

Final Report

1. General Information

- DFG reference number: MU 2907/3-1 – MU 2907/3-2
- Project number: 397230547
- Project title: Analysis on Networks: Encounters of Mathematical Physics, Discrete Mathematics, and Theoretical Computer Science
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2. Summary

The aim of this project was to advance the analysis of elliptic operators on networks, i.e., both combinatorial and metric graphs. Networks serve both as models for complex physical systems and as accessible settings for abstract mathematics. The first period of this project focused on spectral properties, the second on parabolic aspects. Each culminated in a successful doctoral defense.

- Spectral theory (2018–2021). A major achievement was the creation of a toolbox for metric graph surgery, which enables new approaches to spectral problems. Applications include advances in spectral geometry, linking Laplacian eigenvalues and eigenfunctions to structural properties, and the development of clustering methods.
- Parabolic theory (2022–2025). We investigated heat-kernel related quantities, developing a theory of heat content and torsional rigidity with new formulae, bounds, and comparison principles. Further studies addressed parabolic equations rarely treated on metric graphs, such as non-autonomous diffusion, polyharmonic equations, diffusion with dynamic boundary conditions, evolution on ramified time-spaces, and general Sturm–Liouville operators. A striking finding was the discovery of large classes of equations with non-Markovian behavior in both discrete and continuous settings.

We also advanced the spectral theory of infinite metric graphs, complementing results for combinatorial graphs, and studied the eigenvalue comparisons for operators under different boundary conditions.

Another key result was showing that diffusion equations on metric graphs can be interpreted as gradient flows in Wasserstein spaces. This property was known for Euclidean domains and combinatorial graphs, but had not been established for branching metric measure spaces.

Summing up, the project established new tools and results for both spectral and parabolic operators on graphs. Highlights include metric graph surgery, heat-related invariants, non-Markovian diffusion, and the Wasserstein gradient-flow structure. These findings deepen the mathematical understanding of graph-based operators and open avenues for applications in physics, data science, and network analysis.

Ziel des Projekts war die Weiterentwicklung der Analysis elliptischer Operatoren auf Netzwerken, d.h., kombinatorischen und metrischen Graphen. Netzwerke dienen dabei sowohl als Modelle komplexer physikalischer Systeme als auch als handhabbare Umgebungen für abstrakte Mathematik. Die erste Periode dieses Projekts behandelte spektrale, der zweite parabolische Eigenschaften.

- Spektraltheorie (2018–2021). Eine zentrale Leistung war die Entwicklung einer Toolbox zur Chirurgie metrischer Graphen, die neue Zugänge zu spektralen Problemen eröffnet. Anwendungen reichen von Fortschritten in der Spektralgeometrie, wo Eigenwerte und Eigenfunktionen des Laplace-Operators mit strukturellen Eigenschaften verknüpft werden, bis hin zu neuen Clustering-Methoden.
- Parabolische Theorie (2022–2025). Wir untersuchten vom Wärmekern abgeleitete Größen und entwickelten eine Theorie des Wärmehalts und der Torsionssteifigkeit mit neuen Formeln, Schranken und Vergleichsprinzipien. Weitere Arbeiten betrafen selten untersuchte parabolische Gleichungen auf metrischen Graphen, darunter nichtautonome Diffusion, polyharmonische Gleichungen, Diffusion mit dynamischen Randbedingungen, Evolution auf verzweigten Zeiten und allgemeine Sturm–Liouville-Operatoren. Besonders hervorzuheben ist die Entdeckung großer Klassen von Gleichungen mit nicht-Markovschen Eigenschaften im diskreten wie im kontinuierlichen Fall.

Darüber hinaus wurden Erweiterungs- und Spektraltheorie unendlicher metrischer Graphen weiterentwickelt, ebenso Vergleiche von Operatoren mit unterschiedlichen Randbedingungen.

Ein weiteres wichtiges Ergebnis war der Nachweis, dass Diffusionsgleichungen auf metrischen Graphen als Gradientenflüsse in Wasserstein-Räumen verstanden werden können – eine Struktur, die zuvor nur für euklidische Gebiete und kombinatorische Graphen bekannt war.

