
Ambiguities in one-dimensional phase retrieval

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In many scientific areas, such as astronomy, electron microscopy, and crystallography, one is faced with the problem to recover an unknown signal from the magnitudes of its Fourier transform. Unfortunately, this phase retrieval problem is complicated because of the well-known ambiguousness.

Using a novel approach, we give a complete characterization of all occurring ambiguities. Moreover, we show that each further solution of the discrete-time phase retrieval problem can be described by an appropriate convolution representation of the original signal and by suitable rotations, shifts, and conjugations and reflections of the appearing factors.

Based on our characterization of the solution set, we investigate different a priori conditions in order to reduce the number of ambiguities or even to receive a unique solution. For example, if we have access to additional magnitudes of the unknown signal in the time domain, we can show that almost all signals with finite support can be uniquely recovered. An analogous result can be obtained by exploiting additional phase information.