33. Rhein-Ruhr-Workshop

Bestwig, 9.-10. Februar 2024

-Programm, Teilnehmer und Abstracts-

Organisation:

Prof. Dr. G. Plonka-Hoch	Universität Göttingen
Prof. Dr. T. Sauer	Universität Passau
Prof. Dr. M. Skrzipek	FernUniversität in Hagen
Dr. M. Weimar	Universität Würzburg

33. Rhein-Ruhr-Workshop
Bestwig, 9.–10. Februar 2024
PROGRAMM

Freitag, 9. Februar 2024, Vormittag

10.20	*Begrüßung / Organisatorisches*
	Moderation: Tomas Sauer
10.30	**Pascal Schröter** (Technische Universität Chemnitz) Learning Anisotropy for ANOVA approximation
11.00	**Mathias Sonnleitner** (Universität Passau) Lineare Approximation mit zufälligen Messungen
11.30	**Marco Rauscher** (Technische Universität München) Reconstruction of the Shortest Path from a given Signature
12.00	*Gemeinsames Mittagessen*

Freitag, 9. Februar 2024, Nachmittag

	Moderation: Kai Diethelm
14.00	**Jonas Bresch** (Technische Universität Berlin) Denoising of Sphere and $SO(3)$-valued Data by Relaxed Tikhonov Regularization
14.30	**Allesandro Lupoli** (Technische Universität München) Quantization of Bandlimited Functions on the 2D - Torus
15.00	**Frederic Schoppert** (Universität Lübeck) Edge Detection with Polynomial Frames on the Sphere
15.30	*Pause mit Kaffee, Tee, Kuchen*
	Moderation: Joachim Stöckler
16.00	**Martin Ossadnik** (Fernuniversität in Hagen) A posteriori Fehlerschätzer in der Maximumnorm fr BDF-Verfahrenn
16.30	**Safoura Hashemishahraki** (Technische Hochschule Würzburg-Schweinfurt) Stability Properties of Multi-Order Fractional Differential Systems in 3D
17.00	**Marc Hovemann** (Universität Marburg) Multivariate Quarklets in the Context of Bessel-PotentialSpaces on Unit Cubes
18.00	*Gemeinsames Abendessen*

Freitag, 9. Februar 2024, ab 19 Uhr:

Präsentation der Poster
Kai Diethelm (Technische Hochschule Würzburg-Schweinfurt) $(\alpha_1, \alpha_2, \ldots, \alpha_n)$-Eigenvalues of $(n \times n)$ Matrices: Applications, First Results, and Open Questions
Florian Heinrich (Universität Passau) How to make real bases complex?
Moritz Proell (Universität Passau) Darstellungstheorie lokalkompakter Gruppen und C^*-Algebren
Tomas Sauer (Universität Passau) CT for Cultural Heritage
Janina Schmidt (Universität Göttingen) Automated adjustment of the focusing optics of a free electron laser
Michael Stock (Universität Passau) Dirty Tricks for Clean Images Heuristics for 3D CT Reconstruction

Samstag, 10. Februar 2024

7.30	*Frühstück und anschließendes Räumen der Zimmer, Schlüsselabgabe bis 10 Uhr*
	Moderation: Michael Skrzipek
9.15	**Anahita Riahi** (Universität Göttingen) Reconstruction of Undersampled Fourier Data
9.45	**Renu Chaudhary** (Technische Hochschule Würzburg-Schweinfurt) Novel Variants of Diffusive Representaion of Fractional Integrals: Construction and Numerical Computation
10.15	*Pause mit Kaffee, Tee*
	Moderation: Gerlind Plonka-Hoch
10.45	**Michael Juhos** (Universität Passau) Untersuchungen allgemeiner Schattennorm-Kugeln
11.30	**Yannick Riebe** (Universität Göttingen) An Algorithm For Parallel MRI Reconstruction Using Model Based Coil Calibration (MOCCA)
11.45	*Gemeinsames Mittagessen*

Dauer der Vorträge: 30 Minuten, einschließlich Diskussionszeit.