Insgesamt hat das Projekt wesentliche neue Werkzeuge und Resultate für spektrale und parabolische Operatoren auf Graphen geliefert. Hervorzuheben sind die Chirurgie metrischer Graphen, wärme kernbezogene Invarianten, nicht-Markovsche Diffusion und die Wasserstein-Gradientenfluss-Struktur. Diese Ergebnisse vertiefen das theoretische Verständnis und eröffnen Perspektiven für Anwendungen in Physik, Datenwissenschaft und Netzwerktheorie.

3. Progress Report Over the past decades, analysis on networks has become a vibrant meeting point of functional analysis, spectral theory, combinatorics, mathematical physics, and computer science. A paradigmatic example is Google’s PageRank algorithm, which relies on the Perron eigenvector of a Markov chain transition matrix. More broadly, spectral methods for operators on graphs have had far-reaching impact on clustering, image processing, and data analysis. Discrete mathematics and theoretical computer science have thus repeatedly contributed to the development of new operator-theoretic tools, while conversely importing mathematical techniques such as Cheeger inequalities.

Recent years have witnessed a fruitful interplay between spectral graph theory and mathematical physics. Breakthroughs such as the solution of the Kadison–Singer problem or the use of spectral partitioning methods demonstrate the potential of cross-disciplinary approaches. Quantum graphs and discrete Laplacians, once considered primarily as toy models in physics, have now revealed deep structural analogies and are increasingly explored for applications in complex networks and machine learning. Classical graph invariants (chromatic number, mean distance, Wiener index) have been reinterpreted in the quantum setting, yielding new forms of spectral comparison and network design.

A new stage has been marked by the integration of variational methods and surgery principles for graphs, providing systematic tools to study eigenvalues and spectral gaps. Infinite networks have emerged as proxies for manifolds, raising questions on heat kernels, convergence, and stability. Across mathematics, physics, and computer science, the convergence of spectral geometry, diffusion analysis, and network models highlights a dynamic, interdisciplinary “state of the art” in which analysis on networks is both theoretically fertile and practically influential.

Parallel advances have occurred in parabolic theory. Especially heat kernel estimates have been central to understanding both analytic and geometric features of domains and manifolds since the 1950s. It was already known from the case of domains and manifolds that the short-time behavior of the heat kernel, the heat trace (the integral of the heat kernel along its diagonal) and the heat content (the double integral of the heat kernel) encode geometric information, suggesting applications to torsional rigidity, isoperimetric inequalities, and clustering tasks. Nonlinear extensions enrich this landscape. The main goal of this project was to extend these results to networks: on one hand, this is challenging because of the singular nature of combinatorial graphs and, especially, metric graphs (they are metric spaces that are flat almost everywhere, but whose curvature attains the value $-\infty$ at each vertex of degree ≥ 3 , making them a fundamental example of metric measure spaces that are branching and, thus, are not $\text{RCD}(k, N)$ spaces). The initial project (2018–2021) set out to advance spectral geometry of ramified structures. Building on these achievements, the renewal project (2022–2025) expanded the focus toward diffusion processes, heat kernels, and infinite networks.

Let me summarise the most relevant project-related findings: in the following I will refer to the reference list below using its numbering. During the first period (2018–2021), the most relevant findings involved the development of an extensive toolbox of metric graph surgery

[36] that allows to tackle various problems of spectral theory: first and foremost in the realm of spectral geometry (linking properties of Laplacian eigenvalues and eigenfunctions to metric, topologic and combinatorial properties of the underlying structure) [8, 11, 12, 15, 19, 20, 22, 23, 34] but also the development of new clustering methods [21, 23] that may have interesting implications in applied sciences.

