$(\alpha_1, \alpha_2, \ldots, \alpha_n)$-Eigenvalues of $(n \times n)$ Matrices: Applications, First Results, and Open Questions

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Given a real or complex $(n \times n)$ matrix A and a multi-index $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in (0, 1]^n$, the α-characteristic function of the matrix A is defined as

$$p_{A,\alpha}(\lambda) := \det\left(A - \operatorname{diag}(\lambda^{\alpha_1}, \lambda^{\alpha_2}, \ldots, \lambda^{\alpha_n})\right),$$

and a complex number λ with $p_{A,\alpha}(\lambda) = 0$ is called an α-eigenvalue of A. Clearly, in the case $\alpha = (1, 1, \ldots, 1)$, this concept reduces to the classical eigenvalue in the sense of linear algebra.

For an n-dimensional linear system of first oder differential equations, it is well known that the location of the eigenvalues of the coefficient matrix determines the system's stability properties. Therefore, it is important to possess efficient methods for finding the eigenvalues. When the system of differential equations comprises equations of fractional order, the same is true if the classical eigenvalues are replaced by α-eigenvalues.

We present some theoretical results about the location of these α-eigenvalues. Also, we discuss computational strategies that allow their practical calculation in certain special cases. The question for a generally applicable efficient algorithm to compute all α-eigenvalues of a given matrix is still an open problem.

Literature

K. Diethelm, S. Hashemishahraki, H. D. Thai & H. T. Tuan: *A constructive approach for investigating the stability of incommensurate fractional differential systems.* arXiv:2312.00017.