BOOK OF ABSTRACTS

RECENT DEVELOPMENTS IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Ground state properties of interacting bosonic systems (Boccato)

The interacting Bose gas is a system composed of a very large number of quantum particles with totally symmetric wavefunction. Below a critical temperature, a phase transition to a Bose-Einstein condensate is expected to occur, and collective behavior emerges from the underlying many-body theory.

At zero temperature we have precise information on the ground state energy and the low-lying spectrum of excitations (at least in certain scaling limits). However, much less is known close to the critical temperature. In this talk I will discuss how the free energy in the Gross Pitaevskii limit and in the thermodynamic limit can be obtained through a suitable description of correlations. This is a joint work with Giulia Basti, Serena Cenatiempo, Andreas Deuchert.

Studies of Variational Problems on Compact Spaces and Bogoliubov Linearization (Bru)

Building on key results from convex analysis, we present a method that allows nonlinear variational problems on convex compact spaces to be fully studied using a new linearisation process, which we call the "Bogoliubov linearization." We will discuss connections to optimal transport and game theory, as well as an application to the nonlinear thermodynamic formalism of dynamical systems.

The 2-dimensional, dilute Bose gas (Fournais)

In recent years, much progress has been made on the understanding of the energy of the interacting dilute Bose gas. These energy results also allows to prove Bose-Einstein Condensation of large length scales—depending on the density of the gas—but unfortunately not in the thermodynamic limit. In this talk, I will address the special case of the 2-dimensional Bose gas. Here a 2-term formula for the ground state energy in the thermodynamic limit has been derived in recent joint work with Girardot, Junge, Morin and Olivieri. This formula is analogous to the well-known Lee-Huang-Yang formula in 3-dimensions. I will discuss the similarities and differences between the 2 and 3-dimensional situations.

More precise effective operators for graphene (Garrigue)

The macroscopic description of graphene is in general given by a massless Dirac operator. We show how the coupling of three methods (variational approximation, perturbation theory and a multiscale approach) enables to obtain corrected operators, leading to more accurate effective models.

Correlation effects in the dilute spin-1/2 Fermi gas (Giacomelli)

In 1957, Huang and Yang predicted an asymptotic expansion for the ground state energy of a dilute Fermi gas in the thermodynamic limit, accurate up to third order. Their formula revealed remarkable universality: the correlation energy depends solely on the scattering length of the interaction, regardless of the potential's specific details. Establishing the Huang–Yang formula requires a precise analysis of the correlation energy, which is defined as the difference between the ground state energy and that of the free Fermi gas. In this talk we will outline the strategy for proving matching upper and lower bounds on the ground state energy. We will highlight the importance of considering a modified zero-energy scattering equation that takes into account the presence of the Fermi sea when describing the excitations around the Fermi sea, in the spirit of the

Bethe–Goldstone equation. Finally, we will analyze the momentum distribution of the trial state that resolves the ground state energy up to the precision of the Huang–Yang formula.

Approximating the integrated density of states for Poisson distributed random Schroedinger operators (Hasler)

We consider a Schrödinger operator with random potential distributed according to a Poisson process. We show that the integrated density of states can be approximated to arbitrary precision in powers of the coupling constant. Our results are valid for arbitrary strength of the disorder parameter, including the small disorder regime.

On (de)localization of eigenvectors in a Rosenzweig-Porter type model (Henheik)

The Rosenzweig-Porter (RP) model has recently gained a lot of attention as a paradigmatic (toy) model for studying localisation/delocalisation transitions. In this talk, we report on a joint work with G. Cipolloni and L. Erdös, where we study the eigenvectors of a very general RP model, given by a Hamiltonian $H_\lambda = H_0 + \lambda W$. Here, H_0 is a completely arbitrary Hermitian deterministic matrix, $\lambda > 0$ an arbitrary coupling constant, and W a random Wigner matrix. Our results include, in particular, a proof of a mobility edge (for certain H_0) and a version of the Eigenstate Thermalisation Hypothesis (ETH). To deduce these results on eigenvectors, we establish one- and two-resolvent local laws, which we prove by a dynamical method — the Zigzag strategy.

Eigenfunctions and Quantum Transport (Kirsch)

The time evolution of a quantum particle in initial state ψ under the Schrödinger operator $H = -\Delta + V$ is given by $\psi_t = e^{-itH}\psi$. Here Δ may be the Laplacian on $L^2(\mathbb{R}^d)$ or the discrete Laplacian on $\ell^2(\mathbb{Z}^d)$. The long time behavior of ψ_t is intimately connected to spectral properties of H. In this talk we investigate the connection between solutions of the Schrödinger equation $Hf = Ef$, f not necessarily in L^2 and the behavior of ψ_t.

