Attoclock Ultrafast Timing of Inversive Chord-Contact Dynamics

- A Spectroscopic FFT-Glimpse onto Harmonic Analysis of Gravitational Waves -

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According to Bohr's model of the hydrogen atom, it takes about 150 as for an electron in the ground state to orbit around the atomic core. Therefore, the attosecond (1 as = 10^{-18}s) is the typical time scale for Bloch wave packet dynamics operating at the interface of quantum field theory and relativity theory. Attoclock spectroscopy is mathematically dominated by the spectral dual pair of spin groups with the commuting action of the stabilized symplectic spinor quantization

$$\boxed{(\mathrm{Mp}(2, \mathbb{R}), \mathrm{PSO}(1, 3, \mathbb{R}))}$$

In the context of the $\mathrm{Mp}(2, \mathbb{R}) \times \mathrm{PSO}(1, 3, \mathbb{R})$ module structure of the complex Schwartz space $\mathcal{S}(\mathbb{R}^2 \oplus \mathbb{R}^2) \cong \mathcal{S}(\mathbb{C} \oplus \mathbb{C})$ of quantum field theory, the metaplectic Lie group $\mathrm{Mp}(2, \mathbb{R})$ is the two-sheeted covering group $\widetilde{\mathrm{Sp}}(2, \mathbb{R})$ of the symplectic group $\mathrm{Sp}(2, \mathbb{R}) \cong \mathrm{SL}(2, \mathbb{R})$, where the special linear group $\mathrm{SL}(2, \mathbb{R})$ is a simple Lie group, and the two-cycle rotation group $\widetilde{\mathrm{SO}}(2, \mathbb{R}) \cong \widetilde{\mathrm{U}}(1, \mathbb{C})$ is a maximal compact subgroup of $\mathrm{Mp}(2, \mathbb{R})$. The two-fold hyperbolically, respectively elliptically ruled conformal Lie group $\mathrm{SU}(1, 1, \mathbb{C})$ is conjugate to $\mathrm{SL}(2, \mathbb{R}) \cong \mathrm{Spin}(1, 2, \mathbb{R})$ in the complexification $\mathrm{SL}(2, \mathbb{C}) \cong \mathrm{Spin}(1, 3, \mathbb{R})$; the projective Lorentz-Möbius group $\mathrm{PSO}(1, 3, \mathbb{R}) \cong \mathrm{PSL}(2, \mathbb{C}) \cong \mathrm{SL}(2, \mathbb{C})/(\mathbb{Z}_2.\mathrm{id})$ represents a semi-simple Lie group which is isomorphic to the four-dimensional proper orthochroneous Lorentz group $\mathcal{L}_4^{\uparrow +}$ of relativity theory. The relativistic fiber bundle of local space-time frames turns out to be a principal $\mathrm{PSO}(1, 3, \mathbb{R})$ vector space bundle.

The two-sided discrete spectrum of the metaplectic Lie group $\mathrm{Mp}(2, \mathbb{R})$ generates a 2 divisible commutative chord-contact group law of FFT half-size bit-reversed butterfly composition type. The isometry group $\mathrm{PSO}(1, 3, \mathbb{R})$ of the upper sheet $\mathcal{H}^3$ of the two-sheeted unit hyperboloid in the ambient Minkowski space-time manifold $\mathbb{R}^{(1,3)}$ defines the open upper light cone $\mathcal{C}$. The one-sheeted hyperboloid model of isometry group $\mathrm{PSO}(1, 3, \mathbb{R})$, minimal embedded into the flat Clifford torus of the very round sphere $\mathbb{S}_3 \cong \mathrm{Spin}(3, \mathbb{R}) \cong \mathrm{SU}(2, \mathbb{C})$ in the Graßmannian manifold $\mathrm{Gr}(2, \mathbb{R}^2 \oplus \mathbb{R}^2) \cong \mathrm{Gr}(1, \mathbb{C} \oplus \mathbb{C})$, plays a central role in cosmology as the three-dimensional ruled hyperbolic de Sitter space-time variety of constant negative curvature. It was the first