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Denoising of Sphere- and SO(3)-valued Data by Relaxed Tikhonov Regularization

Jonas Bresch
Technische Universität Berlin

Manifold-valued signal- and image processing has received attention due to novel image acquisition techniques. Recently, a convex relaxation of the Tikhonov-regularized nonconvex problem for denoising circle-valued data has been proposed by Condat (2022). In this talk, we show, based on Schur complement arguments, that this variational model can be simplified while leading to the same solution. Our simplified model can be generalized to higher dimensional spheres and to SO(3)-valued data, where we rely on the quaternion representation of the later one. Standard algorithms from convex analysis can be applied to solve the resulting convex minimization problem. As proof-of-the-concept, we use the alternating direction method of minimizers to demonstrate the denoising behavior of the proposed method.

Additionally, we can use our relaxation for TV-regularization on spheres and the SO(3) in an Euclidean-embedded point of view. As in the Tikhonov-like regularization, we can observe the tightness in many cases, but lose the later one in general. Comparisons for the null and one-dimensional sphere, and proof-of-the-concepts for the two-dimensional sphere and the SO(3) are done.

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References

[1] Laurent Condat (2022), *Tikhonov regularization of Circle-Valued signals*, IEEE Transactions on Signal Processing, Vol. 70

NOVEL VARIANTS OF DIFFUSIVE REPRESENTATION OF FRACTIONAL INTEGRALS: CONSTRUCTION AND NUMERICAL COMPUTATION

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Abstract. In this talk, we discuss novel variants of diffusive representations of Riemann-Liouville fractional integrals. These variants aim to offer highly efficient numerical algorithms for the approximate computation of fractional integrals with less computational complexity and memory footprint. For this, we have embarked on an exploration of innovative variations in diffusive representations tailored for fractional integrals. We have approximated the kernel function to the fractional integral by representing it as an exponential sum. This representation can be further optimized by leveraging Prony's method to curtail the number of terms involved. Subsequently, we have harnessed this refined approximation to compute an estimate for the fractional integral. This approach yields a notable reduction in both computational intricacy and memory usage, offering an enticing prospect for practical implementations. In addition to the exponential sum approximation, we have enriched our computational toolkit by developing the Gauss-Laguerre formula as an alternative method for approximating fractional integrals.

References

[1] Diethelm, K. (2023). Diffusive Representations for the Numerical Evaluation of Fractional Integrals. In 2023 International Conference on Fractional Differentiation and Its Applications (ICFDA) (pp. 1-6). IEEE.

$(\alpha_1, \alpha_2, \ldots, \alpha_n)$-Eigenvalues of $(n \times n)$ Matrices: Applications, First Results, and Open Questions

Kai Diethelm & Safoura Hashemishahraki
Faculty of Applied Natural Sciences and Humanities, TH Würzburg-Schweinfurt
Ha Duc Thai & Hoang The Tuan
Institute of Mathematics, Vietnam Academy of Science and Technology

RRW 2024, Bestwig

Given a real or complex $(n \times n)$ matrix A and a multi-index $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in (0, 1]^n$, the α-characteristic function of the matrix A is defined as

$$p_{A,\alpha}(\lambda) := \det \left(A - \operatorname{diag}(\lambda^{\alpha_1}, \lambda^{\alpha_2}, \ldots, \lambda^{\alpha_n}) \right),$$

and a complex number λ with $p_{A,\alpha}(\lambda) = 0$ is called an α-eigenvalue of A. Clearly, in the case $\alpha = (1, 1, \ldots, 1)$, this concept reduces to the classical eigenvalue in the sense of linear algebra.

For an n-dimensional linear system of first oder differential equations, it is well known that the location of the eigenvalues of the coefficient matrix determines the system's stability properties. Therefore, it is important to possess efficient methods for finding the eigenvalues. When the system of differential equations comprises equations of fractional order, the same is true if the classical eigenvalues are replaced by α-eigenvalues.

We present some theoretical results about the location of these α-eigenvalues. Also, we discuss computational strategies that allow their practical calculation in certain special cases. The question for a generally applicable efficient algorithm to compute all α-eigenvalues of a given matrix is still an open problem.

Literature

K. Diethelm, S. Hashemishahraki, H. D. Thai & H. T. Tuan: *A constructive approach for investigating the stability of incommensurate fractional differential systems.* arXiv:2312.00017.

Stability Properties of Multi-Order Fractional Differential Systems in 3D

Kai Diethelm * **Safoura Hashemishahraki** * **Ha Duc Thai** ** **Hoang The Tuan** **

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** *Institute of Mathematics, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, 10307 Ha Noi, Vietnam (e-mail: {hdthai, httuan}@math.ac.vn)*

Abstract: In this talk, we discuss three-dimensional non-commensurate fractional order differential equation systems with Caputo derivatives. Particularly, we focus on necessary and sufficient conditions for the asymptotic stability of such systems.

