ℓ^1-Analysis Minimization and Generalized (Co-)Sparsity: When Does Recovery Succeed?

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Abstract

Our work addresses the problem of signal estimation from undersampled sub-Gaussian measurements under the assumption of a cosparse model. Based on generalized notions of sparsity, we derive novel recovery guarantees for the analysis basis pursuit, enabling accurate predictions of its sample complexity. The corresponding bounds on the number of required measurements do explicitly depend on the Gram matrix of the analysis operator and therefore particularly account for its mutual coherence structure.

Our findings defy conventional wisdom which promotes the sparsity of analysis coefficients as the crucial quantity to study. In fact, this common paradigm seems to break down in many situations of practical interest, for instance, when applying a redundant (multilevel) frame as analysis prior.

By a series of numerical experiments, we demonstrate that, in contrast, our theoretical sampling-rate bounds reliably capture the true recovery performance of various examples, such as redundant Haar wavelets systems, total variation, or random frames. Due to a novel localization argument, it turns out that the presented framework naturally extends to stable recovery, allowing us to incorporate compressible coefficient sequences as well.

This is joint work with M. Genzel and G. Kutyniok (TU Berlin), based on the manuscript: https://arxiv.org/abs/1710.04952