

Explicit Analytical Expression for the Condition Number of Polynomials in Power Form

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Abstract.

The condition of a problem is a basic concept of Numerical Analysis. Condition numbers measure the well- or ill-condition of problems in Linear Algebra (matrices) or Approximation Theory (polynomials). Here we focus on the condition number γ_ω of polynomials P_n of degree $\leq n$ in power form on zero-symmetric intervals $[-\omega, \omega]$, where $\omega > 0$. In his influential papers [1], [2], [3] W. Gautschi has defined and reshaped γ_ω . Basically, he describes γ_ω as the product of two operator norms: an explicit factor times an implicit one (the l_∞ -norm of the coefficient vector of the m -th Chebyshev polynomial of the first kind relative to $[-\omega, \omega]$, where $m \in \{n, n - 1\}$), see [2, Theorem 3.1]. We provide a new sound proof for Gautschi's description of γ_ω , economize the second factor and express it by an explicit analytical formula. In doing so we rely, for the special case $\omega = 1$, on a result from [4, Theorem 5.2.1].

References

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