32. Rhein-Ruhr-Workshop

Bestwig, 10.-11. Februar 2023

-Programm, Teilnehmer und Abstracts-

Organisation:

Prof. Dr. G. Plonka-Hoch	Universität Göttingen
Prof. Dr. T. Sauer	Universität Passau
Prof. Dr. M. Skrzipek	FernUniversität in Hagen
Dr. M. Weimar	Universität Würzburg

TEILNEHMERLISTE RRW 2023

Brockmann, Max	Universität Köln	mbrockm2@smail.uni-koeln.de
Cottin, Claudia	Fachhochschule Bielefeld	claudia.cottin@fh-bielefeld.de
Cottin, Ludwig		ludwig.cottin@t-online.de
Diethelm, Kai	TH Würzburg-Schweinfurt	kai.diethelm@fhws.de
Diederichs, Benedikt	Helmholz Zentrum München	benedikt.diederichs@helmholtz-muenchen.de
Dröge, Chong-Son	Universität Köln	cdroege@smail.uni-koeln.de
Filbir, Frank	Helmholtz-Zentrum München	frank_filbir@web.de
Hashemishahraki, Safoura	TH Würzburg-Schweinfurt	safoura.hashemishahraki@fhws.de
Heilmann, Margareta	Universität Wuppertal	heilmann@uni-wuppertal.de
Heinrich, Florian	Universität Passau	heinrich@forwiss.uni-passau.de
Hielscher, Ralf	Technische Universität Freiberg	Ralf.Hielscher@math.tu-freiberg.de
Hockmann, Mathias	Universität Osnabrück	mahockmann@uos.de
Hovemann, Marc	Universität Marburg	M.Hovemann@gmx.de
Hunhold, Laslo	Universität Köln	hunhold@math.uni-koeln.de
Knoll, Sarah	Universität Köln	knoll@math.uni-koeln.de
Kocurov, Benjamin	Universität Göttingen	b.kocurov@math.uni-goettingen.de
Kopsch, Anne	Universität Marburg	kopscha@mathematik.uni-marburg.de
Kunoth, Angela	Universität Köln	kunoth@math.uni-koeln.de
Lang, Thomas	Universität Passau (FORWISS)	langthom@forwiss.uni-passau.de
Lot, Federico	Universität Marburg	otf@mathematik.uni-marburg.de
Lüttgen, Kai	Technische Universität Chemnitz	kai.luettgen@math.tu-chemnitz.de
Melnyk, Oleh	Helmholtz-Zentrum München	oleh.melnyk@tum.de
Moeller, Moritz	Technische Universität Chemnitz	moritz.moeller@mathematik.tu-chemnitz.de
Plonka-Hoch, Gerlind	Universität Göttingen	plonka@math.uni-goettingen.de
Pöschl, Tim	Technische Universität Freiberg	Tim.Poeschl@math.tu-freiberg.de
Razavi, Raha	Universität Göttingen	r.razavi@math.uni-goettingen.de
Reich, Niklas	Hochschule Ruhrwest	Niklas.Reich@hs-ruhrwest.de
Riebe, Yannick	Universität Göttingen	y.riebe@math.uni-goettingen.de
Rieger, Christian	Universität Marburg	riegerc@mathematik.uni-marburg.de
Sauer, Tomas	Universität Passau	tomas.sauer@uni-passau.de
Schmitz, Lukas	Universität Köln	
Schröter, Pascal	Technische Universität Chemnitz	pascal.schroeter@mathematik.tu-chemnitz.de
Skrzipek, Michael	Fernuniversität in Hagen	michael.skrzipek@fernuni-hagen.de
Stock, Michael	Universität Passau (FORWISS)	stock@forwiss.uni-passau.de
Stiller, David	Universität Köln	dav.stiller@gmail.com
Tüting, Katja	Technische Universität Braunschweig	katja.tueting@tu-braunschweig.de
Vogel, Dorian	Universität Marburg	vogeldor@mathematik.uni-marburg.de
Weller, Anna	Universität Köln	weller@math.uni-koeln.de
Weimar, Markus	Universität Würzburg	markus.weimar@uni-wuerzburg.de
Wünsche, Ehrich	Technische Universität Freiberg	Erik.Wuensche@math.tu-freiberg.de

32. Rhein-Ruhr-Workshop
Bestwig, 10.–11. Februar 2023
PROGRAMM

Freitag, 10. Februar 2023, Vormittag

10.20	*Begrüßung / Organisatorisches*
	Sektionsleitung: T. Sauer
10.30	**M. Moeller** (Technische Universität Chemnitz) Gelfandzahlen und beste trigonometrische m-term Approximation für Wiener Räume mit gemischten Gewichten in L_2
11.00	**A. Weller** (Universität Köln) A Spectral Galerkin Method for Partial Differential Equations on Metric Graphs
11.30	**T. Lang** (Universität Passau) Achieving Quantum Supremacy in Image Processing: Is it Possible?
12.00	*Gemeinsames Mittagessen*

Freitag, 10. Februar 2023, Nachmittag

	Sektionsleitung: F. Filbir
14.00	**M. Hovemann** (Universität Marburg) Adaptive near-best Quarklet Tree Approximation
14.30	**A. Kopsch** (Universität Marburg) Konstruktion von Multiwavelets für allgemeine Skalierungsmatrizen
15.00	**M. Stock** (Universität Passau) LiveTV: Real-time Total Variation Regularization Using Haar Wavelets
15.30	*Pause mit Kaffee, Tee, Kuchen*
	Sektionsleitung: K. Diethelm
16.00	**B. Kocurov** (Universität Göttingen) Mathematical Methods in Parallel MRI
16.30	**L. Hunhold** (Universität Köln) A New Approach to Envelope Estimation for EMD
17.00	**K. Tüting** (Technische Universität Braunschweig) Modeling the Measurement Problem of the Sampling Oscilloscope
17.30	**E. Wünsche** (Technische Universität Freiberg) A Toolbox for Fast Harmonic Approximation on the Rotation Group
18.00	*Gemeinsames Abendessen*

