Aufgabe B0205 — Differentiationsregel

Geben Sie den Definitionsbereich an und bestimmen Sie die 1. Ableitung mit Hilfe der Produkt-, Quotienten- oder Kettenregel.

a) $f(x) = (x+2)\cdot(5x-7)$

b) $f(x) = (\ln(x)+3)\cdot(x-2)$

c) $f(x) = e^x \cdot (2x^2-4x+2)$

d) $f(x) = \dfrac{x^2-3x+5}{x}$

e) $f(x) = \dfrac{e^x}{x-1}$

f) $f(x) = \dfrac{x^3-5}{x^2+2}$

g) $f(x) = (x^3-4x+3)^3$

h) $f(x) = \dfrac{5}{2}\cdot(x^2-3)^4$

i) $f(x) = e^{x^3-x}$

j) $f(x) = \left(\dfrac{1}{x}-5x\right)^5$

Aufgabe B0205 (Lösungshinweise)

a) $D_f = \mathbb{R}$

$$\begin{aligned} u(x) &= x+2 & u'(x) &= 1 \\ v(x) &= 5x-7 & v'(x) &= 5 \end{aligned}$$

$$f'(x) = 1\cdot(5x-7)+(x+2)\cdot 5 = 5x-7+5x+10 = 10x+3$$

b) $D_f = \mathbb{R}_{++}$

$$\begin{aligned} u(x) &= \ln(x)+3 & u'(x) &= \frac{1}{x} \\ v(x) &= x-2 & v'(x) &= 1 \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{1}{x}\cdot(x-2)+(\ln(x)+3)\cdot 1 = 1-\frac{2}{x}+\ln(x)+3 \\ &= -\frac{2}{x}+\ln(x)+4 \end{aligned}$$

c) $D_f = \mathbb{R}$

$$\begin{aligned} u(x) &= e^x & u'(x) &= e^x \\ v(x) &= 2x^2 - 4x + 2 & v'(x) &= 4x - 4 \end{aligned}$$

$$\begin{aligned} f'(x) &= e^x \cdot (2x^2 - 4x + 2) + e^x \cdot (4x - 4) \\ &= e^x \cdot (2x^2 - 4x + 2 + 4x - 4) \\ &= e^x \cdot (2x^2 - 2) = 2e^x \cdot (x^2 - 1) \end{aligned}$$

d) $D_f = \mathbb{R}\backslash\{0\}$

$$\begin{aligned} u(x) &= x^2 - 3x + 5 & u'(x) &= 2x - 3 \\ v(x) &= x & v'(x) &= 1 \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{(2x-3) \cdot x - (x^2 - 3x + 5) \cdot 1}{x^2} = \frac{2x^2 - 3x - x^2 + 3x - 5}{x^2} \\ &= \frac{x^2 - 5}{x^2} = 1 - \frac{5}{x^2} \end{aligned}$$

e) $D_f = \mathbb{R}\backslash\{1\}$

$$\begin{aligned} u(x) &= e^x & u'(x) &= e^x \\ v(x) &= x - 1 & v'(x) &= 1 \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{e^x \cdot (x-1) - e^x \cdot 1}{(x-1)^2} = \frac{e^x \cdot (x - 1 - 1)}{(x-1)^2} \\ &= \frac{e^x \cdot (x-2)}{(x-1)^2} \end{aligned}$$

f) $D_f = \mathbb{R}$

$$\begin{aligned} u(x) &= x^3 - 5 & u'(x) &= 3x^2 \\ v(x) &= x^2 + 2 & v'(x) &= 2x \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{3x^2 \cdot (x^2 + 2) - (x^3 - 5) \cdot 2x}{(x^2+2)^2} \\ &= \frac{3x^4 + 6x^2 - (2x^4 - 10x)}{(x^2+2)^2} \\ &= \frac{x^4 + 6x^2 + 10x}{(x^2+2)^2} \end{aligned}$$

g) $D_f = \mathbb{R}$, $f(x) = u(v(x))$

$$\begin{array}{llll} u(v) = & u(v)^3 & u'(v) = & 3 \cdot v^2 \\ v(x) = & x^3 - 4x + 3 & v'(x) = & 3x^2 - 4 \end{array}$$

$$f'(x) = 3 \cdot (x^3 - 4x + 3)^2 \cdot (3x^2 - 4)$$

h) $D_f = \mathbb{R}$, $f(x) = u(v(x))$

$$\begin{array}{llll} u(v) = & \frac{5}{2} \cdot v^4 & u'(v) = & 10 \cdot v^3 \\ v(x) = & x^2 - 3 & v'(x) = & 2x \end{array}$$

$$f'(x) = 10 \cdot (x^2 - 3)^3 \cdot 2x = 20x \cdot (x^2 - 3)^3$$

i) $D_f = \mathbb{R}$, $f(x) = u(v(x))$

$$\begin{array}{llll} u(v) = & e^v & u'(v) = & e^v \\ v(x) = & x^3 - x & v'(x) = & 3x^2 - 1 \end{array}$$

$$f'(x) = e^{x^3 - x} \cdot (3x^2 - 1)$$

j) $D_f = \mathbb{R}\backslash\{0\}$, $f(x) = u(v(x))$

$$\begin{array}{llll} u(v) = & v^5 & u'(v) = & 5 \cdot v^4 \\ v(x) = & \frac{1}{x} - 5x & v'(x) = & -\frac{1}{x^2} - 5 \end{array}$$

$$\begin{aligned} f'(x) &= 5 \cdot \left(\frac{1}{x} - 5x\right)^4 \cdot \left(-\frac{1}{x^2} - 5\right) \\ &= -5 \cdot \left(\frac{1}{x} - 5x\right)^4 \cdot \left(\frac{1}{x^2} + 5\right) \end{aligned}$$