BOOK OF ABSTRACTS

WORKSHOP ON SPECTRAL GEOMETRY, PDEs AND MATHEMATICAL PHYSICS

The giant component of excursion sets of spherical Gaussian ensembles (Wigman)

Abstract. We establish the existence and uniqueness of a well-concentrated giant component in the supercritical excursion sets of three important ensembles of spherical Gaussian random fields: Kostlan's ensemble, band-limited ensembles, and the random spherical harmonics. Our main results prescribe quantitative bounds for the volume fluctuations of the giant that are essentially optimal for non-monochromatic ensembles, and suboptimal but still strong for monochromatic ensembles.

Our results support the emerging picture that giant components in Gaussian random field excursion sets have similar large-scale statistical properties to giant components in supercritical Bernoulli percolation. The proofs employ novel decoupling inequalities for spherical ensembles which are of independent interest.

This talk is based on a joint work with Stephen Muirhead.

The Hot Spots conjecture is false. How false it? (de Dios Pont)

Abstract. As time evolves, the temperature in a well insulated object converges to the initial average temperature. Which points take the longest to reach this average? The Hot Spots conjecture asserts that, for reasonable domains, these points are the ones further from the bulk of the material, that is to say points in the boundary of the object. We now know the conjecture is false for man reasonable classes of domains. But how false is it? Steinerberger introduced the Hot Spots ratio in order to measure the degree of failure of the conjecture. In this talk we will find the exact value of this ratio in every dimension, and understand the shape of domains for which this conjecture is "as false as possible".

Recent developments around the hot spots conjecture (Rohleder)

Abstract. The hot spots conjecture due to Jeffrey Rauch dates back to the 1970s. Formulated in terms of spectral theory, the conjecture (in its strongest form) is the following: any second eigenfunction of the Laplacian with Neumann boundary conditions on a bounded, simply connected domain should attain its minimum and maximum only on the boundary. Though positive results exist for certain classes of planar domains, to date the hot spots conjecture is open, even for general convex planar domains, and counterexamples exist for certain multiply connected domains.

In this talk we present a recent new approach to the conjecture through non-standard variational principles. We discuss results in dimension 3 and higher (joint work with James B. Kennedy) and results on an analogous problem for mixed Neumann-Dirichlet boundary conditions (joint work with Nausica Aldeghi).

Off-diagonal upper heat kernel bounds on graphs (Rose)

Abstract. The heat kernel as the fundamental solution of the heat equation with respect to a discrete Laplace operator is the distribution of the corresponding jump process on the underlying graph. This talk will review some recent contributions regarding off-diagonal upper bounds with respect to intrinsic metrics.

Spectral comparison results, now and then: An overview (Bifulco)

Abstract. We study Schrödinger operators on compact finite metric graphs subject to δ-coupling and standard boundary conditions often known as Kirchoff-Neumann vertex conditions. We compare the n-th eigenvalues of those self-adjoint realizations and derive an asymptotic result for the mean value of the eigenvalue deviations which represents a generalization to a recent result by Rudnick, Wigman and Yesha obtained for domains in $\mathbb{R}^2$ to the setting of metric graphs. We start this talk by introducing the basic notion of a metric graph and discuss some basic properties of heat kernels on those graphs afterwards. In this way, we are able to discuss a so-called local Weyl law which is relevant for the proof of the asymptotic main result. If time permits, we will also briefly discuss the case of δ'-coupling conditions and some possible generalizations on infinite metric graphs having finite total length or even combinatorial graphs.

This talk is based on joint works with Joachim Kerner (Hagen), Delio Mugnolo (Hagen) and Christian Rose (Potsdam).

Entanglement entropy of non-interacting massless Dirac fermions in dimension one (Spitzer)

Abstract. We present a novel proof of a formula of Casini and Huerta for the entanglement entropy of the ground state of non-interacting massless Dirac fermions in dimension one localized to (a union of) intervals and generalize it to the case of Rényi entropies and to equilibrium states at positive temperature. This is based on joint work with Fabrizio Ferro and Paul Pfeiffer

Macroscopic Thermalization for Highly Degenerate Hamiltonians After Slight Perturbation (C. Vogel)

Abstract. A closed macroscopic quantum system thermalizes if its initial state reaches a suitable equilibrium subspace and stays there for most of the time. A sufficient condition for the thermalization of all initial states is the eigenstate thermalization hypothesis (ETH). Tasaki recently proved the ETH for a perturbation of the Hamiltonian of free fermions on a one-dimensional lattice. The perturbation is needed to remove the high degeneracy of the Hamiltonian. First, we prove the ETH also for the unperturbed Hamiltonian. Second, we show that in general if a Hamiltonian has one eigenbasis in equilibrium, then after adding a small generic perturbation it satisfies the ETH and thus all states thermalize. This is joint work with Barbara Roos, Shoki Sugimoto, Stefan Teufel and Roderich Tumulka.