How to make real bases complex?

Florian Heinrich

Every pair of real, continuous wavelets ψ_1, ψ_2 can be converted to a complex, continuous wavelet $\psi_1 + i\psi_2$. We ask an analogous question for orthonormal bases and Riesz bases and show under which conditions two real orthonormal bases or Riesz bases can be converted to a complex orthonormal basis or Riesz basis, respectively. We approach this problem by converting the problem into an equivalent problem on operators and classify them. As an example, we apply the result to orthonormal wavelet bases.

Multivariate Quarklets in the Context of Bessel-Potential Spaces on Unit Cubes

Marc Hovemann

In this talk it is our main goal to describe multivariate Bessel-Potential Spaces defined on cubes via spline quarklets. For that purpose in a first step we recall the construction of univariate quarklets which have been introduced in the last decade in [3]. Those quarklets are based on biorthogonal compactly supported Cohen-Daubechies-Feauveau spline wavelets that have been enriched with polynomials. Boundary adapted versions of the quarklets can be used to characterize univariate Bessel-Potential spaces $H^s_r((0,1))$ defined on intervals. To obtain multivariate quarklets we apply tensor product methods. It is well-known since many years that multivariate Sobolev spaces $H^s_2(\Omega)$ defined on cubes can be written as an intersection of function spaces which have a tensor product structure, see [4] and [1]. Very recently Hansen and Sickel found that such decompositions also hold in the case of more general Bessel-Potential spaces $H^s_r(\Omega)$ with $1 < r < \infty$. Consequently we can use univariate quarklets in combination with tensor product methods to obtain multivariate quarklet characterizations for Sobolev and Bessel-Potential spaces defined on unit cubes, see [2] for the case of Sobolev spaces.

References

[1] N. Chegini, S. Dahlke, U. Friedrich and R. Stevenson, *Piecewise tensor product wavelet bases by extensions and approximation rates*, Found. Comput. Math. **82** (2013), 2157-2190.

[2] S. Dahlke, U. Friedrich, P. Keding, A. Sieber and T. Raasch, *Adaptive quarkonial domain decomposition methods for elliptic partial differential equations*, IMA J. Numer. Anal. **41**(4) (2021), 2608-2638.

[3] S. Dahlke, P. Keding and T. Raasch, *Quarkonial frames with compression properties*, Calcolo **54**(3) (2017), 823-855.

[4] M. Griebel and P. Oswald, *Tensor product type subspace splittings and multilevel iterative methods for anisotropic problems*, Adv. Comput. Math. **4** (1995), 171-206.

Untersuchungen allgemeiner Schattennorm-Kugeln

Michael Juhos (Universität Passau)

33. Rhein-Ruhr-Workshop
Bestwig, 9.2.2024–10.2.2024

Schattennormen (d. h. die ℓ_p-Normen den Singulärwerte einer Matrix) haben breite Anwendungen gefunden, sei es im Compressed Sensing der Bild- und Signalverarbeitung und in der Matrixvervollständigung oder anderen Gebieten der Data Science, sei es in der nichtkonvexen Optimierung, sei es im Quantencomputing; daneben besteht auch reges theoretisches Interesse, vor allem seitens der Funktionalanalysis, oder in der Wahrscheinlichkeitsrechung als nichtkommutatives Analogon der ℓ_p-Räume, und auch ihre (asymptotische) Geomtrie ist in einigen Aspekten untersucht worden, was sie wieder für dir Komplexitätstheorie interessant macht (etwa Konstruktion von Fooling Functions). Viele Resultate liegen aber nur für Räume quadratischer Matrizen vor, daher widmen wir uns in diesem Vortrag einigen geometrischen und probabilistischen Ergebnissen für die Schattennorm-Einheitskugeln von Matrizen beliebiger Dimensionen; besonderes Gewicht gilt hier der Schatten-∞-Norm, also der Spektralnorm, und wir untersuchen unter anderem Volumen und die Gleichverteilung auf der Einheitskugel.

Der Inhalt des Vortrages ist Teil laufender gemeinsamer Forschung mit Joscha Prochno und Zakhar Kabluchko.