Figure 1: Non-invasive clinical radiology: Zooming onto the opening of the cylindrical channel of genus 1 of a high-definition 3 Tesla gradient controlled magnetic resonance tomography scanner. The Reeb flow of the $\mathbb{S}_1 \cong \mathrm{SO}(2,\mathbb{R}) \cong \mathrm{Spin}(2,\mathbb{R})$-invariant central enveloping channel is conceived as a realization of symplectic spinor geometry which is associated with the spectral dual pair $(\mathrm{Mp}(2,\mathbb{R}), \mathrm{PSO}(1,3,\mathbb{R}))$.
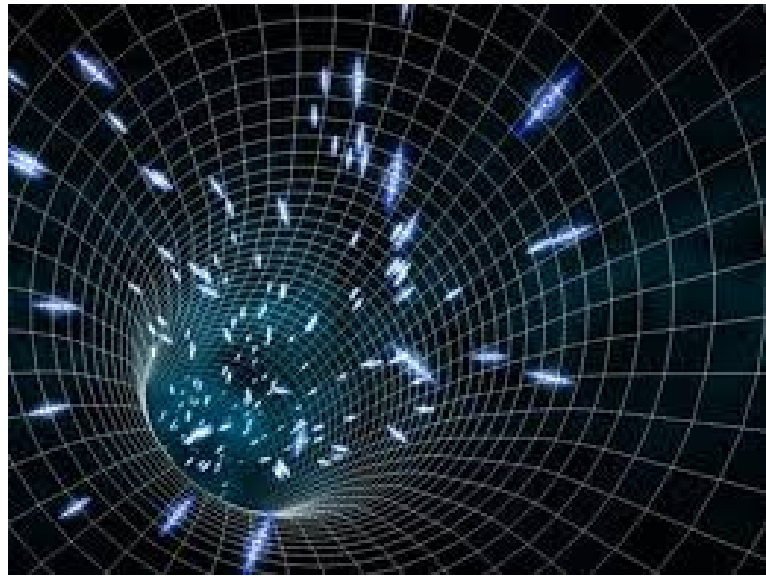

Figure 2: The expanding de Sitter cosmological model: Illustration of the one-sheeted unit hyperboloid model of isometry group $\mathrm{PSO}(1,3,\mathbb{R})$, minimal embedded into the flat Clifford torus of the open-book decomposition of the very round sphere $\mathbb{S}_3 \cong \mathrm{Spin}(3,\mathbb{R}) \cong \mathrm{SU}(2,\mathbb{C}) \hookrightarrow \mathbb{C} \oplus \mathbb{C}$ with $\mathbb{S}_1 \cong \mathrm{SO}(2,\mathbb{R}) \cong \mathrm{Spin}(2,\mathbb{R})$-binding: The symplectic spinor quantization approach to the hyperbolic lattice stabilization of the three fundamental Kepplerian laws of planetary trajectories reveals the hyperbolic de Sitter space-time variety of cosmology. It represented the first expanding world model ever proposed in cosmology, the very one predicting the cosmological redshift of polarized light rays.