One of the main reasons for studying networks is that they are often regarded as toy models for more complicated physical systems. A further, equally interesting feature is that networks make it possible to study elusive abstract mathematical properties in a convenient environment, where computations are made easier by tools from combinatorics, graph theory, Sturm–Liouville theory, or Fourier analysis. This is especially clear when the heat kernel is considered: on metric graphs, this quantity turns out to be remarkably more tractable than on domains or manifolds. In the second period (2022–2025), we have focused on the properties of a few quantities related to the heat kernel: we have developed a comprehensive theory of heat content and torsional rigidity on graphs, deriving new semi-explicit formulae for these quantities that made it possible to derive new bounds and comparison principles [4, 9, 13].

An important line of research throughout the whole lifespan of the project was the development of extension and spectral theoretical aspects of infinite metric graphs, somewhat complementing comparable results achieved in the last years for combinatorial graphs [3, 4, 7, 18, 29]

The project has also delved into studying classes of parabolic equations that have been hitherto seldom studied on metric graphs, including non-autonomous diffusion [33], Ornstein–Uhlenbeck processes [16], polyharmonic equations [29, 30, 31], diffusion with dynamic boundary conditions [17], evolution on ramified timespace [28], general Sturm–Liouville operators [10, 32], first order differential operators [24, 25], random walks in random environments [27]. In particular, we stress the serendipitous discovery of a large class of parabolic equations displaying non-Markovian behavior, both in the discrete and continuous case [9, 26, 30, 32].

One of the potentially most important achievements of this project was the discovery that diffusion equations on metric graphs can also be regarded as gradient flows in Wasserstein spaces: this is classical for Euclidean domains and graphs, but it has been hitherto unknown for any class of branching metric measure spaces [14].

Finally, an originally unexpected but eventually very successful sub-project involved the comparison of operators with different boundary conditions, especially concerning the differences of their eigenvalues [1, 5, 6, 7].

All the articles produced during the lifespan of this project have been published on an open repository (arXiv) and many of them have been then published in journals using the open access agreement of Springer and Wiley (Project DEAL).

During this project, I have co-organised several conferences, minisymposia, and summer schools, including

- Session on “Applied operator theory” at the Joint Meeting of ÖMG and DMV (Linz, September 2025, co-organised with P. Alonso-Ruiz, A. Hussein, N. Nicolussi)
- Master Class in Analysis on “Arithmetics and Geometry: Zeta-functions, Trace formulas, and around” with courses by K. Kirsten, P. Sarnak, S. Jitomirskaya (Stockholm, June 2023, co-organised with H. Hedenmalm, P. Kurasov, J. Rohleder, M. de Courcy-Ireland)
- Conference on “Geometric aspects of evolution and control” (Hagen, April 2023; co-organised with M. Täufer)
- School on “Heat Kernels and Spectral Geometry: From Manifolds to Graphs” with lectures by N. Anantharaman, D. Borthwick, J.B. Kennedy (Bregenz, September 2022, co-organised with P. Kurasov)

- Mini-symposium on “Variational Methods and Equations on Graphs” at the 8th European Congress of Mathematics (Portorož, June 2021, co-organised with A. Puchalska)
- Conference “On mathematical aspects of interacting systems in low dimension” (Hagen, June 2019, co-organised with J. Kerner and W. Spitzer)

The topics of the present project have played a central role in these scientific meetings. This rich outreach activity has been made possible also by my role as Leader of the Working Group on “Variational methods on graphs and networks” of the COST Action CA18232 (`mat-dyn-net.eu`), which has widely enlarged my scientific network since 2019, with a very positive impact on the outcome of the project.

I will now present the main results obtained within the scope of this project by Patrizio Bifulco (PB) and Marvin Plümer (MP)—the two doctoral students supported by this project—and myself, clustered in four areas.

SPECTRAL GEOMETRY, ISOPERIMETRIC INEQUALITIES, AND SURGERY PRINCIPLES

This has been the main area of the first part of this research project. Based on earlier, now-classical work by Nicaise, Exner, Kurasov, Naboko, and others, our working hypothesis was that relevant metric and combinatorial properties of a metric graph are mirrored by the *eigenvalues* of the associated Laplacian with standard (continuity and Kirchhoff-type) vertex conditions. These ideas were later extended to prove subtler estimates for further quantities that are defined using the *heat kernel* instead.

