Spline Representation of One-Dimensional ReLU Neural Networks

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In this talk I will analyze the set of output functions of one-dimensional ReLU neural networks. These functions are always continuous and piecewise linear. I am interested in the connection between the set output functions of such neural networks and the set of continuous piecewise linear functions with a certain number of breakpoints (points of non-differentiability). In particular, I will prove an upper bound for the number of breakpoints of an output function depending on the depth of the neural network and the widths of its layers. Using an explicit construction, I will show that this upper bound is sharp. Further, for neural networks with two hidden layers I will give results on how many breakpoints one can fix a priori and which continuous piecewise linear functions are always an output function of such a neural network.

References

[1] I. Daubechies, R. DeVore, S. Foucart, B. Hanin, G. Petrova, Nonlinear Approximation and (Deep) ReLU Networks, Constructive Approximation (2022) 55:127-172

[2] G. Plonka, Y. Riebe, Y. Kolomoitsev, Spline Representation and Redundancies of One-Dimensional ReLU Neural Network Models, Analysis and Applications, Vol. 21, No. 1 (2023) 127-163

[3] T. Serra, C. Tjandraatmadja, S. Ramalingam, Bounding and Counting Linear Regions of Deep Neural Networks, Proceedings of the 35th International Conference on Machine Learning, Stockholm, Sweden, PMLR 80 (2018)