BPX Preconditioners for B-Spline Discretizations of Operators Arising in Elliptic Variational Inequalities

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Abstract

An important part of mathematical finance is the valuation and pricing of options. Specifically, an American call or put option can be interpreted as a financial contract that allows but not obligates the holder to buy or to sell an underlying asset within a time period $(0, T]$ at a fixed price K. Those options are modelled using the Heston or Black-Scholes model, that differ in their assumptions about the volatility of the underlying asset. The latter assumes that the underlying's volatility is constant. Mathematically, both models are described by a parabolic PDE with a free boundary, which can be reformulated as a variational inequality.

In finance, one is not only interested in the solution of the PDEs arising from modelling the option price but also its first and second order partial derivates. Therefore, we use a spatial discretization based on cubic B-Splines with coinciding knots at the points where the given initial condition is not differentiable, as developed in [B]. In general, the spectral condition number of the stiffness matrix A_J arising from this procedure depends on the discretization's refinement level J, i.e. $\kappa_2(A_J) \sim 2^{2J}$. Here, we apply the so-called BPX preconditioner. It was shown in [DK], [O], that the BPX preconditioner is asymptotically optimal for elliptic PDE operators. This motivates our experiments for the above described variational inequalites.

References

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