Surgery Principles and Spectral Gaps. A central theme of our research has therefore been the investigation of the spectral geometry of quantum graphs: understanding how the metric and topological properties of a network constrain the spectrum of its Laplacian. In a foundational series of papers, MP and I, together with our co-authors, established rigorous *surgery principles* for quantum graphs. We systematically cataloged how local geometric modifications—such as cutting through a vertex, gluing edges, or lengthening an edge—impact the eigenvalues of the Laplacian.

- **Monotonicity and Interlacing:** In order to derive eigenvalue estimates based on the graph structure, an effective approach is to compare a simpler reference graph with other, more general ones: to this aim, we proved that certain local or global modifications of graph induce certain controllable shifts in spectrum or interlacing inequalities, providing a toolkit for manipulating the spectrum, both in the case of discrete and metric graphs ([13, 20, 35, 36]).
- **Impediments to Diffusion:** MP and I, together with our co-authors, applied these principles to derive geometry-based upper bounds on the spectral gap (the first non-zero eigenvalue, λ_1). We identified structural “bottlenecks”—narrow connections between large subgraphs—that trap heat and lower the spectral gap. This allowed us to geometrically quantify “impediments to diffusion” ([9, 10, 16]). Applications to geometric control theory were presented in [11].

Inverse Problems and Scattering. We also tackled inverse problems—determining geometry from spectral data.

- **Distinguishing Cospectral Graphs:** In [12], my co-author and I addressed the problem of *cospectrality*. Using *scattering theory*, we demonstrated that the scattering matrix contains more information than the spectrum alone, allowing us to distinguish between graphs that share the same eigenvalues but have different topologies.
- **Ambarzumian’s Theorem and Spectral Stability:** In [7], PB and his co-author proved a version of Ambarzumian’s theorem for quantum graphs, establishing that if the spectrum of a Schrödinger operator coincides with that of the unperturbed Neumann Laplacian, the potential must be identically zero. This provides a strong links to existing, classical results in inverse spectral theory that go back to the pioneering

study by Ambarzumian in the 1920s. Motivated by these results, in [2, 6, 8] they also provided a number of estimates on the spectral shift for Schrödinger operators on combinatorial or metric graphs induced when singular and/or non-singular potentials are switched on.

Spectral Partitions and Nodal Domains. In literature, several approaches to the assignment of partitioning a domain, or a graph, in a given number k of parts are known; an especially promising one is based on the theory of *spectral minimal partitions*, first developed by Conti, Terracini, Verzini, and then widely popularised by Helffer and his co-authors. Together with my collaborators, I developed a comprehensive theory of spectral minimal partitions for metric graphs.

- **Optimal Cuts:** We analyzed the problem of partitioning a metric graph into k connected clusters to minimize the maximum, or else maximize the minimum, fundamental frequency of the clusters; and proved the existence and regularity of these minimal partitions ([22, 24]). These results represent significant advances over their counterparts on domains; in particular, we were able to prove a graph version of the *hexagonal conjecture* by Caffarelli and Li, which is still wide open on planar domains.
- **Revisiting Pleijel’s Theorem:** A crucial outcome of this project was our investigation of *Courant’s Nodal Domain Theorem*. While we confirmed that Courant’s bound holds for graphs, we demonstrated that *Pleijel’s Theorem* (regarding the asymptotic reduction of nodal domains) generally *fails* in the quantum graph setting ([23]). We constructed counterexamples and provided modified asymptotic estimates, highlighting a fundamental divergence between 1D networks and 2D manifolds. Further information on the geometry of eigenfunctions was deduced by MP and his co-author in [21].

EVOLUTION EQUATIONS AND SEMIGROUP THEORY ON NETWORKS

A second main focus was on diffusion and heat kernel methods. The project especially delved into the dependence of heat kernels on topology and geometry, analyzing short-time asymptotics and establishing a theory of heat content and torsional rigidity on graphs. These investigations aimed to produce new isoperimetric inequalities and comparison principles for diffusion, akin to what has been known for the spectrum since the 1980s, and then for higher eigenvalues since the 2000s. As an intermediate step, my group has previously also focused on the well-posedness and asymptotic behavior of partial differential equations of either hyperbolic or parabolic nature posed on network structures.