More precisely, we consider the quantum transport moments

$$M_p(T)\psi \;=\; \frac{1}{T}\int_0^T \| \, \langle x \rangle^p \, \psi_t(x) \, \|^2 \, dt$$

where $\langle x \rangle = (1 + \|x\|^2)^{1/2}$.

We shall say that there is *dynamical localization* for an initial state ψ if $\sup_{\mathrm{T}} \mathrm{M_p(T)}\,\psi < \infty$ for all $p > 0$. Dynamical localization implies localization, in the sense that ψ belongs to the pure point subspace with respect to H. We also say that there is *dynamical* de*localization* for initial state ψ if $\sup_{\mathrm{T}} \mathrm{M_p(T)}\,\psi = \infty$ for some $p > 0$.

Among other results we prove:

Theorem: *If there are bounded solutions f_E to $Hf_E = Ef_E$ for $E \in \mathcal{E}$ a set of positive Lebesgue measure with $f_E(x_0) \neq 0$ for some x_0 (independent of E) then there is a bounded function φ of compact support such that*

$$M_p(T)\varphi \;\geq\; C\,T$$

for $p > d$. Consequently, there is dynamical delocalization.

The above formulation is for discrete Schrödinger operator on $\ell^2(\mathbb{Z}^d)$, there is an analogous version for the continuous case (on $L^2(\mathbb{R}^d)$).

We apply this theorem to the following two examples.

Our first example is a discrete Schrödinger operator $H = H_0 + V$ on $\ell^2(\mathbb{Z}^{d_1+d_2})$ with $d_2 \geq 1$. Suppose that the potential V vanishes outside the set

$$A = \{n \in \mathbb{Z}^{d_1+d_2} \mid n_i \in p\,\mathbb{Z} \text{ for } i = 1 \ldots d_1\}$$

for some $p \geq 2$. Then $M_p(T) \geq CT$ for $p > d_1 + d_2$ In particular, there is dynamical delocalization for some states. We emphasize that there is *no* restriction on the potential on the set A of 'active sites'.

If the potential on A is random (i.i.d. with a bounded density) it is known that the spectrum of H is pure point below $E_0 = \frac{\pi}{2p}$, in fact there is *dynamical* localization in this energy region. Consequently, there is dynamical localization in some part of the spectrum and dynamical *de*localization in an other.

Our second example is a point interaction in two or three dimensions. Suppose, we have point interactions located on the lattice $\mathbb{Z}^3$ with 'intensity' $\alpha_i, i \in \mathbb{Z}^3$. We denote the corresponding Schrödinger operator by H_α. It is known that the interval $[\frac{\pi}{2}, \infty]$ belongs to the spectrum of H_α. We prove that on this interval we have $M_p(T) \geq CT$ if $p > 3$. This is independent of the values of α. Again, if the coupling constants α_i are random, it is known that there is dynamical localization below some E_0, hence there is a phase transition.

Joint work with Peter Hislop, University of Kentucky, Lexington, USA
and Maddaly Krishna, Ashoka University, India.

On the Entropy-Uncertainty Region for Canonical Quantum States (Leschke)

The quantum Gibbs inequality is used to derive a simple one-parameter variational upper bound on the von Neumann entropy $s \geq 0$ in terms of the (co-)variance-based (and de-dimensionalized) momentum-position uncertainty u. This bound immediately quantifies the intuition that the smallest possible u should increase with increasing s. In particular, it sharpens the famous Kennard-Robertson-Schrödinger inequality $u \geq 1$, which was originally derived only for $s = 0$, that is, for pure states. By optimizing the bound one obtains the more or less well-known interdependence or constraints between the possible values of u and those of s which can be illustrated by a region in the Euclidean plane with coordinates u and s.

Momentum Distributions of Fermi Gases (Lill)

The talk concerns recent progress on the mathematical analysis of the momentum distribution of a fermionic gas in the mean-field and the dilute regime. Properties of the momentum distribution, such as the existence of a discontinuity, may reveal the presence of quasiparticles in the gas, as predicted by Landau's Fermi liquid theory. Although many physical predictions have been achieved assuming this theory, a mathematical proof of the existence of quasiparticles for a 3d Fermi gas in a thermal or ground state remains open since several decades. In our work, we establish formulas for the momentum distribution within trial states that are energetically close to the ground state, both at high and low densities. The formulas agree with predictions from the physics literature and exhibit a discontinuity as expected by Fermi liquid theory. The talk is based on joint works with N. Benedikter, E. Giacomelli, A.B. Lauritsen and D. Naidu.

