Generalizations and Applications of Prony's method

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In many applications the reconstruction of signals which are or can be approximated by a sum of sparse expansion of certain atoms is an often encountered problem. The most well-studied signal model is the is the sparse expansion into complex exponentials, i.e.,

$$f(x) \coloneqq \sum_{j=1}^{m} c_j e^{T_j x} = \sum_{j=1}^{M} c_j z_j^x$$

with $c_j \in \mathbb{C} \setminus \{0\}$,$T_j \in \mathbb{C}$ and $z_j = e^{T_j}$. The signals can be reconstructed using Prony's method and $2M$ equidistant samples $f(\ell)$ for $\ell = 0, \ldots, 2M-1$. In this recent years Peter & Plonka derived a generalized Prony method for the reconstruction of sparse expansions of eigenfunctions of suitable linear operators using only a small number of suitable sample values. Although some examples were given by Peter & Plonka the problem of finding such operators is non-trivial.
In this talk we look at different possible linear operators and their corresponding eigenvalues such as generalizations of shift operators and differential operators and explain the connection between the different models. In particular, we present a way to use these operators in order to reconstruct signal models such as arbitrary linear combinations of Gaussians, non-stationary trigonometric expansions with a special monotone phase function as well as orthogonal polynomials via the generalized Prony method.

This is joint work with Gerlind Plonka (University of Goettingen) & Kilian Stampfer (University of Goettingen).

References

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