.

Example 9. Figure 7 shows the argumentation framework AF_4 for this example.

In the beginning, the AF_4 has an unattacked initial set $\mathsf{IS}^{\not\leftarrow}(AF_4) = \{\{a\}\}$ and the challenged initial sets $\mathsf{IS}^{\leftrightarrow}(AF_4) = \{\{c\}, \{d\}\}$. To begin with a serialisation sequence, we could accept either one of them. If we accept $\{a\}$ first, the challenged initial sets $\{c\}$ and $\{d\}$ remain in the reduct $AF^{\{a\}}$. The serialisation sequences then end by accepting one of these initial sets, which leads to $\mathscr{S}_1 = (\{a\}, \{c\})$ and $\mathscr{S}_2 = (\{a\}, \{d\})$. Otherwise, if we accept c first, only a remain in $AF^{\{c\}}$ ending in the serialisation sequence $\mathscr{S}_3 = (\{c\}, \{a\})$. The last possible sequence begins with accepting d. In $AF^{\{d\}}$ the arguments a and b are still present with a still being an unattacked initial set. After accepting a the sequence $\mathscr{S}_4 = (\{d\}, \{a\})$ ends.

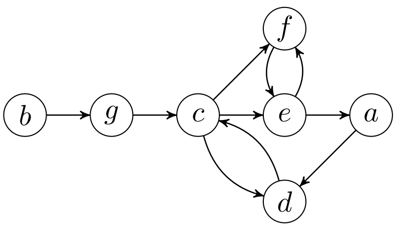
The induced extensions for the sequences found are $\{a, c\}$ and $\{a, d\}$.

Due to the fact that the challenged semantics is the central aspect of this thesis, let us present another example.

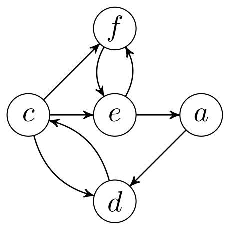
Example 10. Consider the abstract argumentation framework AF_5 from Figure 8a. We have $\mathsf{IS}^{\not\leftarrow}(AF_5 = \{b\})$ and $\mathsf{IS}^{\leftrightarrow}(AF_5) = \emptyset$. Thus, we select argument b first. The reduct is shown in 8b. Since $\mathsf{IS}^{\leftrightarrow}(AF_5^{\{b\}}) = \{\{d, e\}, \{c\}\}$, we can find two serialisation sequences:

$$\mathscr{S}_1 = (\{b\}, \{d, e\})$$
$$\mathscr{S}_2 = (\{b\}, \{c\}, \{a\})$$

Consequently, we have the challenged extensions $\mathsf{c}(AF_5) = \{\{a, b, c\}, \{b, d, e\}\}$.



(a) AF_5



(b) $AF_5^{\{b\}}$

Figure 8: AF_5 and reduct $AF_5^{\{b\}}$ from Example 8

.

Besides the definition of challenged semantics via serialisation sequences, it is also possible to give a recursive definition, as the following theorem demonstrates.

Theorem 2. *Let $AF = (A, R)$ be an abstract argumentation framework and $E \subseteq A$. E is an challenged extension of AF if and only if either*

- $E = \emptyset$ *and* $\mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ *or*
- $E = E_1 \cup E_2, E_1 \in \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ *and* E_2 *is a challenged extension of* AF^{E_1}.

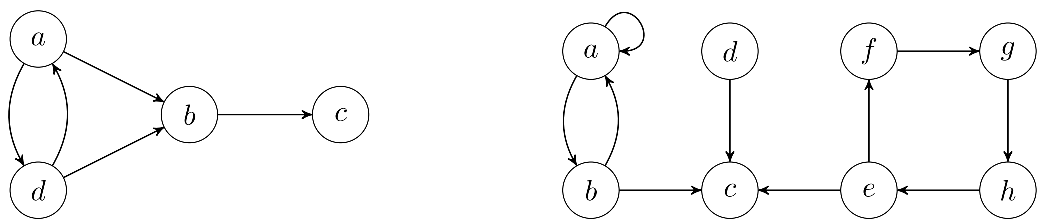
Proof. This proof follows the general structure presented in [11], with modifications to suit the context of the challenged semantics.

Let E be a challenged extension. If $E = \emptyset$, then we have $\mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ and the termination criteria of challenged semantics is met. Assume $E \neq \emptyset$. Therefore we have $E_1 \in \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ such that $E_1 \subseteq E$. We further need to show, that $E_2 = E \setminus E_1$ is a challenged extension of AF^{E_1}. Let $a \in E_2$ and let $b_1, \ldots, b_n \in A$ be the attackers of a in AF. Due to E is a challenged extension, a must be defended by some arguments $c_1, \ldots, c_n$ with c_i attacks b_i for $i = 1, \ldots, n$ (some c_i are possibly identical). Further assume, without loss of generality, $c_1, \ldots, c_k \in E_1$ for some $k \leq n$. It follows, that $b_1, \ldots, b_k$ are not in the reduct AF^{E_1} since they are in E_1^+ and a must only be defended against $b_{k+1}, \ldots, b_n$ in AF^{E_1}. As a consequence of $E_2 = E \setminus E_1$ it applies that $c_{k+1}, \ldots, c_n \in E_2$. Hence, a is defended by E_2 in AF^{E_1} and E_2 is a challenged extension in AF^{E_1}.

We now investigate the other direction. If $E = \emptyset$ and $\mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$, then E is a challenged extension. Assume $E = E_1 \cup E_2, E_1 \in \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ and E_2 is a challenged extension of AF^{E_1}. We have to show that E is a challenged extension. For this, let $a \in E$ and let $b_1, \ldots, b_n \in A$ be the attackers of a in AF. If $a \in E_1$ then there are $c_1, \ldots, c_n$ with c_i attacks b_i for $i = 1, \ldots, n$ because E_1 is per definition either an unattacked or challenged initial set of AF. If $a \in E_2$, suppose there is an attacker b of a such that b is not attacked by any $c \in E$ in AF. It follows that b is also in AF^{E_1} and a is undefended by E_2 in AF^{E_1}, but this is a contradiction to the assumption that E_2 is a challenged extension in AF^{E_1}. □

3.2 Comparison to other Semantics

Now that the challenged semantics is defined, we compare it with other semantics by looking at some examples.



(a) AF_2 (b) AF_3

Figure 9: Reproduction of AF_2 and AF_3 for ease of reference.

Example 11. We first consider AF_2 from Figure 9. Table 1 with extensions for the different semantics for AF_2 is reproduced here as Table 2 for clarity and the extensions for unchallenged and challenged semantics are added.

From the table it is visible that the challenged semantics has the same extensions for AF_2 as stable, preferred and semi-stable, but has different extensions as complete, grounded, ideal, strongly admissible, and unchallenged. We could also see, that the challenged semantics is a subset of admissible and complete extension.

Semantics	**Extensions**
ad	$\{\{a\}, \{a, c\}, \{d\}, \{d, c\}, \emptyset\}$
co	$\{\{a, c\}, \{d, c\}, \emptyset\}$
gr	$\{\emptyset\}$
st	$\{\{a, c\}, \{d, c\}\}$
pr	$\{\{a, c\}, \{d, c\}\}$
id	$\{\emptyset\}$
sst	$\{\{a, c\}, \{d, c\}\}$
sa	$\{\emptyset\}$
uc	$\{\emptyset\}$
c	$\{\{a, c\}, \{d, c\}\}$

Table 2: Extensions of different semantics for AF_2 with unchallenged and challenged semantics.

Example 12. Next, we consider AF_3. The extensions for the semantics are as presented in Table 3.

Here, the extensions of the challenged semantics differs from all others. It is noteworthy that for both abstract argumentation frameworks $\mathsf{c}(AF_3) \subset \mathsf{co}(AF_3)$ and $\mathsf{c}(AF_3) \subset \mathsf{ad}(AF_3)$ also holds. It is also noticeable that the challenged extensions overlap with the preferred, stable and semi-stable extensions.

