Solution paths of convex regularizations

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Abstract: Energy functionals of the form $u \mapsto E_t(u) = \frac{1}{2}\|Au - f\|_2^2 + t\,\|u\|_1$ are frequently used in image and signal analysis to recover a signal $\hat{u}$ from the linear measurements $f \approx A\hat{u}$. An approximate reconstruction is obtained by solving $u(t) \in \operatorname{argmin}_u E_t(u)$. In order to compute a reconstruction close to the original signal $\hat{u}$, the regularization parameter $t \in \mathbb{R}_{\geq 0}$ has to be chosen wisely. Instead of solving the problem just for a fixed t, I will explain how to compute a solution for every $t \in \mathbb{R}_{\geq 0}$ in finite time.
Under a condition on the solution path, called "one-at-a-time", the well-known Homotopy method is able to compute a piecewise linear and continuous solution path by solving a linear system at every kink. To illustrate that this condition is necessary, I will discuss a toy example in which the standard Homotopy method fails.
By replacing the linear system with a nonnegative least squares problem, our method works for arbitrary matrix A and data f. I will give a full characterization of the set of possible directions and discuss the finite termination property of our algorithm.