Freitag, 10. Februar 2023, ab 19 Uhr: **Präsentation der Poster**

Sektionsleitung: M. Skrzipek
M. Brockmann (Universität Köln) Finite Element Method for the Solution of Elliptic Partial Differential Equations on Graphs
B. Diederichs (Helmholz Zentrum München) Advanced Interaction Models in Ptychography
C.-S. Dröge, A. Weller (Universität Köln) Numerical Computation of Equilateral Quantum Graph Spectra
K. Diethelm (Technische Hochschule Würzburg-Schweinfurt) Diffusive Darstellungen für fraktionale Integraloperatoren und ihre Anwendungen
M. Hockmann (Universität Osnabrück) The Rayleigh criterion and well-conditionedness of super resolution
S. Knoll (Universität Köln) BPX Preconditioners for B-Spline Discretizations of Operators Arising in Elliptic Variational Inequalities
K. Lüttgen (Technische Universität Chemnitz) The Transformation $f(x) \to (\cos(x_1), \ldots, \cos(x_n))$ on Function Spaces of Dominating Mixed Smoothness
O. Melnyk (Helmholtz-Zentrum München) Stochastic Amplitude Flow for Phase Retrieval and Ptychography
R. Razavi (Universität Göttingen) Combining Non-Data-Adaptive Transforms for OCT Image Denoising by Iterative Basis Pursuit
N. Reich (Hochschule Ruhrwest) Variational Methods For Parameterized Quadratic Bilinear Differential-Algebraic Equations
L. Schmitz, D. Stiller (Universität Köln) Numerical computation of quantum graph spectra
D. Vogel (Universität Marburg) Adaptive Quarklet Tree Approximation: Numerical Experiments
M. Weimar (Universität Würzburg) Rate-Optimal Sparse Approximation of Compact Break-of-Scale Embeddings

Samstag, 11. Februar 2023

8.00	*Frühstück*
	Sektionsleitung: A. Kunoth
9.00	**T. Pöschl** (Technische Universität Freiberg) Moving Least Squares Approximation on Spheres
9.30	**P. Schröter** (Technische Universität Chemnitz) Hochdimensionale Approximation mit teilweise periodischen Randbedingungen
10.00	**C. Rieger** (Universität Marburg) Kernel Methods for High Dimensional Problems
10.30	*Pause mit Kaffee, Tee*
	Sektionsleitung: G. Plonka-Hoch
10.45	**F. Lot** (Universität Marburg) Numerical Aspects of Multiscale Approximation
11.15	**Y. Riebe** (Universität Göttingen) Spline Representation of One-Dimensional ReLU Neural Networks
11.45	*Gemeinsames Mittagessen*

Dauer der Vorträge: 30 Minuten, einschließlich Diskussionszeit.

Finite Element Method for the Solution of Elliptic Partial Differential Equations on Graphs

Max Brockmann
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
brockmann@math.uni-koeln.de

Abstract

The question of investigating partial differential equations (PDEs) on graphs arises in the context of an interdisciplinary research project of the prediction of protein propagation in the brain network of Alzheimer's Disease patients [WB]. In this context, graphs allow modelling the interconnected structure of the brain network, while PDEs, particularly diffusion equations, describe the protein propagation. As a first attempt to approximate such PDEs on graphs, we will in this work focus on second order elliptic PDEs and study numerical methods for the solution of systems arising from a finite element discretization.

In order to formulate PDEs on graphs, we explain the network structure with the help of metric graphs. Metric graphs use an edgewise parameterization of the graph such that differential operators can be defined on the graph. Additionally for the well-posedness of the PDE on a graph, we require Neumann-Kirchhoff conditions, a kind of flow conservation property, on all vertices.

We discretize the metric graph with a finite element method, as described by Mario Arioli and Michele Benzi [AB]. The discretization of the metric graph can be interpreted as an extended graph with additional vertices, so called internal vertices. We then choose a hat function basis on the extended graph. This motivates the characterisation of the system that arises from the weak formulation of the PDE as

$$\begin{pmatrix} \mathbf{H}_{\mathcal{E}\mathcal{E}} & \mathbf{H}_{\mathcal{E}\mathcal{V}} \\ \mathbf{H}_{\mathcal{V}\mathcal{E}} & \mathbf{H}_{\mathcal{V}\mathcal{V}} \end{pmatrix} \mathbf{u} = \mathbf{f},$$

where $\mathbf{u}$ is the coefficient vector of the solution of the PDE written in the hat function basis. Each submatrix corresponds to different adjacency of hat functions on the extended graph. Thus their size increase with more internal vertices on each of the edges. This is especially important for the matrix $\mathbf{H}_{\mathcal{E}\mathcal{E}}$, because it is a block-diagonal matrix, with each blocks size proportional to the number of internal vertices. Consequently a fine discretization leads to a large system of equations and high computational cost.