Semantic	**Extensions**
ad	$\{\{b\}, \{d, f, h\}, \{b, d, e, g\}, \{d\}, \{b, d, f, h\}, \{b, d\},$ $\{e, g\}, \{f, h\}, \{b, e, g\}, \emptyset, \{d, e, g\}, \{b, f, h\}\}$
co	$\{\{d, f, h\}, \{b, d, e, g\}, \{d\}, \{b, d, f, h\}, \{b, d\}, \{d, e, g\}\}$
gr	$\{\{d\}\}$
st	$\{\{b, d, e, g\}, \{b, d, f, h\}\}$
pr	$\{\{b, d, e, g\}, \{b, d, f, h\}\}$
id	$\{\{b, d\}\}$
sst	$\{\{b, d, e, g\}, \{b, d, f, h\}\}$
sa	$\{\{d\}, \emptyset\}$
uc	$\{\{b, d\}\}$
c	$\{\{d, f, h\}, \{d, e, g\}\}$

Table 3: Extensions from different semantics for AF_3

Example 13. Let AF_5 be the last abstract argumentation framework in consideration. Table 4 show the extensions found for the different semantics. For AF_5, the challenged semantics share the same extensions for stable, preferred and semi-stable, and is a subset of complete and admissible.

Semantic	Extensions
ad	$\{\{b,c\},\{b,d,e\},\{d,e\},\emptyset,\{b,c,a\},\{b\}\}$
co	$\{\{b,d,e\},\{b\},\{b,c,a\}\}$
gr	$\{\{b\}\}$
st	$\{\{b,d,e\},\{b,c,a\}\}$
pr	$\{\{b,d,e\},\{b,c,a\}\}$
id	$\{\{b\}\}$
sst	$\{\{b,d,e\},\{b,c,a\}\}$
sa	$\{\{b\},\emptyset\}$
uc	$\{\{b,d,e\},\{b\}\}$
c	$\{\{b,d,e\},\{b,c,a\}\}$

Table 4: Extensions from different semantics for AF_5

The comparison of the extensions of different semantics for some abstract argumentation frameworks offer first impressions regarding the application of the new semantics. The observations from Example 11 to 13 lead to the upcoming theorems.

At first, it should be noted that the challenged semantics is not already characterised by another semantic.

Theorem 3. *Let AF=(A,R) be an argumentation framework. It does not hold in general that $c(AF) = \sigma(AF)$ for any semantics $\sigma \in \{ad, co, gr, st, pr, id, sst, sa, uc\}$.*

Proof. If another semantics would characterize the challenged semantics, both semantics would have the same extensions for any given abstract argumentation framework. Example 12 shows, that the extensions of the challenged semantics differs from all the other semantics considered in this thesis. Therefore, the challenged sematic is not already characterised by another semantics. □

Hence, it is interesting to analyse it further. We continue with the subset observations for admissible and complete extensions.

Theorem 4. *$c(AF) \subseteq ad(AF)$ but not vice versa.*

Proof. In [11] it is shown that serialisability implies the principle of admissibility. This means, that every extension characterised by serialisability is admissible. Therefore, $c(AF) \subseteq ad(AF)$ follows. □

Table 2 from Example 12 shows, that the other direction $ad(AF) \subseteq c(AF)$ does not hold. We have, for example, $\{a\} \in ad(AF_3)$ but $\{a\} \notin c(AF_3)$.

In other words it can be said that every challenged extension is admissible.

The examples, however, also show, that the challenged semantics is a subset or equal to the complete semantics.

Theorem 5. $\mathsf{c}(AF) \subseteq \mathsf{co}(AF)$ *but not vice versa.*

Proof. We want to proof this theorem in an constructive way by showing that we can construct each serialisation sequence of the challenged semantics also with the complete semantics. We start with $\mathsf{c}(AF) \subseteq \mathsf{co}(AF)$ and have to differentiate between an abstract argumentation framework with no selectable initial sets and abstract argumentation frameworks with selectable initial sets. Afterwards it is shown, that the other direction does not hold. For recap, the selection and termination criteria for complete and challenged semantics are defined in Theorem 1 and Definition 15 respectively.

Let $AF = (A, R)$ be an abstract argumentation framework. Further let $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$. Let $\sigma \in \{c, co\}$ be either the challenged or the complete semantics. Since $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ the termination requirements for challenged semantics is satisfied. From $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ follows $\mathsf{IS}^{\nleftarrow}(AF) = \emptyset$ and therefore the requirements for complete semantics is also satisfied.

We now assume $S \in \mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ instead, which is selectable for challenged semantics. Due to $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) \subseteq \mathsf{IS}(AF) = \mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\nleftrightarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ we can also select S for the complete semantics. Since the reduct AF^S is also an abstract argumentation framework, we can further do the same selection on every reduct produced. The selection of an $S \in \mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF)$ is repeated until the termination criterion for the challenged semantics is met. This is when $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ is reached. As already shown above, we can then also terminate in $\sigma(\mathsf{co})$. □

Table 3 from Example 12 shows that the other direction $\mathsf{co}(AF) \subseteq \mathsf{c}(AF)$ does not hold. For example, there is $\{d\} \in \mathsf{co}(AF_3)$ but $\{d\} \notin \mathsf{c}(AF_3)$.

The proof for Theorem 5 implies that $\emptyset$ may be a challenged extension.

Theorem 6. *Given an abstract argumentation framework* $AF = (A, R)$ *such that* $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ *holds. Then,* $\emptyset$ *is the challenged extension.*

Proof. Assume an abstract argumentation framework $AF = (A, R)$ such that $\mathsf{IS}^{\nleftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset$ holds. We can thus only select $\emptyset$, progress to $AF^{\emptyset} = AF$ and terminate, since the termination criteria are satisfied. □

It can therefore be concluded that there is always at least one extension for the challenged semantics.

Corollary 7. *For any argumentation framework* $AF = (A, R)$*, we have that* $c(AF) \neq \emptyset$*.*

This seems reasonable since the termination requirements are trivially met without any selection if there are no unattacked or challenged initial sets in the first considered abstract argumentation framework.

Finally, no clear patterns can be derived from the structure of the graphs that dictate that the challenged semantics is equal to the complete or preferred semantics. If we, for example, restrict the graphs to those without odd cycles, we find abstract argumentation frameworks where $\mathsf{c}(AF) = \mathsf{pr}(AF)$ (Example 11 with AF_2) as well as $\mathsf{c}(AF) \neq \mathsf{pr}(AF)$ (Example 12 with AF_3) hold. For $\mathsf{c}(AF) \subseteq \mathsf{co}(AF)$ we find abstract argumentation frameworks where $\mathsf{c}(AF) \subseteq \mathsf{co}(AF)$ (Example 12 with AF_3) holds and for $\mathsf{c}(AF) = \mathsf{co}(AF)$ we present Example 14.

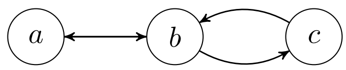


Figure 10: AF_7 with$\mathsf{c}(AF) = \mathsf{co}(AF)$.

Example 14. Consider AF_7 from Figure 10. We find one serialisation sequence $\mathscr{S}_1 = (\{c\}, \{a\})$ that induces the only challenged extension $\{a, c\}$. There is also just one complete extension such that $\mathsf{co}(AF_7) = \{\{a, c\}\}$. Therefore, we have $\mathsf{c}(AF_7) = \mathsf{co}(AF_7)$.

4 Principle-based Analysis of Challenged Semantics

The last section offered some first insights in the behaviour of challenged semantics. But a comparison of semantics via examples offers only a subjective perspective. This is due to the fact, that an example is only one instance and may or may not be a good choice to show that certain properties hold or not. Principles allow an objective perspective for the behaviour of semantics by taking all possible instances of abstract argumentation frameworks in consideration under the principle rules [5]. In the following section, we determine which principles are satisfied by the challenged semantics and which are not.

