Conference

On mathematical aspects of interacting systems in low dimension

Hagen, Germany 24.06. – 27.06.2019

DFG Deutsche Forschungsgemeinschaft

GdF Gesellschaft der Freunde der FernUniversität e.V.

FernUniversität in Hagen

On mathematical aspects of interacting systems in low dimension

Hagen, Germany 24.06. – 27.06.2019

List of already confirmed speakers

R. Band (Haifa)
D. Bianchi (Como)
J. Bolte (London)
N. Benedikter (Vienna)
L. Boßmann (Tübingen)
J. Breuer (Jerusalem)
R. Egger (Düsseldorf)
G. Gallavotti (Rome)
I. Goldsheid (London)
S. Golenia (Bordeaux)
B. Gutkin (Holon)
F. Klopp (Paris)
A. Kostenko (Vienna)
H. Kovařík (Brescia)
K. Kozlowski (Lyon)
E. Le Masson (Cergy-Pontoise)
K. Pankrashkin (Paris)
C.-A. Pillet (Toulon)
A. Posilicano (Como)
A. Sawicki (Warsaw)
S. Sukthaiev (Houston)
A.-S. Sznitman (Zürich)
A. Teta (Rome)
I. Veselic (Dortmund)
J. Yngvason (Vienna)

Organizers

Joachim Kerner, Delio Mugnolo, Wolfgang Spitzer

Programme

Monday, 24th June 2019

09:15 – 09:30
Conference opening
by Prof. Dr. Ada Pellert

09:30 – 10:15
J. Yngvason (Vienna)

10:15 – 11:00
R. Egger (Düsseldorf)

Coffee break

11:30 – 12:15
A. Teta (Rome)

12:15 – 13:00
S. Golenia (Bordeaux)

Lunch break

14:15 – 15:00
M. Pechmann (Hagen)

15:00 – 15:45
K. Kozlowkski (Lyon)

Coffee break

16:15 – 17:00
A. Kostenko (Vienna)

17:00 – 17:45
D. Bianchi (Como)

Tuesday, 25th June 2019

09:30 – 10:15
A.-S. Sznitman (Zurich)

10:15 – 11:00
C.-A. Pillet (Toulon)

Coffee break

11:30 – 12:15
B. Gutkin (Duisburg)

12:15 – 13:00
S. Egger (Haifa)

Lunch break

14:15 – 15:00
L. Boßmann (Tübingen)

15:00 – 15:45
H. Kovarik (Brescia)

Coffee break

16:15 – 17:00
I. Veselic (Dortmund)

Wednesday, 26th June 2019

09:30 – 10:15
F. Klopp (Paris)

10:15 – 11:00
K. Pankrashkin (Orsay)

Coffee break

11:30 – 12:15
A. Sawicki (Warsaw)

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I. Goldsheid (London)

Lunch break

14:15 – 15:00
S. Sukhtaiev (Houston)

15:00 – 15:45
A. Posilicano (Como)

Coffee break

16:15 – 17:00
E. Le Masson (Cergy-Pontoise)

17:00 – 17:45
N. Benedikter (Vienna)

19:00 – 22:00
Conference Dinner

Thursday, 27th June 2019

09:30 – 10:15
J. Breuer (Jerusalem)

10:15 – 11:00
J. Bolte (London)

Coffee break

11:30 – 12:15
R. Band (Haifa)

12:15 – 13:00
G.Gallavotti (Rome)

Lunch break

Monday, 24th June 2019

Holomorphic states and incompressibility bounds for a 2D electron gas in a higher Landau level

J. Yngvason (Vienna)

In the talk I shall first describe how bounds on the particle density for a 2D electron gas in the lowest Landau level in a quantum Hall setting follow from the analyticity of the many-particle wave function. Then I shall explain how these results generalize to higher Landau levels via a represention of states in such levels by holomorhic functions of guiding-center variables.

The first part is based on joint work with Elliott Lieb and Nicolas Rougerie, the second one on joint work with NR.

Electric dipole problem and snake-like interface states for two-dimensional Dirac fermions in graphene monolayers

R. Egger (Düsseldorf)

It is well known that the low-energy physics of graphene monolayers can be accurately described in terms of two-dimensional Dirac fermions. After a basic introduction to this topic, I will discuss two questions of present interest. First, in the presence of a bandgap, we consider bound states induced by an electrostatic potential due to two oppositely charged Coulomb impurities. This potential is equivalent to an electric dipole potential at large distances. We show that bound states exhibit a universal Emov scaling structure near the band edge. Comparing this problem to the corresponding single-impurity "hydrogen" problem, where bound states can dive into the Dirac sea as the coupling constant is increased ("supercriticality"), we will also discuss the conditions for supercriticality to occur in the electric dipole problem. In the second part, I will discuss the interface states near a so-called p-n junction which is formed by a steplike electrostatic potential, $V(x, y) = V_0 \mathrm{sgn}(x)$, in the presence of a homogeneous orbital magnetic eld. Such interface states are shown to represent a mixture of snake orbits and skipping orbits, and they can propagate only along one direction.