In comparison to a domain decomposition approach with suitable PCG-solver proposed in [AB], we use a multigrid method to reduce the size of the matrix, which needs to be solved directly, to a minimum level. The idea of multigrid methods is to iteratively solve a given system of equations with a *coarse grid correction*. We make use of a hierarchical discretization of internal vertices and use suitable intergrid operators in order to transform matrices and vectors from one level to another. We show first numerical results of the convergence rate of the multigrid method on test problems.

References

[AB] M. Arioli, M. Benzi, *A finite element method for quantum graphs*, IMA Journal of Numerical Analysis, Volume 38, Issue 3, Pages 1119-1163, 2017, doi:10.1093/imanum/drx029.

[BP] M. Brockmann, L.Plötzke, *Finite-Elemente-Methode für Quantum-Graphen - Computational Aspects*, Ausarbeitung zum Seminar zur Numerik partieller Differentialgleichungen, Department Mathematik/Informatik, Abteilung Mathematik, Universität zu Köln, Sommersemester 2022.

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Advanced Interaction Models in Ptychography

Benedikt Diederichs

Ptychography became one of the leading techniques for atomically resolved scanning transmission electron microscopy over the last decade. It can be rephrased as a phase retrieval problem for the Short Time Fourier Transform. And while this inverse problem has a compelling mathematical solution, real world data comes with a whole package of problems, including dynamical scattering and partial coherence. Here we elaborate how these can be overcome.

Diffusive Darstellungen für fraktionale Integraloperatoren und ihre Anwendungen

Kai Diethelm

Fakultät Angewandte Natur- und Geisteswissenschaften
Technische Hochschule Würzburg-Schweinfurt

Differential- und Integraloperatoren fraktionaler Ordnung haben sich in den letzten Jahren als leistungsfähiges Werkzeug für die mathematische Modellierung gedächtnisbehafteter Prozesse etabliert. Gerade die Tatsache, dass Gedächtniseffekte modelliert werden können, stellt die Numerik jedoch vor besondere Herausforderungen, denn diese Eigenschaft führt dazu, dass traditionelle Algorithmen im Hinblick auf Laufzeit ebenso wie auf Speicherbedarf eine deutlich höhere Komplexität aufweisen als entsprechende Algorithmen für nicht-fraktionale (gedächtnisfreie) Modelle. Mit den sog. *diffusiven Darstellungen* besteht eine Möglichkeit, die fraktionalen Operatoren in einer Form auszudrücken, die das Gedächtnis nicht mehr explizit beinhaltet, sondern implizit. Dies führt dazu, dass Algorithmen konstruiert werden können, die sowohl bezüglich der Rechenzeit als auch bezüglich des Speicherbedarfs signifikant günstiger sind als konventionelle Ansätze.

Der wesentliche Aspekt einer diffusiven Darstellung ist, dass sich z. B. ein fraktionales Integral einer Funktion f am Datenpunkt t_j darstellen lässt als Integral $\int_J \phi(w, t_j)\,\mathrm{d}w$ über eine Hilfsfunktion ϕ, die sich als Lösung einer von f abhängigen gewöhnlichen Differentialgleichung *erster* Ordnung ergibt. Der Integrationsbereich J ist hierbei üblicherweise $(0, \infty)$ oder $(-\infty, \infty)$, also unbeschränkt. Für die numerische Auswertung der diffusiven Darstellung stellen sich somit die Fragen nach effizienten Methoden (a) zur Lösung der gegebenen Differentialgleichung erster Ordnung, und (b) zur numerischen Bestimmung des genannten Integrals. Zahlreiche in der Literatur vorgeschlagene Ansätze verwenden für den Punkt (b) weitgehend unüberlegt gewählte Quadraturformeln, die die grundlegenden analytischen Eigenschaften des Integranden nicht berücksichtigen und somit relativ langsam konvergieren. Wir stellen eine Konstruktionsmethode [1, 2] vor, die diese Schwäche vermeidet und somit zu sehr effizienten Algorithmen führt. Eine wichtige Frage dabei ist, wie sich die Wahl des Quadraturverfahrens auf die Steifheit der im Rahmen des Gesamtansatzes zu lösenden Differentialgleichung und auf das Verhalten der hierzu gehörigen numerischen Lösungsverfahren auswirkt.

Literatur

[1] K. Diethelm: *A new diffusive representation for fractional derivatives, Part II: Convergence analysis of the numerical scheme.* Mathematics **10** (2022), Article No. 1245.

[2] K. Diethelm: *A new diffusive representation for fractional derivatives, Part I: Construction, implementation and numerical examples.* Erscheint in A. Cardone, M. Donatelli, F. Durastante, R. Garrappa, M. Mazza & M. Popolizio (Eds.): *Fractional Differential Equations: Modeling, Discretization, and Numerical Solvers.* Springer Nature, Singapore (2023).

Numerical Computation of Quantum Graph Spectra

Chong-Son Dröge, Anna Weller
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
droege@math.uni-koeln.de, weller@math.uni-koeln.de