Faber–Krahn Inequalities and Torsional Rigidity. We successfully extended classical isoperimetric inequalities to diffusion phenomena on metric graphs, describing—and in some cases even characterising—the graph topologies that are associated with good insulation.

- **Heat Content:** In [A], PB and I have recently developed the theory of heat content of a metric graph. Our most relevant results have been a closed formula for such heat content that only depends on the length of closed orbits within the metric graph, and which in turn permits to show both an asymptotic expression conceptually similar to Roth’s celebrated trace formula. In [1], PB and his co-author proved a *Faber–Krahn inequality for the heat content* on quantum graphs. By utilizing a novel random walk expansion of the heat kernel, they showed that for small times, the heat content is maximized by the interval among all graphs of fixed volume.
- **Torsional Rigidity:** In [5, 10, 14], my-co-authors and I investigated *torsional rigidity* on discrete and metric graphs, a quantity that was introduced in engineering by Saint-Venant in the 19th century and in mathematics by Pólya in the 1950s and which can be regarded as an integrated version of the heat content. We established a sharp lower bound for the ground-state energy in terms of torsional rigidity; and

proved a remarkable version of the *Kohler-Jobin inequality*—a kind of uncertainty principle first conjectured by Pólya—that was hitherto only known for the Laplacian with Dirichlet boundary conditions on domains. Our results confirm that graphs which have high torsional rigidity necessarily cannot possess arbitrarily low fundamental frequencies.

Hyperbolic Systems and Wave Equations. We addressed the complexities of wave propagation where reflection and transmission occur at vertices.

- **Linear Hyperbolic Systems:** In [25], my co-authors and I treated general linear hyperbolic systems on networks of 1D structures; while in [26] we derived and characterised *dynamic transmission conditions* that ensure well-posedness. A key contribution was our characterization of stability: we determined the specific topological and coefficient conditions under which the system's energy decays to zero.
- **Acoustic Boundary Conditions:** In [3], my co-authors and I studied the wave equation on surfaces with **acoustic boundary conditions**—dynamic conditions modeling locally reacting surfaces. We established well-posedness and energy decay rates, proving how the boundary absorbs energy over time. The case of domains and manifolds poses serious technical difficulties that do not appear in the 1D case. Comparable settings in the parabolic setting were studied in [18].

Autonomous and Non-Autonomous Diffusion and Gradient Flows. We extended the analysis of heat and mass transport to highly abstract settings.

- **Time-Dependent Heat Kernels:** Based on celebrated results by Aronson and Daners on heat kernel estimates for specific classes of non-autonomous operators with specific boundary conditions on Euclidean domains, my co-author and I have presented an abstract approach to proving invariance properties and heat kernel estimates for abstract classes of operators associated with (time-dependent) quadratic forms ([34]). Several applications, especially to lower-dimensional problems, were given.
- **Wasserstein Gradient Flows:** One of the potentially most important achievements of this project was the discovery that diffusion equations on metric graphs can also be regarded as gradient flows with respect to the Wasserstein metric in an appropriate space of probability measures: this is classical for Euclidean domains and graphs, but it was hitherto unknown for any class of branching metric measure spaces ([15]).
- **“Time as a Graph”:** In two conceptually novel works ([28, 29]), my co-authors and I explored evolution equations where the time variable itself is modeled as a graph. This allowed us to analyze “branching time” scenarios, generalizing standard Cauchy problems to systems with multiple, diverging futures; or, alternatively, landscapes of evolution that can hop between different configurations at random time, in the spirit of the theory of *random walks in random environments*.

ANALYSIS ON INFINITE QUANTUM GRAPHS

Standard spectral geometry on metric graphs relies on the compactness of the resolvent operator, which guarantees a discrete spectrum. This compactness may or may not be lost on infinite graphs. My colleagues and I developed new analytical tools to characterize and possibly handle this difficulty.

