Reconstruction of Signals from Truncated Phaseless Measurements

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Abstract

In many applications such as diffraction imaging one aims to reconstruct a compactly supported signal $f \in L^2(\mathbb{R})$ from the absolute value of its Fourier transform $|\hat{f}|$. The data $|\hat{f}(\xi)|$, $\xi \in \mathbb{R}$, represent intensities measured by a CCD sensor. The sensor is obviously unable to cover the entire frequency domain and thus creates a truncation, i.e. the measurement is compactly supported. In this work we examine how a truncation of the Fourier measurements affect the phase reconstruction of f. Mathematically, this problem can be formulated as follows.

Let $f \in L^2(\mathbb{R})$ be compactly supported and consider the decomposition $f(t) = e^{i\theta(t)}|f(t)|$, where

$$\theta : \{t \in \mathbb{R} \mid f(t) \neq 0\} \to \mathbb{R}/2\pi\mathbb{Z}$$

is the phase function of f. For a constant $W > 0$, we define $f_W = (\chi_{[-W,W]}\hat{f})^{\vee}$, i.e. f_W arises from f by bandlimiting. The multiplication by the characteristic function $\chi_{[-W,W]}$ represents a truncation in Fourier space. Again, we write $f_W(t) = e^{i\theta_W(t)}|f_W(t)|$ where θ_W is the corresponding phase function of f_W. The task is to find assumptions on f which yield a bound on $|\theta - \theta_W|$. We give explicit estimates on this distance and state the corresponding assumptions on f. The theory of prolate spheroidal wave functions allows us to construct a class of functions where such an estimate cannot hold. Moreover, we characterize the class of operators which let the phase unchanged when applied to the signal f.