Quantization of Bandlimited Functions on the 2D – Torus

Alessandro Lupoli
TU Munich

The study of quantization techniques for bandlimited functions has become increasingly important both for the results obtainable in some engineering applications (A/D converters, Digital Halftoning, Imaging), and for some Machine Learning and Deep Learning applications. The main issue is highlighted when trying to apply these algorithms to functions defined on closed manifolds, as cuts need to be introduced to apply them, and the schemes will typically not yield good reconstruction along the cuts.

This talk will explore the problem of quantization and reconstruction of a bandlimited signal on the 2D torus, highlighting how changing the sampling scheme can avoid the presence of artifacts generated by Sigma-Delta schemes.

This is joint work with Professor Felix Krahmer.

A posteriori Fehlerschätzer in der Maximumnorm für BDF-Verfahren

Martin Ossadnik*

Wir betrachten für einen zeitunabhängigen elliptischen Operaotr zweiter Ordnung $\mathcal{M}$ und ein Gebiet $\Omega \subset \mathbb{R}$ mit Lipschitzrand eine parabolische Differentialgleichung

$$\mathcal{K}u := \partial_t u + \mathcal{M}u = f \quad \text{in } \Omega \times (0, T], \tag{1a}$$

wobei $f : [0, T] \mapsto L_2(\Omega)$. Zusätzlich haben wir eine Anfangs- sowie eine Dirichletrandbedingung gegeben:

$$u(x, 0) = u_0(x) \quad \text{für } x \in \bar{\Omega} \quad \text{und} \quad u(x, t) = 0 \quad \text{für alle } (x, t) \in \partial\Omega \times [0, T]. \tag{1b}$$

Zunächst leiten wir mittels eines allgemeinen Ansatzes berechenbare a posteriori Fehlerschätzer in der Maximumnorm für Diskretisierungen des Problems (1) her, welche auf den BDF-Verfahren der Ordnung k, $k = 1, \ldots, 5$, (BDF-k-Verfahren) in der Zeit und Finiten Elementen beliebiger Ordnung im Ort basieren. Die zentralen Bausteine der Analysis bilden die elliptische Rekonstrukion, welche von Nochetto und Makridakis eingeführt wurde [1], und L_1-Norm Abschätzungen für die mit dem parabolischen Operator assoziierte Greensche Funktion und deren Ableitung [2].

Anschließend untersuchen wir anhand von numerischen Ergebnissen die Effizienzen der Schätzer und vergleichen diese. Außerdem analysieren wir das Verhalten der verschiedenen Komponenten.

References

[1] Ch. Makridakis and R. H. Nochetto. Elliptic reconstruction and a posteriori error estimates for parabolic problems. *SIAM J. Numer. Anal.*, 41(4):1585–1594, 2003.

[2] N. Kopteva, N. and T. Linß. Maximum norm a posteriori error estimation for a time-dependent reaction-diffusion problem. *Comput. Methods Appl. Math.*, 12(2):189–205, 2012.

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Darstellungstheorie lokalkompakter Gruppen und C^*-Algebren

Moritz Proell
Universität Passau, Fakultät für Informatik und Mathematik
Professur für Angewandte Mathematik

Im Jahr 1822 hat Joseph Fourier in seinem Werk *Théorie Analytic de la Chaleur* die Wärmeleitungsgleichung mit Hilfe von Fourier-Reihen untersucht. Bei genauerer Betrachtung ist diese Methode ein Spezialfall der Dualität der lokalkompakten Gruppen $\mathbb{Z}$ und $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$. In den folgenden Jahren wurde diese Theorie von der Fourier-Analysis zur abstrakten harmonischen Analysis bis hin zur Darstellungstheorie lokalkompakter Gruppen verallgemeinert.
In den 1940er/50er Jahren hat sich gezeigt, dass Darstellungen einer lokalkompakten Gruppe G untersucht werden können, indem man Darstellungen der Gruppen-C^*-Algebra $C^*(G)$ betrachtet.
Mithilfe der linksregulären Darstellung lässt sich die reduzierte Gruppen-C^*-Algebra definieren, wobei sich damit die Mittelbarkeit einer lokalkompakten Gruppe auch darstellungstheoretisch beschreiben lässt.