Extensions and Spectral Phase Transitions. Laplacians on infinite metric graphs have been studied since the 1980s, but always restricted to two classes: Laplacians on metric *graphs of bounded geometry*, or Friedrichs extensions on metric graphs of unbounded geometry. Within this project, a rigorous framework for defining Laplacians on general infinite metric graphs was provided

- **Self-Adjoint Extensions:** My co-authors and I could remove these restrictions and classified in [19] all self-adjoint as well as all Markovian extensions of the minimal Laplacian on a very large class of metric graphs, usually in terms of boundary conditions but in some remarkable cases in terms of different quantities (the dimension of the space of harmonic functions).
- **Compactness Criteria:** My co-authors and I established a new criterion for the compactness of the resolvent in [4]. We characterized exactly when the spectrum remains discrete and when the *essential spectrum* appears, demonstrating “spectral phase transitions” where slight geometric changes at infinity cause a sudden shift from discrete to continuous spectra. In [C] we could present a very elementary, one-parameter model where this phenomenon can be easily described; transition from a classical Weyl law to failure thereof is also observed.

OPERATOR THEORY ON GRAPHS: HIGHER-ORDER MODELS AND POINT INTERACTIONS

The diffusion equation on metric graphs is a macroscopic description of the Brownian motion on network-like systems of 1D channels. Different stochastic processes and pseudo-processes can be studied, too, leading to different, partially unexpected spectral and diffusive properties.

Polyharmonic and Higher-Order Operators. Flexible strings are not the only physical model relevant to networks. This led my co-authors and me to developing the following work package, which has extended known results for operators in higher-dimensional domains making good use of the the simple 1D local geometry of graphs.

- **Ornstein–Uhlenbeck Semigroups:** In [17], we analyzed Ornstein–Uhlenbeck processes on star graphs, providing explicit characterizations of invariant measures, which is essential for understanding stochastic processes on networks with central hubs.
- **Polyharmonic Operators:** In [31, 32] we proved the well-posedness of parabolic and hyperbolic problems associated with these operators, which model stiff beams, the Bi-Laplacian Δ^2 being the prototypical example, under a large class of stationary and dynamic vertex transmission conditions. Associated with these phenomena is the property of eventual positivity, which we characterised in dependence of vertex transmission conditions for elliptic operators of different order [31, 33], including non-self-adjoint, singular transmission conditions; and, more recently studied, of eventual domination of smoothing semigroup, which was first discussed by my co-author and mine in [27] and characterised in terms of spectral properties.
- **Heat kernels in the bounded geometry case:** In [30], my co-authors and I focused on the well-understood case of Laplacians on metric graphs of bounded geometry: while in this case self-adjointness and Markovian properties are immediate to check, we were able to provide an abstract mechanism that allows heat kernels estimates that satisfy appropriate decay estimates on \mathbb{R} to be entailed to general metric graphs. Several associated phenomena, including a Feynman–Kac formula, a Combes–Thomas estimate and strong results on the stability of the spectrum under different realisations could be derived.

Point Interactions and Weyl Laws. We investigated how altering vertex conditions affects spectral asymptotics.

- **δ' -Couplings:** In [2], PB and his co-author focused on graphs with δ' -couplings. They derived a *modified local Weyl law*, showing that these couplings alter the high-frequency asymptotic distribution of eigenvalues compared to standard Kirchhoff conditions.
- **Landscape Functions:** In [10], I applied the *Filoché-Mayboroda landscape function* technique to quantum graphs. I proved that this auxiliary function provides effective pointwise bounds on eigenfunctions, allowing us to predict *localization regions*

without solving the full eigenvalue equation. Substantial applications to nonlinear spectral theory have been put forward in [5].