Monday, 24th June 2019

Universal behaviour in quantum systems at low energy

A. Teta (Rome)

A system of quantum particles with short-range interaction in the low energy limit typically exhibits a universal behaviour, i.e., independent of the details of the interaction, characterised by a single physical parameter known as scattering length. We discuss some aspects of such a behaviour emerging in the study of Efimov effect, Feshbach resonance and neutron scattering.

Monday, 24th June 2019

Spectral properties in the one-commutator theory

S. Golenia (Bordeaux)

In this talk we shall review the recent results that were made in the abstract setting of the Mourre theory. We obtain new spectral results under a one-commutator hypothesis. For instance in the setting of Schroedinger operators and in any dimension, when the potential is $o(1/|x|)$ at innity, we prove some kind of diusion and that the spectral measure is of Rajchmann type. In the case, the absence of singularly continuous spectrum is still an open problem.

Monday, 24th June 2019

Bose-Einstein condensation in the Luttinger-Sy model with contact interaction

M. Pechmann (Hagen)

We study bosons on the real line in a Poisson random potential (Luttinger-Sy model) with contact interaction in the thermodynamic limit at absolute zero temperature. We prove that generalized Bose-Einstein condensation (BEC) almost surely occurs. We also show that the contact interaction alters the type of the condensation, going from a type-I BEC to a type-III BEC as the strength of this interaction is increased. Furthermore, for suciently strong contact interactions we prove that the mean particle density in the largest interval is reduced by the interparticle interaction.

Monday, 24th June 2019

The emergence of the $c = 1$ universality class in the XXZ spin 1/2 chain

K. Kozlowski (Lyon)

It has been argued on formal grounds, starting with the works of Luther and Peschel in 1975 and Haldane in 1981, that the one dimensional XXZ spin 1/2 chain subject to periodic boundary conditions falls with the universality class of the $c = 1$ free boson conformal eld theory. Such heuristics indicate that the XXZ chain should exhibit the same structure of the critical exponents governing the long-distance power-law decay of multi-point correlations functions as the ones arising in the free boson model.

In this talk, I will explain how the $c = 1$ free boson conformal eld theory emerges, directly on the level of the microscopic structure of the XXZ chain's Hilbert space, as an eective theory describing the long-distance behaviour of its static multi-point correlation functions. In particular, I will explain how one can rigorously construct a mapping allowing to put in correspondence the operator content attached to an appropriate subspace of the XXZ chain's Hilbert space with the operator content of the free boson model. The fundamental part of this work is that the correspondence between the two models giving rise to universality is obtained starting from the rst principles, directly at the microscopic level and in a very explicit way.

This is a joint work with J.-M. Maillet (ENS-Lyon, Lyon, France).

Monday, 24th June 2019

On the Essential Self-adjointness of Infinite Quantum Graphs

A. Kostenko (Ljubljana & Vienna)

It is widely known that the pre-minimal Kirchho Laplacian on an infinite metric graph might not be essentially self-adjoint (e.g., if the total length of the underlying metric graph is nite). Our main goal is to provide an overview of necessary and sucient conditions for the essential self-adjointness. In particular, we will focus on recently discovered fruitful connections between quantum graphs and discrete Laplacians on graphs. If time permits we shall also touch upon the problem of deciency indices and a description of self-adjoint extensions.

The talk is based on joint works with P. Exner, M. Malamud, D. Mugnolo, H. Neidhardt and N. Nicolussi.

Monday, 24th June 2019

Graph discretization of the Laplacian and applications

D. Bianchi (Como)

The main goal of the talk is to present ways to discretize the Laplace-Beltrami operator in order to guarantee uniform spectral convergence as the mesh-size goes to zero. We show that standard discretization (local) methods for dierential equations (such as Finite Dierence, collocation methods, Isogeometric Galerkin Analysis, etc.) fail in this task but they present the same limit stencil when the approximation order increases. This limit stencil, called Fourier stencil, guarantees the uniform spectral convergence of its associated graph-Laplacian. As a direct application we make use of the Fourier stencil to study the spectral gap problem for the 1d wave equation.

