From ESPRIT to ESPIRA: Estimation of Signal Parameters by Iterative Rational Approximation

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We consider exponential sums of the form

$$f(t) = \sum_{j=1}^{M} \gamma_j \, \mathrm{e}^{\phi_j t} = \sum_{j=1}^{M} \gamma_j \, z_j^t,$$

where $M \in \mathbb{N}$, $\gamma_j \in \mathbb{C} \setminus \{0\}$, and $z_j = \mathrm{e}^{\phi_j} \in \mathbb{C} \setminus \{0\}$ with $\phi_j \in \mathbb{C}$ are pairwise distinct. The recovery of such exponential sums from a finite set of possibly corrupted signal samples plays an important role in many signal processing applications, see e.g. in phase retrieval, signal approximation, sparse deconvolution in nondestructive testing, model reduction in system theory, direction of arrival estimation, exponential data fitting, or reconstruction of signals with finite rate of innovation.

Often, the exponential sums occur as Fourier transforms or higher order moments of discrete measures (or streams of Diracs) of the form $\sum_{j=1}^{M} \gamma_j \, \delta(\cdot - T_j)$ with $T_j \in \mathbb{R}$, which leads to the special case that $\phi_j = \mathrm{i} T_j$ is purely complex, i.e., $|z_j| = 1$.

We introduce a new method for **E**stimation of **S**ignal **P**arameters based on **I**terative **R**ational **A**pproximation (ESPIRA) for sparse exponential sums. Our algorithm uses the AAA algorithm for rational approximation of the discrete Fourier transform of the given equidistant signal values. We show that ESPIRA can be interpreted as a matrix pencil method applied to Loewner matrices. These Loewner matrices are closely connected with the Hankel matrices which are usually employed for recovery of sparse exponential sums. Due to the construction of the Loewner matrices via an adaptive selection of index sets, the matrix pencil method is stabilized. ESPIRA achieves similar recovery results for exact data as ESPRIT and the matrix pencil method (MPM) but with less computational effort. Moreover, ESPIRA strongly outperforms ESPRIT and MPM for noisy data and for signal approximation by short exponential sums.

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