The *Serialisability* Principle itself satisfies some principles. Hence, if a semantics is serialisable, it also satisfies the principles in the following proposition.

Proposition 8. *The challenged semantics satisfies the principles* Conflict-Freeness*,* Admissibility *and* Modularization*.*

Proof. The challenged semantics is defined through serialisability. Satisfying Serialisability implies directly the satisfaction of the principles *Conflict-Freeness*, *Admissibility* and *Modularization* [11]. Therefore it follows, that the challenged semantics also satisfy these principles. □

From proposition 8 can directly be concluded the following corollary:

Corollary 9. *The challenged semantics satisfies the* Reduct-Admissibility *and the* Semi-Qualified Admissibility *Principles.*

The satisfied principles are presented as the following theorems. We begin with the *Reinstatement* Principle.

Theorem 10. *The challenged semantics satisfies the* Reinstatement *Principle.*

Proof. Given an abstract argumentation framework $AF = (A, R)$, a set $E \subseteq A$ and $E \in c(AF)$. Let us assume that the challenged semantics does not satisfy the *Reinstatement* Principle. Therefore, there is an argument $a \in A$ that is defended by E but $a \notin E$. If $a \in A$ is defended by E, a will be an unattacked initial set in AF^E, because otherwise $AF^E = (A', R')$ has $b \in A'$ with $(b, a) \in R'$, which is a contradiction since a is defended by E. Consequently it has to be shown, that there are no unattacked initial sets in AF^E to conclude that the theorem holds.

Let us now assume there is an $a \in \mathsf{IS}^{\nleftarrow}(AF^E)$. But then E can not be an extension of the challenged semantics since it demands exhaustive selection of unattacked and challenged initial sets by definition. It follows that $\mathsf{IS}^{\nleftarrow}(AF^E) = \emptyset$ and also that the challenged semantics satisfies the *Reinstatement* Principle. □

We continue with the *Directionality* Principle. To proof that the challenged semantics is directional we show, that the challenged semantics satisfies the closure-property as it is done for the unchallenged semantics in [11]. From Theorem 3 in [11] it then follows, that the challenged semantics is directional. For this we switch to the formalisation of serialisability used in [11]. The serialisation algorithm, however, is the same. First, we evaluate the initial sets of an abstract argumentation framework, then we select one of these initial sets and finally progress to the reduct. This is done iterative until eventually some termination criteria are met.

We start by giving definitions for the terms *state*, *selection function* and *termination function* from [29]. Then we give a definition for the challenged semantics in this formalisation. After that, we restate the definition of the closure-property and Theorem 3 from [11] for clarity, followed by a proof, that the challenged semantics satisfy the closure-property.

Let $\mathfrak{A}$ denote a universal set of arguments and $\mathfrak{AF}$ denote the set of all abstract argumentation frameworks.

Definition 16. A state T is a tuple $T = (AF, S)$ with $AF \in \mathfrak{AF}$ and $S \subseteq \mathfrak{A}$.

A state is used to represent the current abstract argumentation framework and the already accepted arguments. The first state is always $(AF, \emptyset)$ representing the original graph without any selections.

Next we need a selection function that returns the initial sets of the current abstract argumentation framework.

Definition 17. A selection function α is any function $\alpha : 2^{\mathfrak{A}} \times 2^{\mathfrak{A}} \times 2^{\mathfrak{A}} \to 2^{\mathfrak{A}}$ with $\alpha(X, Y, Z) \subseteq X \cup Y \cup Z$ for all $X, Y, Z \subseteq \mathfrak{A}$.

The selection function is applied as $\alpha(\mathsf{IS}^{\nleftarrow}, \mathsf{IS}^{\nleftrightarrow}, \mathsf{IS}^{\leftrightarrow})$ to the abstract argumentation framework. This allows for selecting an initial set from the current states abstract argumentation framework and transition to the next state. Finally a termination function is needed, that decides, if the construction of an extension is finished.

Definition 18. A termination function β is any function $\beta : \mathfrak{AF} \times 2^{\mathfrak{A}} \to \{0, 1\}$.

If the termination function evaluates to 1 the current state holds an extension but the construction may be continued.

Selecting an initial argument will transition from one state to another via

$$(AF, S) \xrightarrow{S' \in \alpha(\mathsf{IS}^{\not\leftarrow}(AF), \mathsf{IS}^{\not\leftrightarrow}(AF), \mathsf{IS}^{\leftrightarrow}(AF))} (AF^{S'}, S \cup S').$$

It has to be noted that transition means selecting an intial set S' from AF according to α and step forward to the S'-reduct of AF. If there is a transition within finite steps from state (AF, S) to (AF', S'), including no steps at all, we write $(AF, S) \rightsquigarrow^{\alpha} (AF', S')$ and if (AF', S') also met the termination criteria we write $(AF, S) \rightsquigarrow^{\alpha,\beta} (AF', S')$.

With this formalisation, the challenged semantics can be defined via a function α_c as selection function and a function β_c as termination function.

Theorem 11. *The challenged semantics is serialisable with*
$\alpha_c(X, Y, Z) = X \cup Z$ *and* $\beta_c(AF, S) = \begin{cases} 1, & \textit{if } \mathsf{IS}^{\not\leftarrow}(AF) \cup \mathsf{IS}^{\leftrightarrow}(AF) = \emptyset \\ 0, & \textit{otherwise} \end{cases}$

It is easy to see, that this characterisation is equivalent to that of Definition 15, where α_c corresponds to the selection via S_1 to S_n and β_c corresponds to the exhaustive usage check of unattacked and challenged initial sets in the final reduct.

We proceed to recall the Definition 12 for the closure-property and Theorem 3 from [11] for clarity.

Definition 19. Let σ be serialisable with α_σ and β_σ. We say that σ is $\alpha\beta$-closed for all argumentation frameworks $AF \in \mathfrak{AF}$ if and only if, for every state (AF', S') with $(AF, \emptyset) \rightsquigarrow^{\alpha_\sigma} (AF', S')$ we have that there exists some $AF'' \in \mathfrak{AF}$ and some $S'' \subseteq \mathfrak{A}$ such that $(AF', S') \rightsquigarrow^{\alpha_\sigma,\beta_\sigma} (AF'', S'')$.

The connection between $\alpha\beta$-closure and the *Directionality* Principle then follows in Theorem 3 from [11].

Theorem 12. *If a semantics σ is serialisable via α_σ and β_σ and is $\alpha\beta$-closed, then σ satisfies directionality.*

Finally, with all preparation done, we proof that challenged semantics is $\alpha_c\beta_c$-closed.

Theorem 13. *The challenged semantics is $\alpha_c\beta_c$-closed.*

Proof. The proof is adapted from [11]. Let $AF = (A, R)$ be an abstract argumentation framework and $S \subseteq A$ a set of arguments. We start with a transition $(AF, \emptyset) \rightsquigarrow^{\alpha} (AF_1, S_1)$. Further assume $\beta_c(AF_1, S_1) = 0$. It follows from the definition of β_c, that $\mathsf{IS}^{\not\leftarrow}(AF_1) \cup \mathsf{IS}^{\leftrightarrow}(AF_1) \neq \emptyset$. Consequently it holds that $\alpha_c(\mathsf{IS}^{\not\leftarrow}(AF_1), \mathsf{IS}^{\not\leftrightarrow}(AF_1), \mathsf{IS}^{\leftrightarrow}(AF_1)) \neq \emptyset$ which in turn leads to the conclusion that there must be a transition to a state (AF_2, S_2). Finally this demonstrates, that whenever we reach a state such that β_c is false, we also have the possibility to transition to another state.