Literatur: J. Dixmier, Les C^*-algèbres et leurs Représentations, Gauthier-Villars 1969

Abstract - Shortest-path recovery from signature with an optimal control approach

Marco Rauscher[1,2*], Alessandro Scagliotti[1,3] and Felipe Pagginelli[1]

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In this talk, we consider the signature-to-path reconstruction problem from the control theoretic perspective. Namely, we design an optimal control problem whose solution leads to the minimal-length path that generates a given signature. In order to do that, we minimize a cost functional consisting of two competing terms, i.e., a weighted final-time cost combined with the L^2-norm squared of the controls. Moreover, we can show that, by taking the limit to infinity of the parameter that tunes the final-time cost, the problem Γ-converges to the problem of finding a sub-Riemannian geodesic connecting two signatures. Finally, we provide an alternative reformulation of the latter problem, which is particularly suitable for the numerical implementation.

Reconstruction of Undersampled Fourier Data

Anahita Riahi

Rhein-Ruhr Workshop 2024 Abstract

Magnetic Resonance Imaging is one of the most important tools in the medical world today. From minor joint injuries to detecting cancer, there are no shortage of use cases for this formidable machine. This also means that the quality of the MRI images are of utmost importance. MRI machines take samples in the Fourier domain, which then have to be transformed to the image domain. On the one hand, we wish to take as many samples as possible in the Fourier domain to increase the quality of the images. On the other hand, this increases the time the patient needs to spend in the machine, which could cause movement and hence, inaccurate images. To improve the speed of imaging, a subset of the Fourier domain measurements needs to be taken, but reconstructing undersampled Fourier data is no trivial matter. In this presentation we observe different reconstruction methods of the undersampled Fourier data, including linear reconstruction, Generalised Auto-calibrating Partial Parallel Acquisition (GRAPPA) and the Split Bregman Algorithm and compare these methods. Furthermore, we also attempt to reconstruct the image directly in the image domain instead of the Fourier domain using transfer learning.

An Algorithm For Parallel MRI Reconstruction Using Model Based Coil Calibration (MOCCA)

Yannick Riebe
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Parallel Magnetic Resonance Imaging (MRI) based on simultaneous measurements from multiple receiver coils has been introduced to overcome the relatively slow data acquisition time and at the same time, to achieve improved high-resolution images. To achieve the wanted acceleration of the acquisition time, the goal is to reconstruct the high-resolution proton density (the image) from a subsampled amount of data, thereby exploiting the information from the parallel receiver channels. In the model we use, it is assumed that the given data for each coil is a subsample of the Fourier transform of the product of the proton density and the respective coil sensitivity function. Unfortunately, in general, the coil sensitivity functions are also not known beforehand and have to be estimated from the measured data.

In this talk, I will introduce a new MOdel-based Coil CAlibration (MOCCA) algorithm to reconstruct the coil sensitivities and the proton density from the given (incomplete) measurements. Our new method employs the assumption that the coil sensitivities are smooth functions which can be represented as bivariate trigonometric polynomials of small degree while the proton density is only assumed to be a compactly supported distribution. I will derive fast algorithms for the case of complete and incomplete data that perfectly reconstruct the proton density as well as all sensitivities for the case that they satisfy the considered model exactly. Moreover, I will show that the model fits real MRI data sufficiently well, such that it can be employed for parallel MRI reconstruction in practice.

This talk is based on a joint work with Gerlind Plonka-Hoch.

CT for Cultural Heritage

Tomas Sauer

Universität Passau

Computed Tomography can be used to digitize objects of cultural heritage. I simply show pictures of some outstanding example, no math involved, neither ideals nor the Heisenberg group.

Automated adjustment of the focussing optics of a free electron laser

Janina Schmidt

Georg-August Universität Göttingen
Project with: Prof. Dr. Gerlind Plonka-Hoch, Dr. Klaus Mann, Dr. Bernd Schäfer

FLASH is a free electron laser capable of producing femtosecond short pulses of light in the x-ray spectrum. Before the beam is used in experiments, it should be focused such that the beam profile has desirable properties. This is done by a Kirkpatrick-Baez mirror system which consists of two mirrors that can be bent and rotated. At the moment this mirror system has to be tuned by hand before each experiment, which is very time-consuming. The goal is to find a method to choose the 12 parameters of the KB optics automatically, depending on the varying properties of the incoming beam and on the experiment's requirements. This poster presents a model that approximates the properties of the beam in the experiment plane for given mirror settings. Additionally we present some results of an automated adaption of the shape of the mirrors obtained through a reinforcement learning model.