References

[1] D. Bianchi, *Fourier stencil for the graph discretization of the Laplacian.* preprint.

[2] D. Bianchi and S. Serra-Capizzano, *Spectral analysis of nite-dimensional approximations of 1d waves in non-uniform grids.* Calcolo 55(47) (2018).

[3] D. Burago, S. Ivanov, and Y. Kurylev, *A graph discretization of the Laplace-Beltrami operator.* J. Spectr. Theory 4(4) (2014): 675 – 714.

[4] E. Zuazua, *Propagation, observation, and control of waves approximated by nite dierence methods.* SIAM Rev. 47 (2005): 197 – 243.

Tuesday, 25th June 2019

On bulk deviations for the local behavior of random interlacements

A.-S. Sznitman (Zurich)

In this talk we will discuss some recent large deviation asymptotics concerning the local behavior of random interlacements on Z^d, $d \geq 3$. Among other we will describe the link with previous results concerning macroscopic holes left inside a large box, by the the adequately thickened connected component of the boundary of the box in the vacant sets of random interlacements.

Tuesday, 25th June 2019

Thermodynamics of repeated quantum measurements

C. A. Pillet (Toulon)

Under appropriate circumstances, subjecting a quantum system to repeated measurements with possible outcomes in a nite set $\mathcal{A}$ generates an invariant measure $\mathbb{P}$ of a simple classical dynamical system: the left shift

$$\tau : (\omega_1, \omega_2, \omega_3, \ldots) \mapsto (\omega_2, \omega_3, \omega_4 \ldots)$$

on the set $\mathcal{A}^{\mathbb{N}}$. However, except in very special cases, the resulting dynamical system $(\mathcal{A}^{\mathbb{N}}; \tau ; \mathbb{P})$ does not belong to well studied classes. Viewed as one-dimensional spin systems with long range interactions, these systems exhibit very rich (and sometimes very singular) thermodynamic behaviour. We will outline a general thermodynamic formalism which accommodates these systems and illustrate its unexpected features on a number of examples.

This talk is based on joint works with T. Benoist, N. Cuneo, V. Jakšić and Y. Pautrat.

Tuesday, 25th June 2019

Quantum chaos within integrable systems

B. Gutkin (Duisburg)

It is usually assumed that integrable systems posses simple dynamics, while their energy levels are uncorrelated and follow Poissonian statistics. I will discuss how many-body nature of model changes this perception. In spite of seemingly simple dynamical equations, the set of periodic orbits of many-body integrable systems can be quite complex. I will present a class of integrable spin chains where all periodic orbits are in one-to-one correspondence with periodic orbits of fully chaotic Arnold's cat map and posses non-trivial correlations. As a result, the long range spectral statistics of the corresponding quantum spin chains turn out to be intrinsically connected with the one of chaotic quantum maps.

Tuesday, 25th June 2019

Spectral aspects of 2-particle interactions in 1-dimension

S. Egger (Haifa)

We present various model systems of two particle interactions in one dimension. In particular, scattering properties, heat-kernel asymptotics and the existence of discrete eigenvalues below the essential spectrum are studied. We show similarities and dierences to corresponding one particle operators and demonstrate how spectral properties of two particle operators depend on the shape of the one-dimensional configuration space.

Tuesday, 25th June 2019

Derivation of the one- and two-dimensional Gross-Pitaevskii equation for strongly conned three-dimensional bosons

L. Boßmann (Tübingen)

We study the dynamics of a system of N interacting bosons in a cigar- or disc-shaped trap, which initially exhibit Bose-Einstein condensation and interact via a non-negative interaction potential in the Gross-Pitaevskii scaling regime. The trap is realized by an external potential, which connes the bosons in two/one spatial dimensions to a region of order ε. We study the simultaneous limit $(N, \varepsilon) \to (\infty; 0)$ and show that the N-body dynamics preserve condensation. The time-evolved condensate wave function is the solution of a one/two-dimensional Gross-Pitaevskii equation.

Joint work with Stefan Teufel.

Tuesday, 25th June 2019

Impurity-bound excitons in dimension two

H. Kovařík (Brescia)

We study three-body Schroedinger operators in two dimensions modelling an exciton interacting with a charged impurity. We consider certain classes of multiplicative interaction potentials proposed in the physics literature. We show that if the impurity charge is larger than some critical value, then three-body bound states cannot exist. Our spectral results are confirmed by variational numerical computations based on projecting on a nite dimensional subspace generated by a Gaussian basis.

This is a joint work with H.Cornean and T.G.Pedersen.