Now we show, that there can only be a finite number of transitions until the termination criteria is met. Due to the fact that, if we can not terminate, we can always select and therefore remove an initial set (and the rejected arguments attacked by the initial set) and that initial sets can not be empty by definition, the abstract argumentation framework shrinks with every transition. Consequently, since the abstract argumentation framework is also finite, a state must be reached where no unattacked or challenged initial sets can be found, e.g. the empty argumentation framework. Hence, every possible path will terminate at some state. It follows that α_c and β_c are $\alpha\beta$-closed. □

Finally, with Theorem 3 from [11], we can conclude, that the challenged semantics is directional, because it is $\alpha\beta$-closed.

Theorem 14. *The challenged semantics satisfies the Directionality Principle.*

The other considered principles *Strong Admissibility*, *Naivety*, *Allowing Abstention*, *I-Maximality* and *SCC-Recursiveness* are not satisfied by the challenged semantics, as the following counterexamples show.

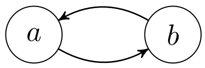


Figure 11: AF_8, a counterexample for *Strong Admissibility* and *Allowing Abstention* used in Examples 15 and 17.

Example 15. Consider AF_8 depicted in Figure 11 with two arguments attacking each other. It applies that $\mathscr{S} = (a)$ is a serialisation sequence of AF_8 and $E = \{a\}$ is the corresponding challenged extension. Since a defends itself against the attack of b there is no set $(E \setminus \{a\}) \subseteq A$ that strongly defends a.

The previous example is consequently a counterexample for the *Strong Admissibility* Principle. This might result from the properties that comes with initial sets. Unattacked initial sets may be strongly admissible, since they are not defending themself. But challenged initial sets do not take strong admissibility into account and allow self defence.

Next, a counterexample for the *Naivety* Principle is shown.

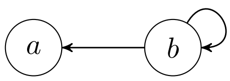


Figure 12: AF_9, a counterexample for *Naivety* used in Example 16.

Example 16. Consider AF_9 with two arguments. We have $\mathsf{c}(AF_9) = \{\emptyset\}$ but $\mathsf{cf}(AF_9) = \{\emptyset, a\}$. Therefore, $\mathsf{c}(AF_9)$ is not maximal w.r.t $\mathsf{cf}(AF_9)$.

It can be concluded that the challenged semantics does not satisfy the *Naivety* Principle. While for a naive view the defence of arguments is not relevant, it is, however, inevitably linked with the challenged semantics. The reason for this lies in the initial sets the challenged semantics operates on, which need to be, by definition, admissible and thus defended. Moreover, the termination criteria does not consider if more naive acceptable arguments are left in the reduct and breaks therefore the maximality criteria of the *Naivety* Principle.

For the *Allowing Abstention* Principle a counterexample can also be found.

Example 17. Reconsider AF_8 already presented for Example 15. It has two arguments attacking each other. We have $\mathsf{c}(AF_8) = \{\{a\}, \{b\}\}$ and therefore exactly two extensions. We also have that $a \in E_1$ and $a \in E_2^+$. Allowing Abstention further requires a third extension with some requirements but there is clearly no third extension at all.

It follows that the challenged semantics does not satisfy the *Allowing Abstention* Principle. This results from the exhaustive selection of unattacked and challenged initial sets as part of the challenged semantics which clearly does not allowing abstention. Maybe a more weak definition of the challenged semantics without exhaustive selection could satisfy the *Allowing Abstention* Principle.

The fact that the challenged semantics does not satisfy the *I-Maximality* Principle is demonstrated by the following counterexample.

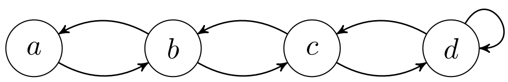


Figure 13: AF_{10}, a counterexample for *I-Maximality* used in Example 18

Example 18. We assume AF_{10}. We have $\mathsf{c}(AF_{10}) = \{\{a\}, \{b\}, \{c, a\}\}$. With $E_1 = \{a\}$ and $E_2 = \{c, a\}$ we have $E_1 \subset E_2$ and therefore $E_1 \neq E_2$.

Thus, the challenged semantics does not satisfy the *I-Maximality* Principle. A reason for this is that the state of an initial set may not be retained as initial set during reduction. For example, the extension $\{c, a\}$ of AF_{10} is only induced by the sequence $\mathscr{S} = (\{c\}, \{a\})$ and there is no other serialisation sequence due to argument c switching from challenged to unchallenged initial set after selecting a beforehand. Besides that, this has already been mentioned in Proposition 4 in [29]. This combined with the non-deterministic character of serialisability induces the violation of this principle. It happens by considering different serialisation sequences on different reducts where the state as an initial set of some arguments sometimes is and sometimes is not retained.

To show that *SCC-Recursiveness* is not satisfied, we first present two quite similar abstract argumentation frameworks and calculate their challenged extensions in detail. Then we point out that there can not be an $\mathscr{BF}_c$ for calculating the extensions of the challenged semantics for both abstract argumentation frameworks via *SCC-Recursiveness* to conclude that no $\mathscr{BF}_c$ exists.

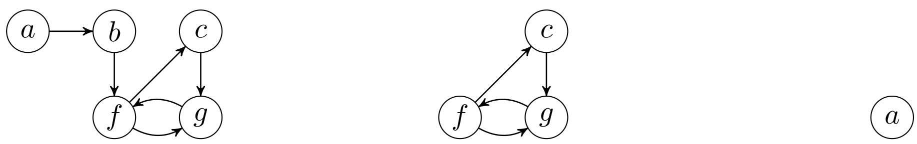
Example 19. Let us assume we have the abstract argumentation framework AF_{11} depicted in Figure 14a and we want to investigate it in regards of the challenged semantics. We have $\mathsf{IS}^{\not\leftarrow}(AF_{11}) = \{\{a\}\}$ and $\mathsf{IS}^{\leftrightarrow}(AF_{11}) = \emptyset$. After selecting $\{a\}$ we face the reduct

(a) AF_{11}. (b) Reduct $AF_{11}^{\{a\}}$.

Figure 14: AF_{11} and the reduct $AF_{11}^{\{a\}}$ from Example 19.

$AF_{11}^{\{a\}}$ depicted in 14b. For the reduct $\mathsf{IS}^{\not\leftarrow}(AF_{11}^{\{a\}}) \cup \mathsf{IS}^{\leftrightarrow}(AF_{11}^{\{a\}}) = \emptyset$ applies. Hence, there is only one challenged extension and thus $\mathsf{c}(AF_{11}) = \{\{a\}\}$.

(a) AF_{12}. (b) Reduct $AF_{12}^{\{a\}}$.



(c) Reduct $AF_{12}^{\{e\}}$. (d) Reduct $AF_{12}^{\{\{e,a\}\}}$. (e) Reduct $AF_{12}^{\{d,f\}}$.

Figure 15: AF_{12} and the reducts $AF_{12}^{\{a\}}$, $AF_{12}^{\{e\}}$, $AF_{12}^{\{\{e,a\}\}}$ and $AF_{12}^{\{d,f\}}$ from Example 20.