expanding world model ever proposed, the very one predicting the cosmological redshift of light rays, and the very one jointly with Minkowski and anti-de Sitter space-time variety to embody the maximal degree of symmetry with respect to both space and time coordinatization.
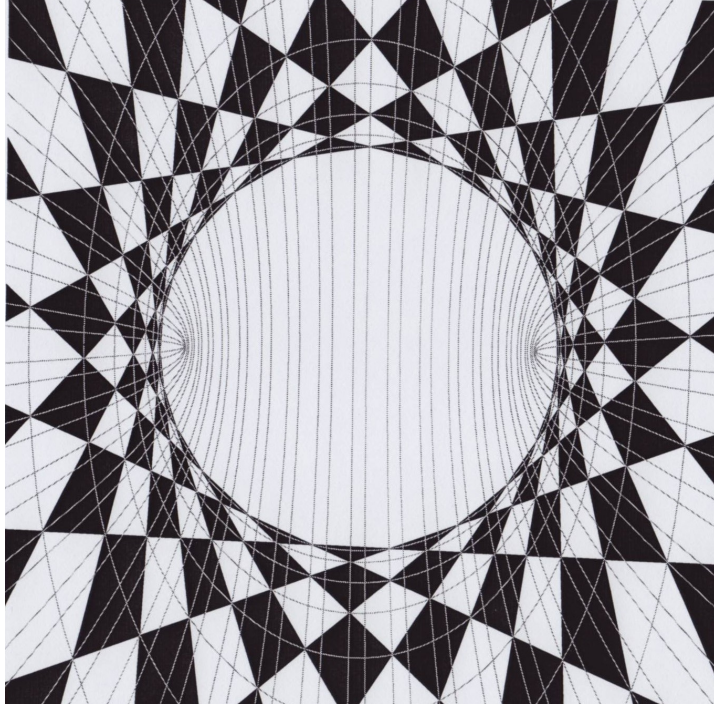
Figure 3: Galois representation of inversive chord-contact dynamics in $\mathbb{P}_\mathbb{C}(\mathrm{Lie}(\mathcal{N})^*)$: The stable Poncelet hyperbolic lattice $\Lambda \hookrightarrow \mathcal{H}^2 \hookrightarrow \mathbb{P}_3(\mathbb{R})$ at the interface of symplectic spinor quantization and relativity theory projected on a page of the open-book foliation of the very round sphere $\mathbb{S}_3 \cong \mathrm{Spin}(3,\mathbb{R}) \hookrightarrow \mathbb{P}_3(\mathbb{R})$. The discrete quadrilateral tessellation defined by a pencil history of two-fold ruled hyperbolic, respectively elliptic paraboloids with central pole on the spinor axis of its joint tangent plane at infinity in the complexified projective dual $\mathbb{P}_\mathbb{C}(\mathrm{Lie}(\mathcal{N})^*)$, forms a $\mathrm{Mp}(2,\mathbb{R})$-stabilized laser beam pavement of the de Sitter world under its Lorentzian structure with the isometry group $\mathrm{PSO}(1,2,\mathbb{R}) \hookrightarrow \mathrm{PSO}(1,3,\mathbb{R})$ of the two-fold absolute spherical circle. The points of order 2, to wit the accessible double nodes of the discrete plane period lattice Λ with $\Lambda = i\,\Lambda = \bar{\Lambda}$ of Poncelet porism or closure theorem, represent the non-cyclic group of order 4, which is the commutative Klein four-group or dihedral group $\mathcal{V}_4 \cong \mathcal{D}_2 \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$. The iterates act on the Dehn twisting chord-contact involution in $\mathcal{H}^2 \hookrightarrow \mathbb{P}_3(\mathbb{R})$; the iterative ascension reveals the structure of the rank-two planar lattice $\frac{1}{N}\Lambda$ which is formed by the Abelian group $\mathbb{Z}_N \oplus \mathbb{Z}_N$ of order $N^2 \geq 4$. The tangent lines perform asymptotic double lines of the involution of conjugate diameters in the two-dimensional ruled Jacobian variety $\mathbb{C}/\Lambda$. The $\mathrm{Mp}(2,\mathbb{R}) \times \mathrm{PSO}(1,3,\mathbb{R})$ module structure of $\mathcal{S}(\mathbb{R}^2 \oplus \mathbb{R}^2)$ mirrors the fact that bipolar stereographic projections within the coadjoint $\mathcal{N}$-orbit model of the unitary dual $\widehat{\mathcal{N}}$ of the three-dimensional real Heisenberg unipotent Lie group $\mathcal{N}$ mutually conjugate inversions on the compact Euclidean sphere $\mathbb{S}_2 \cong \mathrm{SO}(3,\mathbb{R})/\mathrm{SO}(2,\mathbb{R}) \hookrightarrow \mathbb{S}_3 \cong \mathrm{Spin}(3,\mathbb{R}) \cong \mathrm{SU}(2,\mathbb{C})$ into inversions and reflections on horizontal coadjoint $\mathcal{N}$-orbit contact planes of the inversive chord-contact dynamics of $\mathrm{Gr}(2,\mathbb{R}^2 \oplus \mathbb{R}^2) \hookrightarrow \widehat{\mathcal{N}}$: An electron of spin $\frac{1}{2}$ accelerated to 25 000 volts acquires a velocity of 10 % of the speed of light *in vacuo*. The stellar phenomenon of orbital shrinkage at the rate of 1 cm per day at a distance of 21 klyr (1 klyr $= 10^3$ light-years) of the classic relativistic radio pulsar PSR B1913+16, or the astroporism defect of the outstanding binary radio pulsar PSR J0737-3039, or even the massive binary black-hole system OJ 287, due to the gravitational radiation of the lighthouse effect at the expense of spinor energy, prevents the trajectories of the double neutron star or quasar binary to be closed orbits. The omni-directional detection of low-frequency gravitational waves, due to boosts or loxodromic elements of the Lorentz-Möbius group $\mathrm{PSO}(1,3,\mathbb{R})$, requires the mass data of superdense neutron stars and massive quasars.
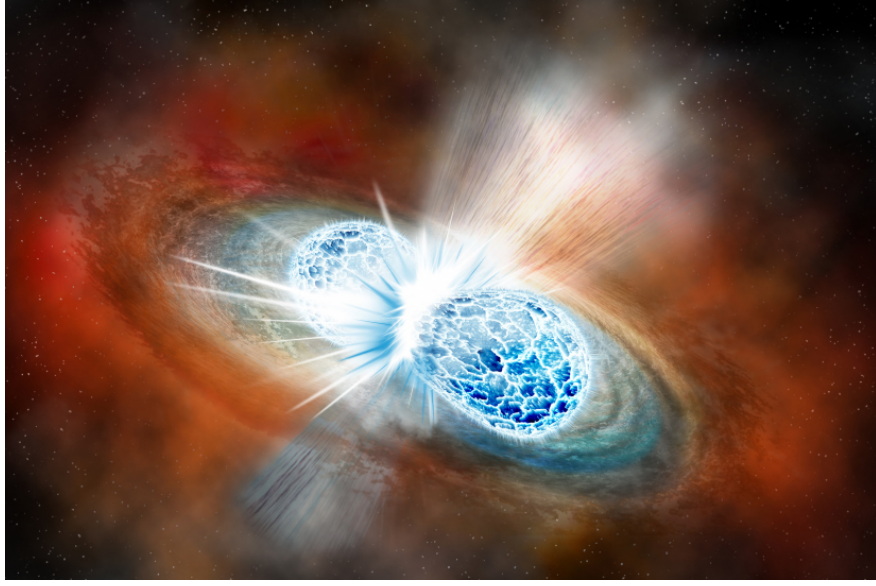
Figure 4: Local frame artwork of the Carnegie Institution for Sciences of the collision of a pair of spinning neutron stars in the elliptic home galaxy NGC 4993 at a distance of more than 130 mlyr (1 mlyr $= 10^6$ light-years): In terms of the spectral dual pair $(\mathrm{Mp}(2,\mathbb{R}), \mathrm{PSO}(1,3,\mathbb{R}))$, the weighted spectra of the observed inspiral of the gravitational waves, described by the principal $\mathrm{PSO}(1,3,\mathbb{R})$ fiber bundle of relativistic space-time frames, are determined by the elements of the Lorentz-Möbius model group without any timelike eigenvector and with two lightlike eigenvectors that are conjugate to an element of the direct product group $\mathrm{PSO}(1,1,\mathbb{R}) \times \mathrm{Spin}(2,\mathbb{R})$. The boosts or loxodromic elements of $\mathrm{PSO}(1,3,\mathbb{R})$ are responsible for the emission of γ-radiation. The spectroscopic detection of the emitted bursts of γ-rays needs the highest density mass data of rapidly spinning, strongly magnetized neutron stars in the universe.