Edge Detection with Polynomial Frames on the Sphere

Frederic Schoppert
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Abstract

The classical orthonormal basis of spherical harmonics provides a powerful tool for analyzing spherical signals. However, a big disadvantage is the fact that the information is only given in terms of global quantities. For this reason, one is often interested in localized polynomial frames to obtain position based information about the frequency content of a given signal. Our results affirm the efficiency of such systems, as we show that they are able to detect jump discontinuities which lie along smooth curves. Specifically, we present upper and lower estimates for the magnitude of the corresponding frame coefficients when the analysis function is concentrated in the vicinity of such a singularity.

Learning Anisotropy for ANOVA Approximation

Pascal Schröter*

09.02.2024

This paper is concerned with learning the anisotropic smoothness of a function based on scattered data. We use this smoothness information in our approximation algorithm improving the convergence rate. In particular, we use the least squares approximation with trigonometric polynomials and frequency boxes with optimized side ratio. Here the NFFT (Nonequispaced Fast Fourier Transform) is applicable to accelerate the computation time of the approximation.

We combine these findings with the truncated ANOVA (analysis of variances) decomposition. This method makes high-dimensional problems feasible. The optimal choice of frequency boxes from above occurs here multiple times for every ANOVA term. With our approach we are able to optimize hundreds of parameters in order to gain approximation accuracy with minimal overhead. Numerical experiments indicate the applicability of our results. This talk based on a joint work with Felix Bartel.

References

[1] Felix Bartel. "Stability and error guarantees for least squares approximation with noisy samples". In: *SMAI J. Comput. Math.* 9 (2023), pp. 95–120. DOI: 10.5802/smai-jcm.96.

[2] Felix Bartel, Daniel Potts, and Michael Schmischke. "Grouped Transformations and Regularization in High-Dimensional Explainable ANOVA Approximation". In: *SIAM J. Sci. Comput.* 44.3 (2022), A1606–A1631. DOI: 10.1137/20M1374547. URL: https://doi.org/10.1137/20M1374547.

[3] Michael Schmischke. *ANOVAapprox.jl Julia Package*. Contributor: P. Schröter, F. Bartel, F. Nestler, L. Weidensager. DOI: 10.5281/zenodo.5657977.

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Lineare Approximation mit zufälligen Messungen

Mathias Sonnleitner (Universität Passau)

Mithilfe von Begriffen der Informations-basierten Komplexitätstheorie (information-based complexity) studieren wir hauptsächlich L_2-Approximation in Funktionenräumen mit Hilbertraumstruktur. Gegeben ein Typ von zulässigen Messungen, etwa Funktionsauswertungen oder Basiskoeffizienten, vergleichen wir optimale Messungen mit zufälligen (unabhängig und identisch verteilt) Messungen, die zum Generieren der Daten verwendet werden. Wir präsentieren kürzlich erzielte Resultate in einheitlicher Manier, wobei wir auf die Theorie der Zufallsmatrizen zurückgreifen. Die vorgestellten Resultate basieren auf einer Kooperation mit M. Ullrich.

Dirty Tricks for Clean Images – Heuristics for 3D CT Reconstruction

A. Michael Stock[1,*], *Tomas Sauer*[1]

[1] University of Passau

The goal is the reconstruction of 3D computed tomography (CT) volumes based on real-world X-ray radiographic measurements. We demonstrate this on an example of locally constant data that is reconstructed in the Haar wavelet basis using only the most important coefficients, while the application of heuristics improves the overall reconstruction quality. In order to achieve the goal of memory-efficient processing, we apply a multilevel reconstruction algorithm where only the relevant coefficients are kept before advancing to the next resolution level. The latter is realized by refining the a recti-linear grid of wavelet coefficients adaptively. Then, the Algebraic Reconstruction Technique (ART) is applied for reconstructing the refined regions. Iterating through the resolution levels, the grid locally approaches the maximal resolution while staying sparse.

However, real-world data introduces several challenges to the compressing reconstruction algorithm. We further employ physics-inspired heuristics for manipulating the weights of the rays, e.g., depending on the quality of the detector for a particular measurement. Furthermore, the local gradient behavior in the projection image of the considered rays may indicate the importance of a particular ray. The refinement step itself can be improved by initializing the newly added wavelet coefficients with subdivision-based predictions.

The results show that the heuristics not only improve the data fidelity compared to the original ART algorithm, but also lower the number of wavelet coefficients used, without a negative impact on the calculation time.

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