Example 20. In this example the abstract argumentation framework AF_{12} from Figure 15a is investigated. The initial sets of the graph are $\mathsf{IS}^{\not\leftarrow}(AF_{12}) = \{\{a\}\}$ and $\mathsf{IS}^{\leftrightarrow}(AF_{12}) = \{\{e\}, \{d, f\}\}$. Consequently we have, at this point, at least three different serialisation sequences to consider: $\mathscr{S}_1 = (\{a\})$, $\mathscr{S}_2 = (\{e\})$ and $\mathscr{S}_3 = (\{d, f\})$. We follow the sequence $\mathscr{S}_1$ first and get the reduct $AF_{12}^{\{a\}}$ from Figure 15b. Since this reduct has no more unattacked or challenged initial sets the sequence finishes. Next we follow the sequence $\mathscr{S}_2$. The reduct $AF_{12}^{\{e\}}$ is shown in Figure 15c. $\mathsf{IS}^{\not\leftarrow}(AF_{12}^{\{e\}}) = \{\{a\}\}$ can be found for this graph. Thus, we get $\mathscr{S}_2 = (\{e\}, \{a\})$ and the resulting reduct $AF_{12}^{\{\{e,a\}\}}$ presented in Figure 15d has no more initial sets. Finally we follow the sequence $\mathscr{S}_3$. The reduct $AF_{12}^{\{d,f\}}$ is shown in Figure 15e. We have $\mathsf{IS}^{\not\leftarrow}(AF_{12}^{\{d,f\}}) = \{\{a\}\}$. After selecting $\{a\}$ for $\mathscr{S}_3 = (\{d, f\}, \{a\})$ no more arguments remain. Therefore the extensions for the challenged semantics are $\mathsf{c}(AF_{12}) = \{\{a\}, \{a, d, f\}, \{a, e\}\}$.

It is to be emphasised that the reducts $AF_{11}^{\{a\}}$ from Figure 14b and $AF_{12}^{\{a\}}$ from Figure 15b are the same graph. This is important for the upcoming view on *SCC-Recursiveness*.

Example 21. We now want to explore the AF_{11} and AF_{12} in terms of *SCC-Recursiveness*. The abstract argumentation framework AF_{11} consists of three strongly connected components $SCCs_{AF_{11}} = \{\{a\}, \{h\}, \{b, c, d, e, f, g\}\}$. A defeat-graph consisting of the SCC-components is always acyclic and is processed from outside in for *SCC-Recursiveness* [6]. We first have to consider

$$E \cap \{a\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{11}|_{\{a\}}, \{a\}) \tag{1}$$

as $\{a\}$ is the first strongly connected component. Since $\mathsf{c}(AF_{11}) = \{\{a\}\}$, as shown in Example 19, a base function $\mathscr{BF}_c$ that calculates the challenged semantics in an SCC-recursive way needs to evaluate to $\{a\}$ for expression 1. Next, we have to consider $\{h\}$ as the second strongly connected component, but it is important to mention, that $\{h\}$ is defeated by the previously accepted $a \in E$. Therefore we have

$$E \cap \{h\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{11}|_{\emptyset}, \emptyset) \tag{2}$$

as the second expression. Here, we can not select anything due to the empty projection, but due to $\mathsf{c}(AF_{11}) = \{\{a\}\}$ we do not want to select more arguments anyway. The third strongly connected component is $\{b, c, d, e, f, g\}$. Note, that $\{b\}$ is also defeated by $a \in E$. We have to evaluate the expression

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{11}|_{\{c,d,e,f,g\}}, \{c, d, e, f, g\}) \tag{3}$$

where the projection itself consists of two strongly connected components. We therefore have to consider

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{11}|_{\{c,f,g\}}, \{c, f, g\}) \tag{4a}$$

which needs to evaluate to $\emptyset$ for a base function $\mathscr{BF}_c$ that calculates the same extensions as the challenged semantics. The last expression to take into account is thus

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{11}|_{\{d,e\}}, \{d, e\}) \tag{4b}$$

and this expression also needs to evaluate to $\emptyset$.

Without giving a definition for $\mathscr{BF}_c$, let us assume, with the already gained knowledge for AF_{11}, there is a base function for the challenged semantics and check it against the similar abstract argumentation framework AF_{12}. Keep in mind that $\mathsf{c}(AF_{12}) = \{\{a\}, \{a, d, f\}, \{a, e\}\} \neq \{\{a\}\} = \mathsf{c}(AF_{11})$. For AF_{12}, we have to consider the strongly connected components $SCCs_{AF_{12}} = \{\{a\}, \{b, c, d, e, f, g\}\}$. The first expression is therefore

$$E \cap \{a\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{12}|_{\{a\}}, \{a\}) \tag{5}$$

which is similar to expression 1. The argument a is in every extension of $\mathsf{c}(AF_{12})$ so that function $\mathscr{GF}_{\mathscr{BF}_c}$ must return $\{a\}$ for expression 5. From AF_{11} we know, that the assumed $\mathscr{BF}_c$ will exactly do that, since $AF_{11}|_{\{a\}} = AF_{12}|_{\{a\}}$. The second strongly connected component leads to

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{12}|_{\{c,d,e,f,g\}}, \{c, d, e, f, g\}) \tag{6}$$

which is exactly the same as expression 3. Since $\{\{c\}\} \notin \bigcup \mathsf{c}(AF_{12})$ expression 6 can safely be continued to

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{12}|_{\{c,f,g\}}, \{c, f, g\}) \tag{7a}$$

and

$$E \cap \{b, c, d, e, f, g\} \in \mathscr{GF}_{\mathscr{BF}}(AF_{12}|_{\{d,e\}}, \{d, e\}) \tag{7b}$$

respectively. If this would not be the case and a $\mathscr{BF}_c$ must also return $\{c\}$ for expression 7a this would change expression 7b due to argument d would not be undefeated any more.

Note again that the expressions 4a and 4b are equivalent to 7a and 7b as their projections are equivalent ($AF_{11}|_{\{c,f,g\}} = AF_{12}|_{\{c,f,g\}}$ and $AF_{11}|_{\{d,e\}} = AF_{12}|_{\{d,e\}}$, respectively). Here, the function $\mathscr{GF}_{\mathscr{BF}_c}$ must not only return $\emptyset$ but also $\{f\}$ for expression 7a and accordingly for expression 7b it must return $\{d\}$ when $\{f\}$ is selected beforehand and $\{e\}$ alongside $\emptyset$. But when a $\mathscr{BF}_c$ is capable of do so, then it would do so for AF_{11} as well. But then $\mathscr{BF}_c(AF_{11}) \neq \mathsf{c}(AF_{11})$ which is a contradiction to the assumption, that $\mathscr{BF}_c$ is the base function for *SCC-Recursiveness* that corresponds to the challenged semantics.

Finally, from Example 21, it follows that $\mathsf{c}(AF) = \mathscr{GF}_{\mathscr{BF}_c}(AF, A)$ does not hold and a base function $\mathscr{BF}_c$ can not exist. Thus, the challenged semantics does not satisfy the *SCC-Recursiveness* Principle. Most likely this is due to the fact that the strongly connected components build a partial order and the *Directionality* Principle directly influenced *SCC-Recursiveness* [6]. This induces a processing order which is not compatible with the non-deterministic character of serialisability for the challenged semantics. Moreover, it can be observed that the projections of expressions 3 and 6 corresponds with the reducts 14b and 15b in Figures 14 and 15. An induced challenged extension E for the corresponding serialisation sequences would be the same for both graphs $E(AF_{11}) = E(AF_{12}) = \{\{a\}\}$ if $\{a\}$ must be chosen first to copy the order of *SCC-Recursiveness*. But since the challenged semantics does not have such restrictions more extensions can be found for AF_{12} which shows that the processing order is a problem for satisfying this principle.

5 Complexity Analysis of Challenged Semantics

The previous sections show a characterisation of the challenged semantics for abstract argumentation frameworks. In computer science, however, solving a problem is only one aspect to consider. Often times it is quite similar important to know how efficiently a solution can be computed. Most of the time the size of the input becomes the key factor that determines how well an algorithm performs. Focal points of investigation are time and space complexity [27].

In this bachelor thesis the time complexity of computing problems related to the challenged semantics for abstract argumentation frameworks is analysed. The results will not show exact measurements of time as this depends on aspects like hardware and implementation that can vary. Instead, the result gives an upper bound for the time needed in a worst case scenario in regards to the specific problem and the input. For this, each problem will be classified with the help of known complexity classes.

