A spectral Galerkin method for the solution of reaction-diffusion equations on metric graphs

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Abstract

We investigate a spectral solution approach for reaction-diffusion equations on graphs interpreted as topological space (*metric graphs*). Of special interest is the numerical computation of eigenfunctions of the negative second order derivative acting on each edge. Remarkably, it is possible to give an explicit characterization of these eigenfunctions and corresponding eigenvalues. Moreover, for equilateral graphs, we will show how to efficiently compute an arbitrary lower part of the spectrum using a very useful relationship to the graph Laplacian matrix of the underlying combinatorial graph. Finally, we can use the basis of eigenfunctions in a Galerkin approach to solve various PDEs on metric graphs, were we here focus on reaction-diffusion equations. This problem is motivated by a recent collaboration with the Institute of Geophysics and Meteorology of the University of Cologne on the modeling of protein distribution in Alzheimer's disease together with the University Hospital Cologne[1]. Part of my numerical results are joint work with Prof. Dr. Mark Ainsworth (Brown University) and with Chong-Son Droege (University of Cologne).

References

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