Abstract

In the context of spectral solution methods for partial differential equations on metric graphs, the efficient computation of *quantum graph* spectra is of utmost importance. Under a quantum graph, we understand a metric graph equipped with a differential operator and for which suitable coupling conditions at the nodes apply. The differential operator of our interest is the negative second order derivative acting on each edge. The spectrum of the quantum graph is then understood as the spectrum of the differential operator acting on the metric graph.
The special case where all edges in the graph have the same length (*equilateral graphs*) has been discussed in [AW] using a well-known connection of quantum and combinatorial graph spectra. Unfortunately, these simplifications do not apply for graphs with different edge lengths. Here, some more work has to be done in order to resolve the eigenvalues and corresponding eigenfunctions.
The simplest situation where all edges have integer lengths can be treated using the observation that the spectrum of a quantum graph does not change when we insert vertices with degree two on the edges. Thus, one can simply modify the graph to an equilateral *extended graph*. For the general case with real, positive edge lengths, we show that we can relate the spectrum to a so-called *Nonlinear Eigenvalue Problem* (NEP): Find eigenvalues $\lambda > 0$ such that a nontrivial $\mathbf{v}$ with

$$\mathbf{H}(\lambda)\mathbf{v} = 0$$

exists. Interestingly, the size of the matrix $\mathbf{H}$ is given by the number of vertices of the graph and the information on the edge lengths only contributes to the entries of the matrix. Conveniently, the corresponding eigenfunctions can be constructed using $\mathbf{v}$.
The solutions of the NEP can be found as the roots of $\det(\mathbf{H}(\lambda))$, which is a highly nonlinear problem. A popular solution method for NEPs is the Newton-trace method [GT]. However, the key to the efficient application of this method are suitable initial guesses. To find these, we propose two different approaches [DW]: First, we investigate a polynomial approximation of $\mathbf{H}$ and apply standard nonlinear eigenvalue solvers for polynomial problems. Secondly, we present a workflow to approximate non-equilateral graphs by equilateral graphs and use the eigenvalues of these approximations as a first guess and improve them using the Newton-trace iteration.

This work was supported by Hypatia.Science, an initiative for the promotion of young female scientists at the Department of Mathematics and Computer Science of the University of Cologne.

References

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[DW] C. Dröge, A. Weller, *Numerical computation of quantum graph spectra as solutions of nonlinear eigenvalue problems,* in preparation.

[GT] S. Güttel, F. Tisseur, *The nonlinear eigenvalue problem*, Acta Numerica, 26, 1-94., 2017.

The Rayleigh criterion and well-conditionedness of super resolution

Mathias Hockmann

University of Osnabrück

In microscopy applications, numerous definitions of the term "resolution limit" or "diffraction limit" exist. While they have the common reasoning to describe the smallest resolvable distance between two objects, they differ by a constant and lack in a clear definition of what is meant by resolvability. We address this issue by considering super resolution (SR) as the mapping of Fourier coefficients of a discrete measure on $[0, 1)^d$ to its support and weights. In practice, the question of resolvability is then linked to the condition number of this map. The diffraction limit can be seen as an assumption on the separation of the involved measures similar to the Rayleigh criterion. In fact, we can prove that SR is well-conditioned if the Rayleigh criterion holds and this improves a bound on the assumed separation by Chen and Moitra. This is joint work with Stefan Kunis.

Adaptive near-best Quarklet Tree Approximation

Marc Hovemann
(joint work with S. Dahlke, T. Raasch and D. Vogel)

This talk is concerned with near-optimal approximation of a given function $f \in L_2([0,1])$ with elements of a polynomially enriched wavelet frame, a so-called quarklet frame. For that purpose we introduce the concepts of quarklet indices and quarklet trees. We use the underlying tree structure of the frame elements to derive an adaptive algorithm, that can be used to create approximations with an error close to the best tree approximation error for a given cardinality, as long as standard assumptions concerning the local errors are fulfilled. In connection with that some of our proofs are inspired by *hp*-approximation techniques of Binev [1]. To conclude the talk, we support our findings by numerical experiments, demonstrating, that our approach can be used to achieve inverse-exponential convergence rates. More information concerning adaptive near-best quarklet tree approximation can be found in our recent preprint [2].

References

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[2] S. Dahlke, M. Hovemann, T. Raasch and D. Vogel, *Adaptive Quarklet Tree Approximation*, preprint, 2023, arXiv:2301.04111.

A New Approach to Envelope Estimation for EMD

Laslo Hunhold
University of Cologne
hunhold@math.uni-koeln.de

Abstract

The Empirical Mode Decomposition (EMD) is a self-adaptive and data-driven method for additively separating multi-component, non-linear, non-stationary signals into Intrinsic Mode Functions (IMFs) (see [Hua+98]). The separation process, called sifting, is based on the estimation of lower and upper envelopes. The established B-Spline-based method proposed in [Che+06] has the disadvantage that the estimated envelopes can intersect with the original signal (which violates the envelope-property).

There have been numerous approaches to improve the mathematical formulation and estimation of envelopes (see for example [HPH12], [HK13] and [Niu+21]) with their own downsides in regard to precision and efficiency.

This talk presents a new iterative method for envelope estimation that solves the intersection-problem and has interesting properties in regard to the EMD algorithm itself.

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BPX Preconditioners for B-Spline Discretizations of Operators Arising in Elliptic Variational Inequalities

Sarah Knoll
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
knoll@math.uni-koeln.de

Abstract

An important part of mathematical finance is the valuation and pricing of options. Specifically, an American call or put option can be interpreted as a financial contract that allows but not obligates the holder to buy or to sell an underlying asset within a time period $(0, T]$ at a fixed price K. Those options are modelled using the Heston or Black-Scholes model, that differ in their assumptions about the volatility of the underlying asset. The latter assumes that the underlying's volatility is constant. Mathematically, both models are described by a parabolic PDE with a free boundary, which can be reformulated as a variational inequality.

In finance, one is not only interested in the solution of the PDEs arising from modelling the option price but also its first and second order partial derivates. Therefore, we use a spatial discretization based on cubic B-Splines with coinciding knots at the points where the given initial condition is not differentiable, as developed in [B]. In general, the spectral condition number of the stiffness matrix A_J arising from this procedure depends on the discretization's refinement level J, i.e. $\kappa_2(A_J) \sim 2^{2J}$. Here, we apply the so-called BPX preconditioner. It was shown in [DK], [O], that the BPX preconditioner is asymptotically optimal for elliptic PDE operators. This motivates our experiments for the above described variational inequalites.