The following decision problems are analysed [23]:

- $\textsc{Ver}_c(AF, E)$: The *Verification* Problem answers the question whether the set of arguments S is a challenged extension for the given abstract argumentation framework AF.
- $\textsc{Exists}_c^{\neg\emptyset}(AF)$: The *Existence* Problem answers the question whether there is a non empty challenged extension for a given abstract argumentation framework AF or not.
- $\textsc{Skept}_c(AF, a)$: The *Skeptical Acceptance* Problem answers the question whether the argument a is skeptically accepted under the challenged semantics in the given abstract argumentation framework AF or not. An argument $a \in A$ for an abstract argumentation framework $AF = (A, R)$ is skeptically acceptable, if the argument a is accepted in every extension of the considered semantics
- $\textsc{Cred}_c(AF, a)$: The *Credulous Acceptance* Problem answers the question whether the argument a is credulously accepted under the challenged semantics in the given abstract argumentation framework AF or not. An argument $a \in A$ is credulously acceptable, if the argument a is accepted in at least one extension of the considered semantics.

It has to be noted that $\textsc{Exists}_c^{\emptyset}(AF)$ is not analysed. The answer to this problem would be whether there is any challenged extension for a given abstract argumentation framework AF or not. Due to Corollary 7, this problem is trivial.

Example 22. Recall AF_{12} from Figure 15. The challenged extensions of AF_{12} are $\mathsf{c}(AF_{12}) = \{\{a\}, \{a, d, f\}, \{a, e\}\}$. We have a in each challenged extension of AF_{12}. Therefore a is *skeptically acceptable* under the challenged semantics for AF_{12}, because it appears in every set of $\mathsf{c}(AF_{12})$. In contrast, the arguments d, f, e and also a are *credulously acceptable*, as they appear in at least one extension.

We now give a short introduction into the complexity classes used in this thesis. For more information on those classes, please refer to [27] or [23]. With P (*polynomial-time*) we denote the set of problems that are solvable in polynomial time. These problems are considered to be efficiently solvable, even if the exponent is a large number. A subclass of P is L (*logarithmic space*), which also limits the used space. The complexity class NP (*non-deterministic polynomial-time*) includes problems, for which no polynomial time algorithm is known, but for which a given solution or a potential witness can be verified in polynomial time. The complexity class coNP consists of all decision problems, for which the complement problem lies in NP. This means that if the answer to a problem instance is "no", there is a witness that can be used to efficiently verify this. In this thesis we make use of *oracle* machines. Such an oracle machine, or simply oracle, allows to provide an answer to a decision problem in a single computational step [23]. Oracles are used in the classes Π_2^P, Σ_2^P and $\mathsf{P}_{\|}^{\mathsf{NP}}$. $\Pi_2^P = \mathsf{co}\Sigma_2^P = \mathsf{coNP}^{\mathsf{NP}}$ is the class of decision problems that are in coNP with access to an NP-oracle, $\Sigma_2^P = \mathsf{NP}^{\mathsf{NP}}$ is the class of decision problems that are in NP with access to a NP-oracle and $\mathsf{P}_{\|}^{\mathsf{NP}}$ is the class of decision problems that are in P with access to logarithmically many *adaptive* NP-oracle calls [27].

For ease of reference we repeat the results for runtime complexity in regards to the different initial sets from [29] in Table 5:

σ	IS	$\mathsf{IS}^{\not\leftarrow}$	$\mathsf{IS}^{\not\leftrightarrow}$	$\mathsf{IS}^{\leftrightarrow}$
$\textsc{Ver}_\sigma$	in P	in P	coNP-c	NP-c
$\textsc{Exists}_\sigma$	NP-c	in P	$\mathsf{P}_{\|}^{\mathsf{NP}}$-c	NP-c
$\textsc{Cred}_\sigma$	NP-c	in P	$\mathsf{P}_{\|}^{\mathsf{NP}}$-c	NP-c
$\textsc{Skept}_\sigma$	cpNP-c	in P	$\mathsf{P}_{\|}^{\mathsf{NP}}$-c	coNP-c

Table 5: Repetition of Table 1 from [29]. Runtime complexity of decision problems with regards to initial sets. Attached "-h" for hardness, attached "-c" for completeness.

We start the complexity analysis with the *Verification* problem. All proofs are inspired by [11].

Theorem 15. *$\textsc{Ver}_c(AF, E)$ is in Σ_2^P and coNP-hard.*

Proof. To show the Σ_2^P-membership we start by guessing an integer k representing the length of the serialisation sequence inducing E. For $i = 1, \ldots, k$ we iteratively guess a set $S_i \subseteq A$ and verify that it is either an unattacked or a challenged initial set for $AF^{S_1 \cup \cdots \cup S_{i-1}}$. The latter can be done in NP since verification of $\mathsf{IS}^{\leftrightarrow}$ is NP-complete, while the former is not relevant as it is in P [29]. For $AF^{S_1 \cup \cdots \cup S_k}$ we then verify that no unattacked or challenged initial sets remain. The latter can be accomplished by an NP-oracle. We have verified that E is a challenged extension when $E = S_1 \cup \cdots \cup S_k$ holds.

For coNP-hardness we reduce from the existence problem for challenged initial sets, which has been proven to be NP-complete [29]. It is assumed that AF has no unattacked initial sets. Then AF has a challenged initial set if and only if $\emptyset$ is not an challenged extension. □

Due to the guessing steps, the given algorithm for *Verification* is non-deterministic. For the absence of further initial sets in the last reduct, we need to call an NP-oracle, since we want to get an answer to the complement of the *Existence*-Problem for challenged initials sets, which itself is a NP-Problem.

Next, we address the *Existence* problem.

Theorem 16. *$\textsc{Exists}_c^{\neg\emptyset}(AF)$ is NP-complete.*

Proof. Since the challenged semantics terminates for every path as shown for the *Directionality* Principle in Theorem 13, we only need to find an unattacked or challenged initial set in AF. For this problem the existence of unattacked initial sets can be ignored since it is in P while verifying existence of a challenged initial set is in NP [29]. Therefore it follows that $\textsc{Exists}_c^{\neg\emptyset}(AF)$ is in NP.

Since existence of unattacked initial sets is not relevant and existence of challenged initial sets is NP-complete, it can be concluded directly, that $\textsc{Exists}_c^{\neg\emptyset}$ is NP-hard and hence NP-complete. □

From the proof it can be seen that, if the abstract argumentation framework AF has at least one unattacked initial set, the problem could be solved in polynomial time.

We progress to *Skeptical-* and *Credulous Acceptance* that questions the credibility of single arguments.

Theorem 17. *$\textsc{Skept}_c(AF, a)$ is in Π_2^P.*

Proof. To show the Π_2^P-membership we demonstrate that the complement problem $\neg\textsc{Skept}_c$ is in $\mathsf{NP}^{\mathsf{coNP}}$. $\neg\textsc{Skept}_c$ is the problem that answers the question if an argument a is not skeptically acceptable with regards to the challenged semantics in a given abstract argumentation framework AF. The following non-deterministic algorithm solves $\neg\textsc{Skept}_c$ in $\mathsf{NP}^{\mathsf{coNP}}$: We start by guessing an integer k representing the length of a serialisation sequence. For $i = 1, \ldots, k$ we iteratively guess a set $S_i \subseteq A$ and verify that it is either an unattacked or a challenged initial set for $AF^{S_1 \cup \cdots \cup S_{i-1}}$. The former can be done in P while the latter is in NP [29]. For $AF^{S_1 \cup \cdots \cup S_k}$ we then verify that no unattacked or challenged initial sets remain. The latter can be accomplished by an coNP-oracle. It follows that $E = S_1 \cup \cdots \cup S_k$ is a challenged extension. We have verified that a is not skeptically acceptable under the challenged semantics for the abstract argumentation framework AF when $a \notin E$. Since this runs in $\mathsf{NP}^{\mathsf{coNP}}$, the original problem $\textsc{Skept}_c(AF, a)$ is in Π_2^P. □

The last problem we consider is the *Credulous Acceptance* Problem.