Relaxation to nonequilibrium (Maes)

We describe the structure of relaxation for a steadily driven macroscopic body. The time-evolution is characterized as the zero-cost flow for a nonequilibrium and nonlinear extension of the Onsager-Machlup action governing the dynamical fluctuations. The approach hinges on two main elements: the principle of local detailed balance, which identifies the relevant thermodynamic forces, and the canonical decomposition of the frenesy into a Legendre pair. Notably, it is the time-symmetric component of the Lagrangian, the frenesy, that shapes the structure of the macroscopic evolution for given forcing. We add a simple argument for why the nonequilibrium entropy, which governs the static macroscopic fluctuations of the system, is monotone in time. The results can be interpreted as the steady nonequilibrium extension of GENERIC where relaxation to equilibrium is governed by a dissipative gradient flow superimposed on a Hamiltonian flow. Based on: Christian Maes and Karel Netočný, Relaxation to nonequilibrium. arXiv:2603.03490v2

On the Quantum AT Line in the Quantum SK Model (Manai)

The classical Sherrington–Kirkpatrick model (SK) is the paradigmatic mean-field model of spin glasses. The phase boundary between the replica-symmetric and replica symmetry breaking (RSB) phases in the presence of a longitudinal field was predicted by de Almeida and Thouless, and has long stood as a central conjecture in statistical mechanics. In this talk, we investigate the behavior of the model in the presence of a transverse field. This non-commutative quantum extension—the quantum SK model—is well motivated, and a key conjecture is that the corresponding quantum AT (QAT) line, in contrast to its classical counterpart, predicts a ground-state transition. Specifically, there exists a critical field b_c such that for $b > b_c$, the glass phase disappears at all temperatures. I will review the current state of knowledge on the QAT line, including important results due to Wolfgang Spitzer and collaborators. I will then present two joint works with Simone Warzel. First, we derive a generalization of the Parisi formula for the quantum SK model, which leads to an infinite-dimensional variational problem with a sup–inf structure, in contrast to the purely infimum-based variational principle of the classical SK model. In the second part, I introduce a simplified variant—the so-called self-overlap corrected quantum SK model. In this setting, we establish an exact QAT line that captures all essential features of the phase diagram of the full quantum SK model. Moreover, we characterize the behavior of the replica overlap and show that the phase transition is indeed a genuine glass transition.

Random Schrödinger operators on manifolds (Merz)

We consider random Schrödinger operators on compact Riemannian manifolds with Anderson-type potentials. We prove high-probability spectral inclusion bounds showing that eigenvalues remain close to those of the Laplacian, with deviations controlled by the ℓ^{d+1}-norm of the potential coefficients. This improves upon deterministic bounds, which involve the $\ell^{(d+1)/2}$-norm, and matches the scaling predicted by Euclidean models. Based on joint work with Jean-Claude Cuenin and Eduard Stefanescu.

Logarithmic and anomalous enhancements of the entanglement entropy associated with a half-filled lowest Landau level (Müller)

We consider the bipartite entanglement entropy of the ground state of a quasi-free Fermi gas in two dimensions subject to a constant magnetic field. The particle density is chosen such that the ground state corresponds to a half-filled lowest Landau level. We find that, for suitable half fillings, the entanglement entropy may feature any asymptotic growth between a logarithmically enhanced area law and a volume law. This is joint work with Leo Wetzel.

Stability of spectral gaps and the Local Topological Quantum Order property (Nachtergaele)

A characteristic feature of topological insulators is the spectral gap above their ground state(s) in the bulk. The robustness of topological phases of quantum many-body systems requires a spectral gap above the ground states that is stable under gentle perturbations of the Hamiltonian. This stability property is closely related to the so-called Local Topological Quantum Order (LTQO) property. We report a recent proof of the LTQO property for some two-dimensional AKLT models and explain the connection with spectral gap stability.

A quantum correlation inequality and its application to the Bose gas (Nam)

I will discuss a general inequality that allows for controlling the variance of a Gibbs state through first moment estimates of suitably perturbed states. This result is derived from Stahl's theorem on Laplace transforms for matrix functions and provides a helpful tool for understanding the behavior of the interacting Bose gas at the critical temperature of Bose-Einstein condensation. This is recent joint work with Andreas Deuchert and Marcin Napiórkowski, inspired by a previous result with Mathieu Lewin and Nicolas Rougerie.