References

[B] S. Boschert, *B-Spline Based Methods: From Monotone Multigrid Schemes for American Options to Uncertain Volatility Models*, PhD thesis, University of Cologne, Faculty of Mathematics and Natural Sciences, 2020.

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Mathematical Methods in Parallel MRI

Benjamin Kocurov
University of Göttingen
b.kocurov@math.uni-goettingen.de

Magnetic Resonance Imaging (MRI) is an important technique in medical imaging. In this talk I aim to give an introduction to the mathematical background of parallel MRI. After stating the continuous mathematical model for the associated image reconstruction problem I will talk about sampling of the data and discretization of the problem. I will then present some reconstruction algorithms for this problem. In the end I will discuss some of the implicit assumptions of these algorithms.

References

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Konstruktion von Multiwavelets für allgemeine Skalierungsmatrizen

Anne Kopsch
Philipps-Universität Marburg

In diesem Vortrag wird es um die Konstruktion von Wavelets und Multiwavelets gehen. Insbesondere sind wir daran interessiert, minimale Anforderungen zu identifizieren, unter denen eine Konstruktion möglich ist. Dazu werden wir das Konzept der Multiresolution Analysis genauer betrachten und aufzeigen, auf welche Bedingungen wir für unsere Konstruktion verzichten können. Darüber hinaus möchten wir die Anzahl an Mother Wavelets minimieren. Dies kann man erreichen, indem man allgemeine Skalierungsmatrizen verwendet, deren Determinante die Anzahl an Mother Wavelets bestimmt.

Achieving Quantum Supremacy in Image Processing: Is it Possible?

Thomas Lang

(Fraunhofer Institute of Integrated Circuits IIS,
Division Development Center X-ray Technology)

Quantum computing is currently one of the fastest emerging branches of information processing due to its theoretical computation powers exceeding conventional computers by far. However, currently well-known quantum-powered algorithms are of theoretical nature and its effect on practical problems has yet to be discovered. Here, we coarsely review the history of quantum computing and what the term *quantum supremacy* means. Furthermore, we discuss the problem of encoding grayscale images and demonstrate how - theoretically - the encoding on a quantum hardware is exponentially more efficient than any known classical encoding. At the same time, we demonstrate why retrieving the image from the quantum encoding is the bad part and we validate this statement by experiments performed both on simulators and on actual quantum hardware.

Numerical Aspects of Multiscale Approximation

Lot Federico

Fachbereich Mathematik und Informatik, Philipps-Universität Marburg, 35032 Marburg, Germany
lotf@mathematik.uni-marburg.de

Abstract

In many practical applications naturally arise the needs to construct high-fidelity models from discretely given data. Kernel methods are a popular choice as they can cope with unstructured point clouds as data. The main drawback is, however, that usually a dense linear system with high condition number has to be solved.
Kernel-based multiscale methods are an attempt to overcome those limitations. These methods are characterised by an appropriate choice of a scaled kernel and an hierarchical organisation of the data. The resulting approximant is known to be accurate as well, but its computation is substantially more stable. In this talk, we will introduce this method, present a novel numerical solution strategy and give some numerical examples.
Collaborators and funding will be achnowledged during the talk.

The Transformation $f(x) \to f(\cos(x_1,), \ldots, \cos(x_n))$ on Function Spaces of Dominating Mixed Smoothness

Kai Lüttgen (joint work with Tino Ullrich),

Chemnitz University of Technology, 09111 Chemnitz, Germany

Abstract

Imagine folding a strip of paper into a zig-zag-like shape. If we are careful during the folding, then the paper strip will end up with sharp edges. It describes a surface which we might intuitively call *rough* or *non-smooth*. Next grab both ends of the strip and bend them towards each other, turning the strip into a star-like or cog-like shape. Did the paper strip become *more rough* during this process? Or did it perhaps become *smoother*? Going purely by intuition we might conclude that no, although the strip might be a little bent and stretched the overall *roughness* or *smoothness* of the strip did not significantly change.

It is one aim of the present work to confirm our intuition, that is, to show that a specific operation of periodisation does not completely destroy the smoothness properties of functions. More precisely, we study the composition operator

$$T_{\cos} : f(x_1, \ldots, x_n) \to f(\cos(x_1), \ldots, \cos(x_n))$$

taking functions from the euclidean space $\mathbb{R}^n$ to the torus $\mathbb{T}^n$. Composition operators on function spaces have been well-studied in the context of *changes of variable*. The literature knows numerous results for composition operators induced by bijective functions. Changes of variable with non-bijective functions are more difficult to handle (see for example [1] for results in this direction).

The operator above is clearly induced by a non-bijective function, making its analysis slightly complicated at times. Nonetheless, using techniques from [3, 4, 5, 6], we show that $T_{\cos}$ is a bounded operator on certain Besov and Triebel-Lizorkin spaces of dominating mixed smoothness. In more detail, we obtain the result that

$$T_{\cos} : S^r_{p,q}A(\mathbb{R}^n) \to S^r_{p,q}A(\mathbb{T}^n)$$

is bounded if

- $A = F$ and $r > 1$, $1 < p < \infty$, $1 < q \leq \infty$,
- or if $A = B$ and $r > 1/p$, $1 < p < \infty$, $0 < q \leq \infty$.