Theorem 18. *$\textsc{Cred}_c(AF, a)$ is in Σ_2^P.*

Proof. We again start by guessing an integer k representing the length of a serialisation sequence. For $i = 1, \ldots, k$ we iteratively guess a set $S_i \subseteq A$ and verify that it is either an unattacked or a challenged initial set for $AF^{S_1 \cup \cdots \cup S_{i-1}}$. The former can be done in P while the latter is in NP [29]. For $AF^{S_1 \cup \cdots \cup S_k}$ we then verify that no unattacked or challenged initial sets remain. The latter can be accomplished by an coNP-oracle. It follows that $E = S_1 \cup \cdots \cup S_k$ is a challenged extension. We have verified that a is credulously acceptable under the challenged semantics for the abstract argumentation framework AF when $a \in E$. □

Due to time constraints we could not provide proofs for the hardness of $\textsc{Skept}_c(AF, a)$ and $\textsc{Cred}_c(AF, a)$. However, based on the results that are in line with the unchallenged semantics, it is to be expected that the problems are Π_2^P- and Σ_2^P-complete. Presumably, the reductions from [11] for both problems can possibly be adapted to the challenged semantics.

6 Discussion

The results presented above form the basis for the following discussion guided by the Research Questions.

The goals of Research Question 1 was a definition of the challenged semantics and a comparison to other semantics. In Section 3 we characterised challenged semantics in Definition 15 on the basis of serialisability. Theorem 2 also gives a recursive definition. These definitions are possible since a non-empty admissible set can be written as the union of pairwise disjoint initial sets as proposed in Corollary 1 from [29] based on the idea of modularization from [7]. The comparison between extensions of the challenged semantics and extensions of different semantics for some examples showed, that the challenged semantics forms a subset of the complete semantics, which itself is a subset of the admissible semantics. This is reasonable, because as already mentioned, the challenged semantics adds some further restrictions in terms of selection and termination compared to the definition of the complete semantics. The last result in this regard are that there is always at least one extension for the challenged semantics.

The focus of Research Question 2 is on the principle-based analysis of the challenged semantics. Interestingly, *serialisability* implies the principles *Conflict-Freeness*, *Admissibility* and *Modularization* [11]. Thus, the challenged semantics satisfies these principles by definition. The challenged semantics also satisfies the *Reinstatement* Principle, which is expected due to the subset relationship to the complete semantics. Then, the challenged semantics satisfies the *Directionality* Principle. This has been proven via the closure-property. It is, however, due to the dependencies between the selection and termination criteria, expected that every path for the challenged semantics terminates.

There are, on the other hand, the principles *Naivety*, *Strong Admissibility*, *Allowing Abstention*, *I-Maximality* and *SCC-Recursiveness* that are not satisfied by the challenged semantics. Since conflict-freeness is a component of admissibility, it might have been understandable that challenged semantics would satisfy the naivety principle. However, challenged semantics only considers initial sets, which by definition also require defence. This leads to the construction of sets that are not necessary maximal while being conflict-free. The defence for *Strong Admissibility*, nevertheless, goes beyond the definition of defence for inital sets, as it does not allow self defence. Therefore it is understandable that the challenged semantics does not satisfy this principle. Equally understandable is that the challenged semantics does not satisfy the *I-Maximality* Principle, since it is a subset of the complete semantics that does not satisfy the *I-Maximality* Principle, either. However, it was not necessarily expected that the challenged semantics would not satisfy the *SCC-Recursiveness* Principle due to the observation from [29], that conflicting initial sets are always within a singe strongly connected component.

To get an overall view, Table 6 shows the considered semantics and their satisfied principles. A close connection can be seen between challenged and unchallenged semantics, as their definitions are pretty close to each other. Due to the subset relationship to complete semantics it is reasonable that the challenged semantics satisfies most of the principles that the complete semantics satisfy. Only the possibility to abstain arguments is not

Principle \ σ	co	gr	st	pr	id	sst	uc[6]	c
Admissibility	✓[1]	✓[1]	✓[1]	✓[1]	✓[1]	✓[1]	✓	✓
Strong Admissibility	✗[1]	✓[1]	✗[1]	✗[1]	✗[1]	✗[1]	✗	✗
Reinstatement	✓[1]	✓[1]	✓[1]	✓[1]	✓[1]	✓[1]	✓	✓
I-Maximality	✗[1]	✓[1]	✓[1]	✓[1]	✓[1]	✓[1]	✗	✗
Directionality	✓[1]	✓[1]	✗[1]	✓[1]	✓[1]	✗[1]	✓	✓
Allowing Abstention	✓[3]	✗[3]?	✗[3]	✗[3]	✗[3]	✗[3]	✗	✗
Modularization	✓[2]	✓[2]	✓[2]	✓[2]	?	?	✓	✓
SCC-Recursiveness	✓[5]	✓[5]	✓[5]	✓[5]	?	?	✗	✗
Naivety	✗[2]	✗[2]	✓[2]	✗[2]	✗[4]	✗[4]	✗	✗

Table 6: Overview over the considered semantics and their compliance with selected principles. Results with a superscript 1 are from [5], 2 from [7], 3 from [3], 4 from [31], 5 from [6] and the results for the unchallenged semantics with superscript 6 are taken from [11]. Note, that conflict-free, reduct- and semi-qualified-admissibility are omitted, since every semantics is admissible.

given for the challenged semantics, as well as a recursive construction via evaluation over strongly connected components. It is also noticeable that most of the other semantics satisfy *I-Maximality*, which the challenged, unchallenged and complete semantics do not.

For Research Question 3, the runtime complexity of the challenged semantics was investigated. The results are depicted in Table 7. Since unchallenged and challenged semantics are closely related and just differs in the selected initial sets, it is reasonable, that they are similar complex. The difference in complexity for the Exists Problem results from different complexity for the existence problem for unchallenged and challenged initial sets (shown in Table 5). Moreover, the unchallenged and challenged semantics seem to be more complex in comparison to the classical semantics for most of the problems. This maybe come from the construction via initial sets for which the considered problems are already complex.

Problem \ σ	ad[1]	co[1]	pr[1]	uc[2]	c
Ver_σ	in L	in L	coNP-c	in Σ_2^P and $\mathsf{P}_{\parallel}^{\mathsf{NP}}$-h	in Σ_2^P and coNP-h
$\text{Exists}^{\neg\emptyset}$	NP-c	NP-c	NP-c	$\mathsf{P}_{\parallel}^{\mathsf{NP}}$-c	NP-c
Skept_σ	trivial	P-c	Π_2^P-c	Π_2^P-c	in Π_2^P
Cred_σ	NP-c	NP-c	NP-c	Σ_2^P-c	in Σ_2^P

Table 7: Overview over the complexity of the considered problems for the semantics ad, co, pr, uc and c. The simple Exists is omitted, since it is trivial for every shown semantics. A "-c" suffix stands for complete, while a "-h" suffix means hard.

7 Future Work

This bachelor thesis lays the foundation for future research. It could be interesting to experiment with slightly different definitions for challenged semantics. For example, the termination condition could be softened to converge towards the complete semantics or allowing abstention. It might also be worth examining the behaviour of challenged semantics under other structural constraints like bipartiteness.

In the complexity analysis no proof for the hardness of skeptical and credulous acceptance of an argument could be found and thus are still open questions. Since the computational complexity for challenged semantics is high in comparison to other semantics, it would be interesting to find efficient implementation approaches. For example, the use of parallelism as shown in [12] could be an option.

Another possibility would be to examine whether challenged semantics can be transferred to other formalisms like WAF or PrAF.

Still an open problem is the question, whether the challenged semantics can be defined without referring to initial sets.

