Real Sparse Fast DCT for Vectors with Short Support

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There are many well-known fast algorithms for the discrete Fourier transform (DFT) of sparse input functions or vectors, both deterministic and randomized ones. Under the assumption that the N-length input vector has a short support of length m, the fastest of these algorithms achieve runtimes of $\mathcal{O}(m \log N)$.

For the closely related discrete cosine transform of type II (DCT-II), given by

$$\mathbf{x}^{\widehat{\text{II}}} := \sqrt{\frac{2}{N}} \left(\varepsilon_N(j) \cos\left(\frac{j(2k+1)\pi}{2N} \right) \right)_{j,k=0}^{N-1} \mathbf{x}, \quad \mathbf{x} \in \mathbb{R}^N,$$

where $\varepsilon_N(0) = 1/\sqrt{N}$ and $\varepsilon_N(j) = 1$ for $j \neq 0$, there also exists a fast algorithm with a runtime of $\mathcal{O}(m \log m \log N)$ if $\mathbf{x}$ has a short support of length m (see Bittens and Plonka, Sparse Fast DCT for Vectors with One-block Support, http: //arxiv.org/abs/1803.05207, 2018). However, this algorithm employs complex arithmetic utilizing the close relation between the DCT-II and the DFT, even though the DCT itself is a completely real transform.

In this talk we present a new fast and deterministic IDCT-II algorithm that reconstructs the input vector $\mathbf{x} \in \mathbb{R}^N$, $N = 2^J$, with short support of length m from $\mathbf{x}^{\widehat{\text{II}}}$ using only real arithmetic if an upper bound M on m is known. The resulting algorithm has a runtime of $\mathcal{O}\left(M \log M + m \log_2 \frac{N}{M}\right)$, requires $\mathcal{O}\left(M + m \log_2 \frac{N}{M}\right)$, and does not employ inverse FFT algorithms to recover $\mathbf{x}$.

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