Uncertainty relations and null control for the heat conduction problem on domains with multiscale structure

I. Veselic (Dortmund)

I discuss uncertainty relations (aka spectral inequalities) for the Laplace and Schroedinger operators on bounded and unbounded domains. The subset of observation is assumed to be a thick or an equi-distributed set. A new result on the control cost allows to apply the first mentioned results and study the behaviour of the control cost in several asymptotic regimes, both regarding time and geometry. Methodical analogies to the study of random Schroedinger operators are highlighted.

Wednesday, 26th June 2019

Exponential decay for the 2 particle density matrix of disordered many-body fermions at zero and positive temperature

F. Klopp (Paris)

We will consider a simple model for interacting fermions in a random background at zero and positive temperature. At zero temperature, we prove exponential decay for the 2 particle density matrix of a ground state. At positive temperature we prove exponential decay for the 2 particle density matrix of the density operator in the grand canonical ensemble.

Wednesday, 26th June 2019

Two-dimensional Dirac operators with interactions supported by curves

K. Pankrashkin (Orsay)

Two-dimensional Dirac operators with interactions supported by curves. We study the two-dimensional Dirac operator with an arbitrary combination of electrostatic and Lorentz scalar interactions of constant strengths supported by curves. For any combination of coupling constants we give a rigorous description of the self-adjoint realization of the operators and give describe the spectral properties. It particular, for a critical combination of parameters the operator shows a loss of regularity, which results in a new accumulation point of eigenvalues in the gap of the essential spectrum, and we express the position of this points in terms of the coupling constants.

Based on a joint work with Jussi Behrndt (Graz), Markus Holzmann (Graz) and Thomas Ourmieres-Bonafos (Paris-Dauphine).

Wednesday, 26th June 2019

Quantum statistics on graphs

A. Sawicki (Warsaw)

The discovery of anyonic statistics has been one of the most exciting developments in physics in the last 50 years. It has led to improving our understanding of the phenomenon of superconductivity, fractional quantum Hall eect and brought new ideas to the area of quantum computing. During my talk I will discuss recent results concerning both abelian and non-abelian quantum statistics on graphs.

This is a joint work with J. Harrison J. Keating , T. Maciazek and J. Robbins.

References

[1] Maciazek, T., Sawicki, A., *"Non-abelian quantum statistics on graphs"*, arXiv:1806. 02846 (2018).
[2] Maciazek, T., Sawicki, A., *"Homology groups for particles on one-connected graphs"*, J. Math. Phys. 58 (2017), no. 6.
[3] Harrison, J. M., Keating, J. P., Robbins, J. M., Sawicki, A., *"n-Particle Quantum Statistics on Graphs"*, Comm. Math. Phys. 330, 1293 (2014).
[4] Leinaas, J. M., Myrheim, J., *"On the theory of identical particles"*, Nuovo Cim. 37B, 1 (1977).

Wednesday, 26th June 2019

Top Lyapunov exponents for products of non-homogeneous Markov-dependent matrices

I. Goldsheid (London)

Consider a sequence of of matrices from $SL(m,R)$ of the form $g_n = F(x_n)$, where x_n is a non-homogeneous Markov chain with phase space X and $F: X \mapsto SL(m,R)$. I shall discuss conditions under which the norm of the product $g_n g_{n-1} ... g_1$ of such matrices grows exponentially as $n \mapsto \infty$. The main result generalizes the well known theorem proved by A. Virtser in 1980 who considered the case of homogeneous Markov chains.

Wednesday, 26th June 2019

Anderson localization for Kirchhoff Laplacians on trees

S. Sukhtaiev (Houston)

This talk concerns Anderson localization for continuum and discrete graph Laplacians on random trees. I will show that the dynamical localization holds for random branching, random lengths, and random Kirchho models.

This is based on joint work with D. Damanik and J. Fillman.

Wednesday, 26th June 2019

The semi-classical limit with delta potentials

A. Posilicano (Como)

We consider the semi-classical limit of the quantum evolution of Gaussian coherent states whenever the Hamiltonian H is given, as sum of quadratic forms, by $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \alpha\delta_0$, with $\alpha \in \mathbb{R}$ and δ_0 the Dirac mass at $x = 0$. We show that the quantum evolution can be approximated, uniformly for any time away from the collision time and with an error of order $\hbar^{1+\lambda}$, $\lambda < \frac{1}{2}$, by the quasi-classical evolution generated by a self-adjoint extension of the restriction to $C_c^\infty(\mathscr{M}_0)$, $\mathscr{M}_0 := \{(q, p) \in \mathbb{R}^2 : q \neq 0\}$, of $(-i$ times) the generator of the free classical dynamics; such a self-adjoint extension does not correspond to the classical dynamics describing the complete reflection due to the innite barrier. Similar approximation results are also provided for the wave and scattering operators.