On the other hand, the precision of ultrahigh frequency measurements in inversive chord-contact dynamics depends on the laser light ray control by a stable atomic lattice. The existence proof of the $\mathcal{V}_4$-stabilized Poncelet hyperbolic lattice Λ of laser light rays in the projection $\partial\mathcal{H}^2$ of the boundary $\partial\mathcal{H}^3 \cong \mathcal{C}/\mathbb{R}_+$ of the ambient hyperbolic plane $\mathcal{H}^2 \hookrightarrow \mathbb{P}_2(\mathbb{R}) \hookrightarrow \mathbb{P}_3(\mathbb{R})$ follows from Abel's theorem for conformally represented parabolic Riemann surfaces of genus 1 and their Jacobian varieties modulo Λ. Their universal covering is the parabolic Riemann surface $\mathbb{C} \cong \mathbb{R} \oplus \mathbb{R}$. In special relativity, an element of the three-dimensional hyperbolic manifold $\mathcal{H}^3 \hookrightarrow \mathbb{P}_3(\mathbb{R})$ represents the velocity of a relativistic particle of unit mass. Boosts or loxodromic elements of the ambient hyperbolic manifold $\mathcal{H}^3$ can be conceived as an acceleration of cosmic radiation particles with respect to a fixed reference frame; they admit no fixed point on $\mathcal{H}^3$, and two fixed points on the boundary $\partial\mathcal{H}^3$. In the modality of magnetic resonance tomography, the Hopf fibration induced Reeb flow through the scanner's cylinder channel of genus 1 with orbits tangent to the plane at infinity of the three-dimensional real projective space $\mathbb{P}_3(\mathbb{R})$ is implemented by cylindrical wired coil configurations of the Dehn twisted Poncelet hyperbolic lattice Λ of the flat Clifford torus of the Eisenstein-Weierstraß elliptic function pair $(\wp_\Lambda, \wp'_\Lambda)$, and its associated Eisenstein meromorphic based $(\cot = \frac{1}{\tan}, \csc = \frac{1}{\sin})$ calculus. Then the two-sheeted iterates of the complex gradient controlled action of the non-cyclic Klein four-group $\mathcal{V}_4 \cong \mathcal{D}_2$ form a subgroup of the cyclic Fourier-Wiener torsion group $\mathbb{Z}_4$ of the inversive chord-contact dynamics which is inherent to the ruled hyperbolic de Sitter space-time variety.