Contact Holonomy of Foucault's Spherical Pendulum: A Spinor Gauge Approach

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To the memory of the great mathematician Ernst Witt (1911 - 1991)

Abstract. It needed more than 300 years to visualize the Copernican idea of the rotational motion per sidereal day of the spinning earth. The famous spherical pendulum holonomy experiment performed by Jean Bernard Léon Foucault (1819-1867) took place in 1851 beneath the dome of the Panthéon, Paris, where he suspended a 28-kg brass-coated lead bob with a 67-m-long wire. For this choice of spinor gauge of gravitation, at $\varphi_0 = +41°$ N Latitude, the phase angle displacement $\Delta\theta$ of the symplectic swing-plane, which is caused by the parallel transport or parallel translation of the initial swing-plane along the latitude of the globe $\mathbb{S}_2 \cong \mathrm{SO}(3, \mathbb{R})/\mathrm{SO}(2, \mathbb{R})$, is about $11°$ per hour, or about $272°$ per sidereal day.

The purpose of the present paper is to elucidate why one might expect the Foucault spherical pendulum's swing-plane to follow a parallel vector field and to Lie group theoretically establish an associate Hopf-Rinow type theorem on the existence of a minimizing geodesic segment joining dynamically any given pair of points on a connected complete smooth surface.

In the context of Foucault's spherical pendulum holonomy experiment, parallel vector fields on the principal fiber bundle $\mathrm{SO}(3, \mathbb{R}) \longrightarrow \mathrm{SO}(3, \mathbb{R})/\mathrm{SO}(2, \mathbb{R})$ are annihilated by the covariant derivative operator ∇. The radially scaled disc model, which is endowed with the Poincaré metric of the hyperbolic plane structure, exact gnomonically inherited at the origin from the two-sheeted conformal anti-de-Sitter space-time, traces its geodesic open diameters from the open-book foliation by means of its symplectic swing-plane pages. In terms of contact topology of the three-dimensional spinor group $\mathrm{Spin}(3, \mathbb{R}) \cong \mathrm{SO}(4, \mathbb{R})/\mathrm{SO}(3, \mathbb{R}) \cong \mathrm{SU}(2, \mathbb{C})$ or very round sphere $\mathbb{S}_3 \cong \mathrm{SL}(1, \mathbb{H})$ of the Hopf principal circle bundle $\mathbb{S}_1 \hookrightarrow \mathbb{S}_3 \xrightarrow{\eta} \mathbb{S}_2$, the binding is represented by the special orthogonal group $\widehat{\mathrm{W}}(\mathbb{R}) \cong \mathrm{SO}(2, \mathbb{R}) \cong \mathrm{U}(1, \mathbb{C}) \cong \mathbb{S}_1$, dual to the wideband Witt group $\mathrm{W}(\mathbb{R}) \cong \mathbb{Z} = \mathbb{N} \cup -\mathbb{N}$, of tracking the diurnal motion of the spinning earth; the group $\widehat{\mathrm{W}}(\mathbb{R})$ acts by turning over the symplectic leaves. The spinor gauge group modulo squares $\det : \mathrm{W}(\mathbb{R}) \longrightarrow \mathbb{R}^\times/\mathbb{R}^{\times 2} \cong \mathrm{SO}(4, \mathbb{R})/\mathrm{PSO}(1, 3, \mathbb{R})$ admits as its law of composition the multiplication of squares or the orthogonal sum $\perp$, respectively. Integral reduction of equivalence classes of quadratic vector spaces by the free Abelian group $\mathrm{W}(\mathbb{R})$ projects stepwise the two-sheeted conformal anti-de-Sitter world and affords the Sagnac velocity-cone spinorial calibration of hyperbolic parallel transport of laser ring gyroscopes inside Clifford contact tori.