Joint work with Claudio Cacciapuoti and Davide Fermi.

Wednesday, 26th June 2019

Quantum chaos in the Benjamini-Schramm limit

E. Le Masson (Cergy-Pontoise)

One of the fundamental problems in quantum chaos is to understand how high-frequency waves behave in chaotic environments. A famous but vague conjecture of Michael Berry predicts that they should look on small scales like Gaussian random elds. We will show how the notion of Benjamini-Schramm convergence of manifolds (originally defined for graphs) can be used to formulate Berry's conjecture precisely, and give some partial results using this formalism.

Based on joint works with Miklos Abert, Nicolas Bergeron and Tuomas Sahlsten.

Wednesday, 26th June 2019

Bosonization beyond One Dimension: Correlation Energy of a Fermi Gas

N. Benedikter (Vienna)

While bosonization is a rigorously established and exact method to analyze one-dimensional systems, it is difficult to generalize to higher dimensional systems. I am going to discuss how bosonization can be generalized to higher dimensions as an approximation valid for example in the mean-field scaling regime of an interacting three-dimensional Fermi gas. In particular I am going to show how bosonization can be used to systematically derive an upper bound on the correlation energy of a Fermi gas.

Thursday, 27th June 2019

Fluctuations of linear statistics for Schroedinger operators with a random decaying potential

J. Breuer (Jerusalem)

Linear statistics provide a tool for the analysis of fluctuations of random measures and have been extensively studied for various models in random matrix theory. In this talk we discuss the application of the same philosophy to the analysis of the nite volume eigenvalue counting measure of one dimensional Schroedinger operators and demonstrate it with some interesting results in the case of a random decaying potential. This is joint work with Yoel Grinshpon and Moshe White.

Thursday, 27th June 2019

Semiclassics for difference operators

J. Bolte (London)

Weyl quantisation on a compact classical phase space leads to nite dimensional quantum Hilbert spaces, with the dimension of the Hilbert space serving as a semiclassical parameter. The kinematical aspects of this setting are well known from the quantisation of torus maps. In this talk we consider semiclassical approximations of continuous-time dynamics generated by a Schroedinger equation, in which the Hamiltonian is a difference operator. We prove a Gutzwiller trace formula, and as examples discuss an inverted harmonic oscillator on a two-dimensional toroidal phase space and a magnetic Laplacian on a four-dimensional torus.

The talk is based on joint work with Sebastian Egger, Stefan Keppeler, and with Lewis Proctor.

Thursday, 27th June 2019

Nodal statistics of quantum graph eigenfunctions

R. Band (Haifa)

We discuss the number of zeros of Laplacian eigenfunctions on a metric (quantum) graph. The n-th eigenfunction has at least $n-1$ zeros and at most $n-1+\beta$ zeros, where β is the number of graph cycles (graph's first Betti number). The nodal surplus of an eigenfunction is dened as the number of its zeros minus $(n-1)$. For a given graph, one might study the distribution of the nodal surplus of its eigenfunctions. This distribution is interestingly connected to the graph's topology and we show some recent results in this direction. Furthermore, numerical studies suggest that the nodal surplus distribution has a universal form: it converges to a normal distribution as the number of cycles grows. We state this conjecture and discuss our recent progress in proving it.

The talk is based on joint works with Lior Alon and Gregory Berkolaiko.

Thursday, 27th June 2019

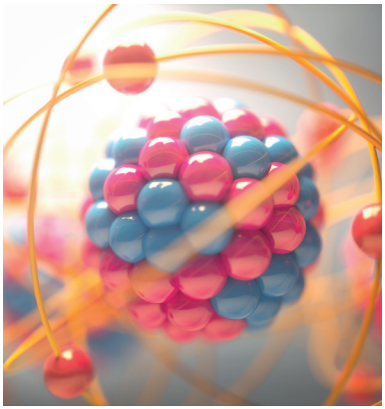
The Kondo problem: as a paradigm of the RG application

G. Gallavotti (Rome)

The susceptibility of a two level impurity interacting with fermions on a one dimensional lattice is studied via a RG transformation. Its non divergence at $T = 0K$ is discussed.

Notes

Notes



Conference

Further information:
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FernUniversität in Hagen

GdF Gesellschaft der Freunde der FernUniversität e.V.

DFG Deutsche Forschungsgemeinschaft