Construction and Approximation Properties of Shearlet Frames for Sobolev Spaces

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One of the most important areas of applied mathematics is the numerical solution of partial differential equations (PDEs). In recent years a new approach to tackle this task emerged, using tools from harmonic analysis. For example, wavelet frames for the Sobolev space $H^s(\Omega)$ were developed as an important tool for solving elliptic PDEs defined on a bounded domain $\Omega \subset \mathbb{R}^2$. Unfortunately, wavelets fail to provide good approximation rates when considering functions with curvilinear singularities. However, it was proven that shearlets exhibit optimal approximation rates for functions of this type. However, there are no straightforward constructions of pure shearlet systems on bounded domains. Therefore, in this talk, we present a frame for the Sobolev space $H^s(\Omega)$ which is mainly (but not fully) built out of shearlets. Since boundary conditions on the PDEs should be incorporated, we will also make use of wavelets adapted to the boundary of Ω. A discussion of approximation rates of the novel construction shall be underlined by numerical experiments which highlight the advantages of this new system, e.g. compared to pure wavelet systems. At the end of the talk we discuss some challenges which we need to resolve before employing these systems to the numerical solution of PDEs.

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