Interacting many-particle systems in the random Kac-Luttinger model and proof of Bose-Einstein condensation (Pechmann)

Following a model originally considered by Kac and Luttinger, we study interacting many-particle systems in a random background. The background consists of hard spherical obstacles with fixed radius and that are distributed via a Poisson point process with constant intensity on R^d , $2 \leq d \in \mathbb{N}$. Interactions among the (bosonic) particles are described through repulsive pair potentials of mean-field type. As a main result, we prove (complete) Bose–Einstein condensation (BEC) in the thermodynamic limit and into the minimizer of a Hartree-type functional, in probability or with probability almost one depending on the strength of the interaction. As an important ingredient, we use very recent results obtained by Alain-Sol Sznitman regarding the spectral gap of the Dirichlet Laplacian in a Poissonian cloud of hard spherical obstacles in large boxes. To the best of our knowledge, our paper provides the first proof of BEC for systems of interacting particles in the Kac-Luttinger model, or in fact for some higher-dimensional interacting random continuum model. This is a joint work with Chiara Boccato and Joachim Kerner.

Eigenvalue statistics and (de)localization for randomly perturbed long-range hopping operators (Rojas-Molina)

We report on recent work on the spectral and dynamical properties of operators with (weakly) decaying long-range hopping terms with diagonal disorder or Anderson type. We show that under certain conditions on the parameters of the model, the operator exhibits a form of dynamical delocalization in a region of the spectrum where pure point spectrum and Poisson eigenvalue statistics hold. These results apply in particular to the discrete fractional Anderson model, under certain conditions on the fractional exponent and the strength of disorder. This work is in collaboration with P. Hislop (Kentucky) and R. Matos (PUC Rio).

Time-frequency analysis: eigenvalue asymptotics (Sobolev)

We study discrete spectrum of self-adjoint Weyl pseudodifferential operators with discontinuous symbols of the form $\mathbf{1}_\Omega\phi$ where $\mathbf{1}_\Omega$ is the indicator of a domain in $\Omega \subset \mathbb{R}^2$, and $\phi \in C_0^\infty(\mathbb{R}^2)$ is a real-valued function. It was known that in general, the singular values s_k of such an operator satisfy the bound $s_k = O(k^{-3/4})$, $k = 1, 2, \dots$. We show that if Ω is a polygon, the singular values decrease as $O(k^{-1}\log k)$. In the case where Ω is a sector, we obtain an asymptotic formula which confirms the sharpness of the above bound.

Joint with A. Derkach.

Ferromagnetic Ordering of Energy Levels (Starr)

With Bruno Nachtergaele and Wolfgang Spitzer, we showed that for an $\mathrm{SU}_q(2)$-symmetric spin-1/2 XXZ spin chain, there is ferromagnetic ordering of energy levels (FOEL). This means that letting $E(j)$ be the lowest energy among total-spin j multiplets we have $E(L/2) < E((L/2)-1) < \dots$. But on an even ring this type of inequality may be violated, as one sees numerically. By using the Hulthen bracket basis, popularized by Temperley and Lieb, we can give some heuristics for why the FOEL is violated on the spin ring.

Anderson localization in high-contrast media with random spherical inclusions (Täufer)

We study spectral properties of partial differential operators modelling composite materials with highly contrasting constituents, comprised of soft spherical inclusions with random radii dispersed in a stiff matrix. Such operators have recently attracted significant interest from the research community, including in the context of stochastic homogenization. In particular, it has been proved that the spectrum of these operators may feature a band-gap structure in the regime where heterogeneities take place on a sufficiently small scale. However, the nature of the limiting (as the small scale tends to zero) spectrum in the above setting is non-classical and not completely understood. We prove that Anderson localization occurs near band edges, thus shedding light on the limiting spectral behaviour. Our results rely on recent nontrivial advancements in quantitative unique continuation for PDEs, in combination with assumptions on the model that are standard in the Anderson localization literature. This is based on joint work with Matteo Capoferri.

The Feynman representation for the interacting Bose Gas (Vogel)

In this talk, we review the Feynman representation for the (interacting) Bose gas. We highlight some recent results on the short-loop/interlacement transition for interactions on cycles. We then pivot to the path-interacting case and discuss the problem posed by interacting infinite loops, with respect to the construction of Gibbs measures and large deviations.

Resolvent algebras and many-body quantum physics (Yngvason)

An approach to quantum many-body physics based on algebras of resolvents of field operators was initiated by Detlev Buchholz and Hendrik Grundling in 2008 and has been further developed in a number of papers since then. In contrast to other algebraic formulations (Weyl algebras or polynomial algebras), resolvent algebras have the crucial advantage of being invariant under a large family of time evolutions, acting as automorphisms of the algebra. Hence they form a natural arena for the definition and study of thermal equilibrium and ground states of infinite systems. In particular, the important concept of locally normal states requires an algebraic setting. In the talk a brief survey of these developments will be presented.