Spectral properties of Schrödinger Operators with δ'-interactions via Robin-Laplacians (M. Vogel)

Abstract. We discuss various spectral properties of two-dimensional Schrödinger operators with δ'-potentials supported on star graphs. In particular, we review some elementary proof ideas and demonstrate how Robin Laplacians can assist in determining the spectral properties of Schrödinger operators with δ'-potentials. The main observation is that a rather detailed spectral picture in the δ'-case can be obtained by comparison with both Robin boundary conditions and δ-interactions. We also discuss possibilities for generalization beyond star graphs.

Geometric excess of the fundamental gap (Bucur)

Abstract. On convex sets of $\mathbb{R}^N$ the Payne-Weinberger and Andrews-Clutterbuck inequalities provide sharp lower bounds of the fundamental gap of the Laplace operator with Neumann and Dirichlet boundary conditions, respectively, in terms of the diameter. For $N \geq 2$ the bounds are not attained and are improved with a geometric term related to the flatness of the convex set. This is a joint work with V. Amato and I. Fragala.

Eigenvalues of the Neumann magnetic Laplacian in the unit disk (Léna)

Abstract. We consider the first eigenvalue of the magnetic Laplacian with Neumann boundary conditions in the unit disk. The well developed asymptotic analysis of Fournais-Helffer (when the field strength goes to infinity), known inequalities and numerical computations lead to rather simple conjectures. We explore these questions by revisiting, theoretically and numerically, a classical picture obtained by the physicist D. Saint-James. On the way, we refine the known asymptotic results by combining them with a formula stated by Saint-James.

Spectral minimal partitions of unbounded domains (Kennedy)

Abstract. We study the problem of partitioning possibly unbounded domains by minimising functionals built out of p-norms of Dirichlet Laplacian (or Schrödinger) eigenvalues, analogous to the "classical" spectral minimal partition (SMP) problems on bounded domains considered by Helffer, Terracini and others.

We will show that there is a special threshold value, related to the essential spectrum of the operator on the whole domain, which acts as an upper bound for the energies attained by the minimal partitions. Moreover, if there is a partition with energy strictly below this threshold value then a minimising partition always exists; while otherwise existence is not guaranteed.

There are also several surprises compared with the bounded case; for example, the strong (set) and weak (function-based) formulations of the problem are not always equivalent; and even in the ∞-norm case (that is, where one seeks a partition for which the maximum of the first eigenvalues among all partition cells is minimal) there may be minimising partitions for which not all cells have the same first eigenvalue.

This is based on joint work with Matthias Hofmann and Hugo Tavares.

Spectral Partition Problems with Volume Constraint: An Overview (Andrade)

Abstract. In this talk, we discuss spectral partition problems with volume constraints, a class of variational problems arising at the intersection of spectral theory, shape optimization, and partial differential equations. We establish the existence of an optimal open partition by proving that the corresponding eigenfunctions are locally Lipschitz continuous.

On the electron distribution of relativistic atoms and heat kernel bounds (Merz)

Abstract. The study of the electron distribution in atoms and molecules is paramount in quantum physics and chemistry. By the uncertainty principle, the innermost electrons move with velocities which are a substantial fraction of the speed of light. Hence, a relativistic description is mandatory. In this talk, we present new pointwise upper bounds for the sum of the squares of the eigenfunctions of the relativistic Chandrasekhar operator, in particular for each angular momentum channel separately. Our proof is concise and primarily relies on recently established heat kernel bounds for Hardy perturbations of subordinated Bessel heat kernels. This talk is based on joint works with Krzysztof Bogdan and Tomasz Jakubowski, and with Rupert Frank.

Multifractality in Quantum Star Graphs (Nietschmann)

Abstract. In the field of quantum chaos, much attention has been given to localization and delocalization properties of chaotic dynamical systems. In the physics literature, it has been conjectured that intermediate systems which lie at the transition point between two physical regimes such as the Anderson model, feature eigenfunctions that exhibit a “multifractal” structure: a self-similarity in a certain scaling regime which cannot be described by a single fractal exponent, but instead requires a continuous spectrum of exponents. In this talk, I will give an introduction into quantum star graphs before showing that multifractality exists for a generic choice of edge lengths.

