Aufgabe B0305 — Differentiation

Bestimmen Sie die partiellen Ableitungen erster und zweiter Ordnung.

a) $f(x,y) = 5x^4 + 3y^5$

b) $f(x,y) = 3x^2 \cdot e^{5y}$

c) $f(x,y) = 2e^{\frac{x}{y}}$

d) $f(x,y) = 1{,}5x^4 + 2x^2y^3 - y^3$

e) $f(x_1,x_2) = (x_1 - 2)^3 + 5x_1^2x_2^4 - (x_2^2 + 3x_1)^2$

f) $f(x_1,x_2) = 2x_1^3a - 1{,}5x_1x_2^2 + 6x_2^4b^3 \quad$ mit $a,b \in \mathbb{R}$

Aufgabe B0305 (Lösungshinweise)

a) $f(x,y) = 5x^4 + 3y^5$

$f_x(x,y) = 20x^3$ $\qquad$ $f_y(x,y) = 15y^4$

$f_{xx}(x,y) = 60x^2$ $\qquad$ $f_{yy}(x,y) = 60y^3$

$f_{xy}(x,y) = 0 = f_{yx}(x,y)$

b) $f(x,y) = 3x^2 \cdot e^{5y}$

$f_x(x,y) = 6x \cdot e^{5y}$ $\qquad$ $f_y(x,y) = 15\,x^2 \cdot e^{5y}$

$f_{xx}(x,y) = 6e^{5y}$ $\qquad$ $f_{yy}(x,y) = 75\,x^2 \cdot e^{5y}$

$f_{xy}(x,y) = 30x \cdot e^{5y} = f_{yx}(x,y)$

c) $f(x,y) = 2e^{\frac{x}{y}}$

$f_x(x,y) = \frac{2}{y}e^{\frac{x}{y}}$ $\qquad$ $f_y(x,y) = -\frac{2x}{y^2}e^{\frac{x}{y}}$

$f_{xx}(x,y) = \frac{2}{y^2}e^{\frac{x}{y}}$ $\qquad$ $f_{yy}(x,y) = \frac{4x}{y^3}e^{\frac{x}{y}} + \frac{2x^2}{y^4}e^{\frac{x}{y}} = 2xe^{\frac{x}{y}} \cdot (\frac{2}{y^3} + \frac{x}{y^4})$

$$f_{xy}(x,y) = -\frac{2e^{\frac{x}{y}}}{y^2} - \frac{2xe^{\frac{x}{y}}}{y^3} = -2e^{\frac{x}{y}} \cdot (\tfrac{1}{y^2} + \tfrac{x}{y^3}) = f_{yx}(x,y)$$

d) $f(x,y) = 1{,}5x^4 + 2x^2y^3 - y^3$

$f_x(x,y) = 6x^3 + 4xy^3$ $\qquad$ $f_y(x,y) = 6x^2y^2 - 3y^2$

$f_{xx}(x,y) = 18x^2 + 4y^3$ $\qquad$ $f_{yy}(x,y) = 12x^2y - 6y$

$f_{xy}(x,y) = 12xy^2 = f_{yx}(x,y)$

e) $f(x_1,x_2) = (x_1 - 2)^3 + 5x_1^2 x_2^4 - (x_2^2 + 3x_1)^2$

$f_{x_1}(x_1,x_2) = 3(x_1 - 2)^2 + 10x_1 x_2^4 - 6(x_2^2 + 3x_1)$ $\quad f_{x_2}(x_1,x_2) = 20x_1^2 x_2^3 - 4x_2(x_2^2 + 3x_1)$

$f_{x_1x_1}(x_1,x_2) = 6(x_1 - 2) + 10x_2^4 - 18$ $\quad f_{x_2x_2}(x_1,x_2) = 60x_1^2 x_2^2 - 12(x_2^2 + x_1)$

$f_{x_1x_2}(x_1,x_2) = 40x_1 x_2^3 - 12x_2 = f_{x_2x_1}(x_1,x_2)$

Hinweis: Zum Ableiten müssen Ketten- und Produktregel angewendet werden!

f) $f(x_1,x_2) = 2x_1^3 a - 1{,}5x_1 x_2^2 + 6x_2^4 b^3$

$f_{x_1}(x_1,x_2) = 6x_1^2 a - 1{,}5x_2^2$ $\quad f_{x_2}(x_1,x_2) = -3x_1 x_2 + 24x_2^3 b^3$

$f_{x_1x_1}(x_1,x_2) = 12x_1 a$ $\quad f_{x_2x_2}(x_1,x_2) = -3x_1 + 72x_2^2 b^3$

$f_{x_1x_2}(x_1,x_2) = -3x_2 = f_{x_2x_1}(x_1,x_2)$