8 Conclusion

In this bachelor thesis we analysed the new challenged semantics based on serialisability. It is defined by selecting unattacked and challenged initial sets exclusively, until a reduct is reached, that does not provide any more initial sets of these types to select. By comparison of extensions from different semantics for some example abstract argumentation frameworks, we could confirm, that the challenged semantics is not yet characterised by another semantics. Moreover, we proofed, that the challenged semantics always has at least one extension and is a subset of the complete semantics.

We then analysed the challenged semantics in regard of common principles from the literature. Our results show, that the challenged semantics satisfies the principles *Conflict-Freeness*, *Admissibility*, *Reduct-Admissibility*, *Semi-Qualified Admissibility* and *Modularization* due to serialisability. Challenged semantics also satisfies *Reinstatement* and *Directionality.* It, however, does not satisfy *Naivety*, *Allowing Abstention*, *I-Maximality* and *SCC-Recursiveness.*

Finally we considered the runtime complexity for challenged semantics with regards to the typical reasoning problems VERIFICATION, if a set is a challenged extension, EXISTENCE of a challenged, non-empty extension, SKEPTICAL ACCEPTANCE and CREDULOUS ACCEPTANCE of an argument. We found, that $\textsc{Ver}_c$ is in Σ_2^P and coNP-hard, $\textsc{Exists}_c^{\neg\emptyset}$ is NP-complete and for $\textsc{Skept}_c$ and $\textsc{Cred}_c$ we proofed the Π_2^P- and the Σ_2^P-membership, respectively.

References

[1] Leila Amgoud and Jonathan Ben-Naim. Ranking-based semantics for argumentation frameworks. In *International Conference on Scalable Uncertainty Management*, pages 134–147. Springer, 2013.

[2] Pietro Baroni, Martin Caminada, and Marco Giacomin. Abstract argumentation frameworks and their semantics. In Pietro Baroni, Dov Gabbay, Marco Giacomin, and Leendert van der Torre, editors, *Handbook of Formal Argumentation*, pages 159–236. College Publications, 2018.

[3] Pietro Baroni, Martin Caminada, and Massimiliano Giacomin. An introduction to argumentation semantics. *Knowledge Eng. Review*, 26:365–410, 12 2011.

[4] Pietro Baroni and Massimiliano Giacomin. Solving semantic problems with odd-length cycles in argumentation. In Thomas Dyhre Nielsen and Nevin Lianwen Zhang, editors, *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, pages 440–451, Berlin, Heidelberg, 2003. Springer Berlin Heidelberg.

[5] Pietro Baroni and Massimiliano Giacomin. On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence*, 171:675–700, 07 2007.

[6] Pietro Baroni, Massimiliano Giacomin, and Giovanni Guida. Scc-recursiveness: A general schema for argumentation semantics. *Artificial Intelligence*, 168:162–210, 10 2005.

[7] Ringo Baumann, Gerhard Brewka, and Markus Ulbricht. Comparing Weak Admissibility Semantics to their Dung-style Counterparts – Reduct, Modularization, and Strong Equivalence in Abstract Argumentation. In *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning*, pages 79–88, 9 2020.

[8] Ringo Baumann, Gerhard Brewka, and Markus Ulbricht. Revisiting the foundations of abstract argumentation–semantics based on weak admissibility and weak defense. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 2742–2749, 2020.

[9] Ringo Baumann, Gerhard Brewka, and Markus Ulbricht. Revisiting the foundations of abstract argumentation – semantics based on weak admissibility and weak defense. *Proceedings of the AAAI Conference on Artificial Intelligence*, 34:2742–2749, 04 2020.

[10] Lars Bengel, Julian Sander, and Matthias Thimm. Characterising serialisation equivalence for abstract argumentation. In *Proceedings of the 27th European Conference on Artificial Intelligence (ECAI'24)*, October 2024.

[11] Lars Bengel and Matthias Thimm. Serialisable semantics for abstract argumentation. In *COMMA*, pages 80–91, 2022.

[12] Lars Bengel and Matthias Thimm. Towards parallelising extension construction for serialisable semantics in abstract argumentation. In *Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning (KR'23)*, September 2023.

[13] Lydia Blümel and Matthias Thimm. A ranking semantics for abstract argumentation based on serialisability. In *Proceedings of the 9th International Conference on Computational Models of Argument (COMMA'22)*, September 2022.

[14] Martin Caminada. Strong admissibility revisited. In *Computational Models of Argument*, pages 197–208. IOS Press, 2014.

[15] Martin W. A. Caminada, Walter A. Carnielli, and Paul E. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 2012.

[16] Martin WA Caminada and Dov M Gabbay. A logical account of formal argumentation. *Studia Logica*, 93:109–145, 2009.

[17] C. Cayrol and M. C. Lagasquie-Schiex. On the acceptability of arguments in bipolar argumentation frameworks. In Lluís Godo, editor, *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, pages 378–389, Berlin, Heidelberg, 2005. Springer Berlin Heidelberg.

[18] Lisa A Chalaguine and Anthony Hunter. A persuasive chatbot using a crowd-sourced argument graph and concerns. In *Computational Models of Argument*, pages 9–20. IOS Press, 2020.

[19] Jérémie Dauphin, Tjitze Rienstra, and Leon van der Torre. A principle-based analysis of weakly admissible semantics. In *Proceedings of COMMA 2020*, pages 167–178, 2020.

[20] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial intelligence*, 77(2):321–357, 1995.

[21] P.M. Dung, P. Mancarella, and F. Toni. Computing ideal sceptical argumentation. *Artificial Intelligence*, 171(10):642–674, 2007. Argumentation in Artificial Intelligence.

[22] Paul E. Dunne, Anthony Hunter, Peter McBurney, Simon Parsons, and Michael Wooldridge. Weighted argument systems: Basic definitions, algorithms, and complexity results. *Artificial Intelligence*, 175(2):457–486, 2011.

[23] Wolfgang Dvorák and Paul E Dunne. Computational problems in formal argumentation and their complexity. *JOURNAL OF APPLIED LOGICS-IFCOLOG JOURNAL OF LOGICS AND THEIR APPLICATIONS*, 4(8):2557–2622, 2017.

[24] Anthony Hunter and Matthias Thimm. Probabilistic reasoning with abstract argumentation frameworks. *Journal of Artificial Intelligence Research*, 59:565–611, 2017.

[25] Hengfei Li, Nir Oren, and Timothy J. Norman. Probabilistic argumentation frameworks. In Sanjay Modgil, Nir Oren, and Francesca Toni, editors, *Theorie and Applications of Formal Argumentation*, pages 1–16, Berlin, Heidelberg, 2012. Springer Berlin Heidelberg.

[26] Sanjay Modgil and Henry Prakken. The aspic+ framework for structured argumentation: a tutorial. *Argument & Computation*, 5(1):31–62, 2014.

[27] C.H. Papadimitriou. *Computational Complexity*. Theoretical computer science. Addison-Wesley, 1994.

[28] Henry Prakken and Giovanni Sartor. Law and logic: A review from an argumentation perspective. *Artificial Intelligence*, 227:214–245, 2015.

[29] Matthias Thimm. Revisiting initial sets in abstract argumentation. *Argument & Computation*, 13(3):325–360, 2022.

[30] Francesca Toni. A tutorial on assumption-based argumentation. *Argument & Computation*, 5(1):89–117, 2014.

[31] Leon van der Torre and Srdjan Vesic. The principle-based approach to abstract argumentation semantics. *IfCoLog Journal of Logics and Their Applications*, 2017.

[32] Frans H. van Eemeren, Rob Grootendorst, and Tjark Kruiger. *Handbook of Argumentation Theory: A Critical Survey of Classical Backgrounds and Modern Studies*. De Gruyter Mouton, Berlin, Boston, 1987.

[33] Yuming Xu and Claudette Cayrol. Initial sets in abstract argumentation frameworks. *Journal of Applied Non-Classical Logics*, 28(2-3):260–279, 2018.