These results were obtained during visits paid at important European research groups and presented at numerous colloquia and international conferences by MP, PB and myself: among these investigation and dissemination activities let me mention

- October 2018: Conference “Semigroups of Operators – Theory and Applications”; Kazimierz Dolný (Poland)
- January 2019: Research visit in Lisbon
- July 2019: Research visit in Lisbon
- February 2019: Conference “Differential Operators on Graphs and Waveguides”; Graz (Austria)
- July 2021: Conference “Research on Mathematical Analysis and Semigroups” and “PDEs and Semigroups in Applied Analysis”; both Bari (Italy)
- September 2021: Conference “Mathematical Models for Interacting Dynamics on Networks”; Rome (Italy)
- October 2021: Conference “PDE, Analysis and Applications”; Valenciennes (France)
- May 2022: Research visit in Stockholm
- July 2022: Research visit in Lisbon
- June 2022: Conference and training school “Spectral Theory for Operators and Semigroups”; Agropoli (Italy)
- September 2022: Training school “Heat Kernels and Spectral Geometry: From Manifolds to Graphs”; Bregenz (Austria)
- April 2024: Conference “Probability and Analysis 2024”; Będlewo (Poland)
- June 2025: Conference “Walkshop on Mathematical Physics 2025”; Bremen (Germany)

With the financial support of this project I could also invite to Hagen distinguished colleagues, including

- August 2018: Noema Nicolussi
- September 2018: Marcel Schmidt
- January 2022: Olaf Post
- April 2022: Alessandro Duca
- May 2022: Fabian Rupp
- June 2022: Robert Denk
- June 2022: Joachim von Below
- March 2023: Giulia Meglioli
- June 2023: Tommaso Rossi

4. Published Project Results

4.1 Category A – Articles in peer-reviewed journals, contributions to peer-reviewed conferences or to anthology volumes, and book publications

- (1) P. Bifulco, M. Täufer, *Faber–Krahn inequality for the heat content on quantum graphs via random walk expansion*, Electron. J. Probab., 2025, DOI: 10.1214/25-EJP1414
- (2) P. Bifulco, J. Kerner, *A modified local Weyl law and spectral comparison results for δ^l -coupling conditions*, Journal of Mathematical Physics, 2025, DOI: 10.1063/5.0239937
- (3) D. Mugnolo, E. Vitillaro: *The wave equation with acoustic boundary conditions on non-locally reacting surfaces*, Notices of the American Mathematical Society, 2025, DOI: 10.1090/memo/1526
- (4) M. Düfel, J.B. Kennedy, D. Mugnolo, M. Plümer, M. Täufer, *On the spectrum of infinite quantum graphs*, Analysis and Mathematical Physics, 2025, DOI: 10.1007/s13324-025-01131-0