In the setting $A = B$ we also present some results for the limiting cases. Lastly, we also draw a connection between the Sobolev spaces $H^s([-1, 1])$ and spaces $L^{2,s}_{-\frac{1}{2},-\frac{1}{2}}([-1, 1])$ of Sobolev-type related to Chebyshev polynomials as they are considered in [2].

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Stochastic Amplitude Flow for Phase Retrieval and Ptychography

Oleh Melnyk* †

10.02.2023

We consider Stochastic Amplitude Flow (SAF) for phase retrieval [1, 2], a stochastic gradient descent for the amplitude-based squared loss. While the convergence to a critical point of (nonstochastic) Amplitude Flow is well-understood, SAF is a much less studied algorithm. We close this gap by deriving the convergence guarantees for SAF based on the contributions for Amplitude Flow and analysis for stochastic gradient descent. These results are then applied to two more algorithms, which can be seen as instances of SAF. The first is an extension of the Kaczmarz method for phase retrieval [5]. The second is Ptychographic Iterative Engine [4], which is a popular algorithm for ptychography [3], a special case of phase retrieval with the short-time Fourier transform.

It is based on our preprint [6].

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*Mathematical Imaging and Data Analysis, Helmholtz Center Munich, 85764 Neuherberg, Germany
†Department of Mathematics, Technical University of Munich, 85748 Garching bei München, Germany (oleh.melnyk@tum.de)

Gelfandzahlen und beste trigonometrische m-term Approximation für Wiener Räume mit gemischten Gewichten in L_2

Moritz Moeller

Fakultät für Mathematik, TU Chemnitz

Abstract

Die beste m-term Approximation ist seit ihrer Formulierung ein eher theoretisches Forschungsobjekt. Ein neues Resultat von Jahn, T.Ullrich und Voigtlaender liefert jedoch praktische Anwendungen dieser Größe zur Abschätzung von Sampling Zahlen. Der Vortrag beschäftigt sich mit einer Klasse von Räumen in denen damit neue Erkenntnisse gewonnen werden können: Wiener Räume mit gemischten Gewichten. Die Resultate für beste m-term Approximationen nutzen Techniken für Approximationen auf hyperbolische Kreuzen sowie übliche Dekompositionsmethoden. Zusätzlich wird das asymptotische Verhalten der Gelfandzahlen untersucht, die eine theoretische Optimalitätsgrenze, für nichtlineare Rekonstruktion, in diesen Räumen sind.

Moving Least Squares Approximation on Spheres

Tim Pöschl 

TU Bergakademie Freiberg

In this talk we consider the Moving Least Squares Approximation scheme on spheres. We introduce a result from Wendland which states, that the error of the MLS approximation decays with order δ^{L+1}, if the ansatz space consists of all spherical harmonics up to degree L. Here δ denotes the support radius of the weight function.

Later in the talk we will show, that the same order of approximation is attained, if the ansatz space only consists of spherical harmonics of every second degree up to L. It will turn out, that this also leads to significantly higher numerical stability.

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Title: Combining Non-Data-Adaptive Transforms for OCT Image Denoising by Iterative Basis Pursuit

Abstract:
"Optical Coherence Tomography (OCT) images, as well as a majority of medical images, are imposed to speckle noise while capturing. Since the quality of these images is crucial for detecting any abnormalities, we develop an improved denoising algorithm that is particularly appropriate for OCT images.
The essential idea is to combine two non-data-adaptive transform-based denoising methods that are capable to preserve different important structures appearing in OCT images while providing a very good denoising performance.
Based on our numerical experiments, the most appropriate non-data-adaptive transforms for denoising and feature extraction are the Discrete Cosine Transform (DCT) (capturing local patterns) and the Dual-Tree Complex Wavelet Transform (DTCWT) (capturing piecewise smooth image features). These two transforms are combined using the Dual Basis Pursuit Denoising (DBPD) algorithm. Further improvement of the denoising procedure is achieved by total variation (TV) regularization and by employing an iterative algorithm based on DBPD."

Supervisor: Prof. Dr. Gerlind Plonka-Hoch

Presenter (poster): Raha Razavi

Variational Methods For Parameterized Quadratic Bilinear Differential-Algebraic Equations

Niklas Reich, Universität Ulm / Hochschule Ruhr West

We investigate variational formulations for (parameterized) quadratic bilinear differential-algebraic equations. To prove the well-posedness of such variational formulations we adapt the Brezzi-Rappaz-Raviart theory for problems, where trial and test space do not coincide. Within the Brezzi-Rappaz-Raviart theory we use a Petrov-Galerkin method to derive detailed approximate solutions and model order reduction via the Reduced Basis Method to derive efficient, reduced solutions for the variational problem. For the Reduced Basis Method online-efficient error estimates are presented. We then use the requirements of the Brezzi-Rappaz-Raviart theory to develop well-posed variational formulations for (parameterized) quadratic bilinear differential-algebraic equations. We start with a standard variational formulation, which is shown to be conform with the Brezzi-Rappaz-Raviart theory, i.e. well-posedness is ensured. Additionally we develop an ultraweak variational formulation, that yields an optimally stable system, but is unfortunately not conform with Brezzi-Rappaz-Raviart theory in its presented form.