On the fundamental eigenvalue gap of Sturm-Liouville operators (Ahrami)

Abstract. We use methods of direct optimization as in [Z. El Allali and E. M. Harrell II , Optimal bounds on the fundamental spectral gap with single-well potentials, Proc. Amer. Math. Soc., 150,(2022), 57-587] to characterize the minimizers of the fundamental gap of Sturm-Liouville operators on an interval, under the constraint that the potential is of single-well form and that the weight function is of single-barrier form, and under similar constraints expressed in terms of convexity. This is based on joint work with Zakaria El Allali and Evans Harrell.

Flat bands, atypical spectra and dynamics on non-locally finite crystals (Täufer)

Abstract. We investigate the spectral theory of periodic graphs which are not locally finite but carry non-negative, symmetric and summable edge weights. These periodic graphs are shown to have rather intriguing behaviour. We shall discover "partly flat bands" which are only flat for certain quasimomenta. We construct a periodic graph whose Laplacian has purely singular continuous spectrum. We prove that motion remains ballistic along at least one layer under quite general assumptions. We construct a graph whose Laplacian has purely absolutely continuous spectrum, exhibits ballistic transport, yet fails to satisfy a dispersive estimate. We believe that this class of graphs can serve as a playground to better understand exotic spectra and dynamics in the future. Based on joint work with Joachim Kerner, Olaf Post and Mostafa Sabri (Comm. Math. Phys., 2025).

λ Vs h (Brasco)

Abstract. For open subsets of the Euclidean space, it is known that we can bound from below the first eigenvalue of the Dirichlet-Laplacian, in terms of its Cheeger's constant only. Is it possible to revert this estimate? In the talk we will try to give some answers to this question.

Convergence of first order operators on thick graphs (Post)

Abstract. In this talk we discuss the convergence of first order operators on a thickened graph (a graph-like space) towards a similar operator on the underlying metric graph. On the graph-like space, the first order operator is of the form exterior derivative (the gradient) on functions and its adjoint (the negative divergence) on closed 1-forms (irrotational vector fields). Under the assumption that each cross section of the tubular edge neighbourhood is convex, that each vertex neighbourhood is simply connected and under suitable uniformity assumptions (which hold in particular, if the spaces are compact) we establish generalised norm resolvent convergence of the first order operator on the graph-like space towards the one on the metric graph. The square of the first order operator is of Laplace type; on the metric graph, the function (0-form) component is the usual standard (Kirchhoff) Laplacian. A key ingredient in the proof is a uniform Gaffney estimate: such an estimate follows from an equality relating here the divergence operator with all (weak) partial derivatives and a curvature term, together with a (localised) Sobolev trace estimate (joint work with P. Exner).

Exponential tail estimates for quantum lattice dynamics (Cedzich)

Abstract. We consider the quantum dynamics of a particle on a lattice for large times. Assuming translation invariance, and either discrete or continuous time parameter, the distribution of the ballistically scaled position Q(t)/t converges weakly to a distribution that is compactly supported in velocity space, essentially the distribution of group velocity in the initial state. We show that the total probability of velocities strictly outside the support of the asymptotic measure goes to zero exponentially with t, and we provide a simple method to estimate the exponential rate uniformly in the initial state. Near the boundary of the allowed region the rate function goes to zero like the power 3/2 of the distance to the boundary. The method is illustrated in several examples.

Spectral Flow and Eigenvalue Comparison for Schrödinger Operators on Metric Graphs (Sofer)

Abstract. When comparing the spectra of two self-adjoint operators, it is often useful to compare their eigenvalue counting functions. A well-known result in this direction for Schrödinger operators on metric graphs is known as Dirichlet-Neumann bracketing, which essentially states that the eigenvalue counting functions of the Dirichlet and Neumann Laplacians differ by at most the size of the boundary.

The goal of this talk is to introduce a useful tool for such comparison results, known as the spectral flow, which is a topological invariant associated with one-parameter families of self-adjoint operators. We show that for Schrödinger operators on metric graphs, the spectral flow can be effectively computed using the associated scattering matrices. We then present several applications, in the form of nodal index theorems and eigenvalue interlacing results.

The talk is partially based on joint work with Ram Band and Marina Prokhorova.

Optimal Poincaré-Hardy inequalities on graphs (Fischer)

Abstract. We review a method to obtain optimal Poincaré-Hardy inequalities on the hyperbolic spaces. Then we show how to transfer the basic idea to the discrete setting. This yields optimal Poincaré-Hardy-type inequalities on weakly spherically symmetric graphs which include fast enough growing trees and anti-trees. Moreover, this method yields optimal weights which are larger at infinity than the optimal weights constructed via the Fitzsimmons ratio of the square root of the minimal positive Green's function. Joint work with Christian Rose.