- (5) P. Bifulco, D. Mugnolo, *On the p -torsional rigidity of combinatorial graphs*, *Nonlinear Analysis*, 2025,
DOI: 10.1016/j.na.2024.113694
- (6) P. Bifulco, J. Kerner, *Comparing the spectrum of Schrödinger operators on quantum graphs*, *Proceedings of the American Mathematical Society*, 2024,
DOI: 10.1090/proc/16578
- (7) P. Bifulco, J. Kerner, *A note on Ambarzumian's theorem for quantum graphs*, *Archiv der Mathematik*, 2024,
DOI: 10.1007/s00013-024-01997-9
- (8) P. Bifulco, J. Kerner, *Some spectral comparison results on infinite quantum graphs*, *Journal of Mathematical Physics*, 2024,
DOI: 10.1063/5.0178226
- (9) L.N. Baptista, J.B. Kennedy, D. Mugnolo, *Mean Distance on Metric Graphs*, *Journal of Geometric Analysis*, 2024,
DOI: 10.1007/s12220-024-01574-0.
- (10) D. Mugnolo, *Pointwise eigenvector estimates by landscape functions: Some variations on the Filoche–Mayboroda–van den Berg bound*, *Mathematische Nachrichten*, 2024,
DOI: 10.1002/mana.202300239.
- (11) M. Egidi, D. Mugnolo, A. Seelmann, *Sturm-Liouville problems and global bounds by small control sets and applications to quantum graphs*, *Journal of Mathematical Analysis and Applications*, 2024,
DOI: 10.1016/j.jmaa.2024.128101.
- (12) D. Mugnolo, V. Pivovarchik, *Distinguishing cospectral quantum graphs by scattering*, *Journal of Physics A: Mathematical and Theoretical*, 2023,
DOI: 10.1088/1751-8121/acbb44.
- (13) G. Berkolaiko, J.B. Kennedy, P. Kurasov, D. Mugnolo, *Impediments to diffusion in quantum graphs: Geometry-based upper bounds on the spectral gap*, *Proceedings of the American Mathematical Society*, 2023,
DOI: 10.1090/proc/16322.
- (14) D. Mugnolo, M. Plümer, *On torsional rigidity and ground-state energy of compact quantum graphs*, *Calculus of Variations and Partial Differential Equations*, 2023,
DOI: 10.1007/s00526-022-02363-9.
- (15) M. Erbar, D. Forkert, J. Maas, D. Mugnolo, *Gradient flow formulation of diffusion equations in the Wasserstein space over a metric graph*, *Networks and Heterogeneous Media*, 2022,
DOI: 10.3934/nhm.2022023.
- (16) E. Estrada, D. Mugnolo, *Hubs-biased resistance distances on graphs and networks*, *Journal of Mathematical Analysis and Applications*, 2022,
DOI: 10.1016/j.jmaa.2021.125728.
- (17) D. Mugnolo, A. Rhandi, *Ornstein–Uhlenbeck semigroups on star graphs*, *Discrete and Continuous Dynamical Systems - Series S*, 2022,
DOI: 10.3934/dcdss.2022030.
- (18) A. Khoutaibi, L. Maniar, D. Mugnolo, A. Rhandi, *Parabolic equations with dynamic boundary conditions and drift terms*, *Mathematische Nachrichten*, 2022,
DOI: 10.1002/mana.201900348.
- (19) A. Kostenko, D. Mugnolo, N. Nicolussi, *Self-adjoint and Markovian extensions of infinite quantum graphs*, *Journal of the London Mathematical Society*, 2022,
DOI: 10.1112/jlms.12539.
- (20) M. Plümer, *Upper eigenvalue bounds for the Kirchhoff Laplacian on embedded metric graphs*, *Journal of Spectral Theory*, 11(4), 1857–1894, 2021,
DOI: 10.4171/JST/388
- (21) M. Plümer, M. Täufer, *On fully supported eigenfunctions of quantum graphs*, *Letters in Mathematical Physics*, 2021,
DOI: doi.org/10.1007/s11005-021-01489-9

- (22) J.B. Kennedy, P. Kurasov, C. Léna, D. Mugnolo, *A theory of spectral partitions of metric graphs*, Calculus of Variations and Partial Differential Equations, 2021, DOI: 10.1007/s00526-021-01966-y.
- (23) M. Hofmann, J.B. Kennedy, D. Mugnolo, M. Plümer, *On Pleijel's Nodal Domain Theorem for Quantum Graphs*, Annales Henri Poincaré, 2021, DOI: 10.1007/s00023-021-01077-6.
- (24) M. Hofmann, J.B. Kennedy, D. Mugnolo, M. Plümer, *Asymptotics and Estimates for Spectral Minimal Partitions of Metric Graphs*, Integral Equations and Operator Theory, 2021, DOI: 10.1007/s00020-021-02635-7.
- (25) M. Kramar Fijavž, D. Mugnolo, S. Nicaise, *Dynamic transmission conditions for linear hyperbolic systems on networks*, Journal of Evolution Equations, 2021, DOI: 10.1007/s00028-021-00715-0.
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4.2 Category B – Any other form of published results:

- (A) P. Bifulco, D. Mugnolo, *On the heat content of compact quantum graphs*, 2025, arXiv:2502.09461
- (B) D. Mugnolo, *The heat flow driven by the Laplacian of a directed hypergraph*, 2025, arXiv:2510.17497

- (C) J.B. Kennedy, D. Mugnolo, and M. Täufer, *Towards a theory of eigenvalue asymptotics on infinite metric graphs: the case of diagonal combs*, 2024, arXiv:2403.10708
- (D) P. Bifulco and D. Mugnolo, *On the Lipschitz continuity of the heat kernel*, 2023, arXiv:2307.08889