Dieser Vortrag befasst sich mit Variationsformulierungen für quadratisch bilineare differential-algebraische Gleichungen. Um die Wohlgestelltheit solcher Variationsformulierungen zu zeigen, wird die sogenannte Brezzi-Rappaz-Raviart-Theorie adaptiert für Probleme, deren Ansatz- und Testraum nicht übereinstimmen. Innerhalb der Brezzi-Rappaz-Raviart-Theorie wird eine Petrov-Galerkin-Methode für detaillierte Approximationslösungen und die Reduzierte-Basis-Methode für effiziente, reduzierte Lösungen der Variationsformulierungen genutzt. Dazu werden auch online-effiziente Fehlerschätzer für die Reduzierte-Basis-Methode vorgestellt. Die Anforderungen aus der Brezzi-Rappaz-Raviart-Theorie werden dann genutzt, um wohlgestellte Variationsformulierungen zu entwickeln. Begonnen wird zunächst mit einer Standard-Variationsformulierung. Für diese kann gezeigt werden, dass sie alle notwendigen Anforderungen der Brezzi-Rappaz-Raviart-Theorie erfüllt. Zusätzlich wird eine sehr schwache Variationsformulierung entwickelt. Diese führt zu einem optimalstabilen System, erfüllt aber nicht die notwendigen Anforderungen der Brezzi-Rappaz-Raviart-Theorie.

Spline Representation of One-Dimensional ReLU Neural Networks

Yannick Riebe
University of Göttingen
y.riebe@math.uni-goettingen.de

In this talk I will analyze the set of output functions of one-dimensional ReLU neural networks. These functions are always continuous and piecewise linear. I am interested in the connection between the set output functions of such neural networks and the set of continuous piecewise linear functions with a certain number of breakpoints (points of non-differentiability). In particular, I will prove an upper bound for the number of breakpoints of an output function depending on the depth of the neural network and the widths of its layers. Using an explicit construction, I will show that this upper bound is sharp. Further, for neural networks with two hidden layers I will give results on how many breakpoints one can fix a priori and which continuous piecewise linear functions are always an output function of such a neural network.

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KERNEL METHODS FOR HIGH DIMENSIONAL PROBLEMS

CHRISTIAN RIEGER

Kernel methods are a common tool in many approximation problems as they can be used in a mesh-free setting. Moreover, the dimensionality of the approximation problem appears only mildly in the numerics but of course is present in theoretical assumptions on the point cloud which are needed to derive an error analysis. In this talk, we discuss kernel methods for potentially high-dimensional Bayesian inverse problems and present a problem-adapted error analysis for those problems.

The talk is based on joint work with several people who will be acknowledged during the talk.

Philipps-Universität Marburg, Fachbereich 12 - Mathematik und Informatik, Hans-Meerwein-Str. 6, D-35032 Marburg

Email address: riegerc@mathematik.uni-marburg.de

Numerical Computation of Equilateral Quantum Graph Spectra

Lukas Schmitz, David Stiller
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
lschmi99@smail.uni-koeln.de, dstille1@smail.uni-koeln.de

Abstract

Under an equilateral quantum graph, we understand a metric graph equipped with a differential operator, suitable coupling conditions, and such that all edge lengths are equal. The differential operator of our interest is the negative second-order derivative, often called *Hamiltonian* acting on each edge. The spectrum of the quantum graph Γ is then understood as the spectrum of the differential operator acting on the metric graph. We are interested if there is any relation between the spectra of quantum graphs and discrete operators on the underlying combinatorial graph. And in fact, there exists a very useful and prominent relation to the spectrum of the combinatorial graph Laplacian matrix if we assume the graph to be equilateral. Yet, this relation only holds true for a specific part of the spectrum, which we will refer to as the vertex spectrum. The remaining non-vertex part of the spectrum needs separate consideration. By using a simple trick of inserting artificial vertices on the edges, we will see, that we can again reduce the non-vertex eigenvalues to a discrete problem and draw conclusions like specifying their multiplicity [AW]. In this context, we will speak of an *extended graph Laplacian system*.

It remains to develop an efficient method to compute the associated eigenfunctions. To do so, we only need the eigenvectors of the discrete graph Laplacian eigenvalue problem. In the case of non-vertex eigenfunctions, we present an improved method to calculate the required eigenvectors of the extended graph Laplacian system by using theoretically derived information on their values at the vertices of the quantum graph.

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Hochdimensionale Approximation mit teilweise periodischen Randbedingungen

Pascal Schröter

Fakultät für Mathematik, TU Chemnitz

In diesem Vortrag betrachten wir orthonormale Basen, die als Tensorprodukt von Fourier-Basen zusammen mit Kosinus-Basen entstehen. Wir entwickeln einen schnellen Algorithmus, um mit der dazugehörigen Matrix, zusammengesetzt aus Zeilen einer nicht äquidistanten Fourier-Matrix und Zeilen einer nicht äquidistanten Kosinus-Matrix, und ihrer Transponierten, zu multiplizieren. Des Weiteren wird die Idee der gruppierten Fourier-Transformationen auf solche Matrizen übertragen. Dies führt zu einer ANOVA (analysis of variance)-Dekomposition mit einer Basis, die die Fourier-Basis und die Kosinus-Basis in verschiedenen Dimensionen kombiniert. Darauf aufbauend erhalten wir aus der Transformation in d-dimensionale Polarkoordinaten orthonormale Basen auf d-dimensionalen Kugeln und Sphären und können somit die effizienten Algorithmen zur hochdimensionalen Approximation auf den Torus übertragen.

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LiveTV: Real-time Total Variation Regularization Using Haar Wavelets

A. Michael Stock (University of Passau), Tomas Sauer*

Haar tensor wavelet analysis enables sparse representation of locally constant data. This can be exploited when compressing very large computed tomography (CT) scans: Even when keeping just a small percentage of the original wavelet coefficients, high image quality is preserved.

Now, for multi-dimensional data, we consider only coefficients created by applying one wavelet and one or two scaling functions for the two-dimensional or three-dimensional case, respectively. Properly weighted, this provides us with local gradient estimates. Moreover, thresholding these particular coefficients can at the same time approximately reduce the total variation (TV) norm. Finally, we present results for two-dimensional image data and three-dimensional CT data.

stock@forwiss.uni-passau.de

Modeling the measurement problem of the sampling oscilloscope

- measurement data, uncertainty and reconstruction

Katja Tüting
Technische Universität Braunschweig
Institute for Partial Differential Equations

Oscilloscope measurements are never exact, they are subject to aleatoric and epistemic measurement uncertainties. An oscilloscope maps a time-varying voltage, which is assumed to exist and to be continuous, to a discrete measurement vector. We discuss approaches to model the measurement process, and in doing so, we provide an insight into the measurement problem of the sampling oscilloscope.

Because the real voltage is not available for comparison with the measured values, a high-fidelity (HiFi) measurement, that serves as a reference for the low-fidelity (LoFi) measurement of the sampling oscilloscope, is considered in modeling. A calibration operator is discussed based on the modeling of the HiFi and LoFi measurements.

Uncertainties in measurements and calibration lead to uncertainties in reconstructions, and it is necessary to quantify them. We evaluate and compare the uncertainties from a frequentist and a Bayesian point of view.

Adaptive Quarklet Tree Approximation: Numerical Experiments

Dorian Vogel
(joint work with S. Dahlke, M. Hovemann and T. Raasch)
Philipps-Universität Marburg

We are concerned with near-optimal approximation of a given function $f \in L_2([0,1])$ with elements of a polynomially enriched wavelet frame, a so-called quarklet frame. Inspired by *hp*-approximation techniques of Binev, we use the underlying tree structure of the frame elements to derive an adaptive algorithm that, under standard assumptions concerning the local errors, can be used to create approximations with an error close to the best tree approximation error for a given cardinality. This poster provides information on the practical implementation and the performance of the quarklet algorithm. In particular, we demonstrate that our approach can be used to achieve inverse-exponential convergence rates (with respect to the degrees of freedom spent) for models of typical solutions of partial differential equations where we expect that adaptive schemes outperform classical uniform schemes.

Rate-Optimal Sparse Approximation of Compact Break-of-Scale Embeddings

Markus Weimar
Julius-Maximilians-Universität Würzburg (JMU)
MARKUS.WEIMAR@UNI-WUERZBURG.DE

The poster addresses the approximation problem of functions in new scales of function spaces with hybrid smoothness. In these scales we combine classical (isotropic) regularity measured in L_p with so-called dominating mixed smoothness which arises in high-dimemsional real-world applications, e.g., related to the electronic Schrödinger equation. Sharp dimension-independent rates of convergence for linear and non-linear best approximations using n hyperbolic wavelets are presented. Important special cases include the approximation of function having dominating mixed smoothness w.r.t. L_p in the norm of the (isotrpic) energy space H^1.

The presented results are based on a recent paper [1] which represents the first part of a project in joint work with Janina Hübner (RUB), Glenn Byrenheid (FSU Jena), and Markus Hansen (PU Marburg).

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A spectral Galerkin method for the solution of reaction-diffusion equations on metric graphs

Anna Weller
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
weller@math.uni-koeln.de

Abstract

We investigate a spectral solution approach for reaction-diffusion equations on graphs interpreted as topological space (*metric graphs*). Of special interest is the numerical computation of eigenfunctions of the negative second order derivative acting on each edge. Remarkably, it is possible to give an explicit characterization of these eigenfunctions and corresponding eigenvalues. Moreover, for equilateral graphs, we will show how to efficiently compute an arbitrary lower part of the spectrum using a very useful relationship to the graph Laplacian matrix of the underlying combinatorial graph. Finally, we can use the basis of eigenfunctions in a Galerkin approach to solve various PDEs on metric graphs, were we here focus on reaction-diffusion equations. This problem is motivated by a recent collaboration with the Institute of Geophysics and Meteorology of the University of Cologne on the modeling of protein distribution in Alzheimer's disease together with the University Hospital Cologne[1]. Part of my numerical results are joint work with Prof. Dr. Mark Ainsworth (Brown University) and with Chong-Son Droege (University of Cologne).

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A Toolbox for Fast Harmonic Approximation on the Rotation Group

Erik Wünsche

TU Bergakademie Freiberg

Abstract

Band-limited functions on the rotation group $\mathcal{SO}(3)$ can be described by the Fourier coefficients of its harmonic series. Therefore, operations like addition or convolution are directly computable on frequency domain.

For the evaluation of such harmonic series there exist $\mathcal{O}(N^3 \log^2 N)$ algorithms [2] [3]. Moreover, following Risbo [4] we get a polynomial transformation directly from a representation property of Wigner-D functions. This implies a practically faster $\mathcal{O}(N^4)$ algorithm. A combination of Gauß and Clenshaw-Curtis quadrature yields an inverse algorithm to compute the Fourier coefficients from the function evaluations on a (w.r.t Euler angles) equispaced grid. Using inner symmetries of functions on $\mathcal{SO}(3)$ we can improve these algorithms. The tangent space of a function on $\mathcal{SO}(3)$ is isomorphic to $\mathbb{R}^3$. By a clever choice of the basis of the tangent space, the gradient of a harmonic series can be described by 3 harmonic series. This only requires scaling and shifted addition of the Fourier coefficients.

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