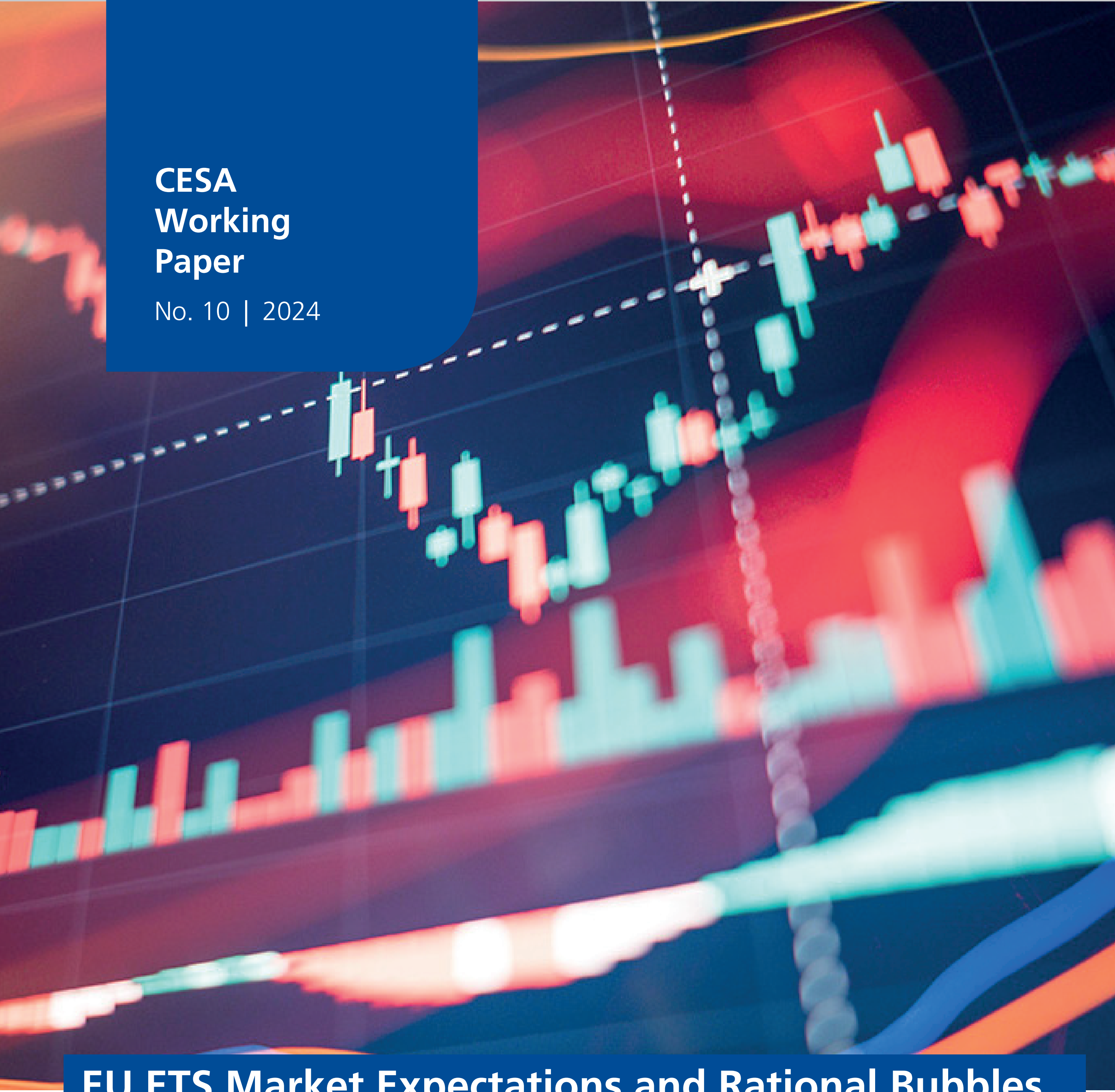

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Serious concerns about the existence of a price bubble within the European Union Emissions Trading System (EU ETS) emerged during its third trading period. Existing bubble tests based on costs for switching from cheap, polluting to costly, clean energy sources are restricted to situations of market certainty. This limitation is unrealistic, considering the ongoing CO_2 reduction measures. Additionally, the fundamental value is not uniquely identified, leading to inconclusive empirical findings. We apply a robust approach to infer bubbles in the EU ETS. Our findings do not support the presence of a bubble in the third or fourth trading period.

Key words: allowance pricing • bubbles • cap-and-trade • EU ETS

JEL classification: C15 • G14 • Q48 • Q56

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1. Introduction

The European Union (EU) introduced the European Emissions Trading System (EU ETS) in 2005 as a key climate protection instrument. Overall, the EU ETS covers around 40% of total greenhouse gas emissions within the European Union. This renders the EU ETS the most comprehensive cap-and-trade system in the world. The participating states issue emission allowances partly free of charge, partly through auctions. A single allowance permits the emission of one ton of CO_2 equivalents. Annually, firms subject to reporting obligations are required to submit an emissions report for the previous year and they are obligated to prove ownership of the corresponding number of allowances by the compliance date.[1] Market participants have the option of banking allowances, given that the certificates maintain their validity not only within the compliance period but also extending into a subsequent trading period and beyond. The right to freely trade emission allowances establishes market prices for greenhouse gas emissions in both the spot and futures market. A cap determines the total amount of greenhouse gas emissions allowed per trading period.

Insert Figure 1 here.

Since 2018, there have been rapid and substantial price increases for the market price of emission allowances (see Figure 1). In this respect, several studies have explored the role of rational bubbles in the EU ETS. For the latest research supporting the hypothesis that rational bubbles are driving these price surges, see the studies of Friedrich, Fries, Pahle, and Edenhofer (2020), Wei, Li, and Wang (2022), Huang and Wang (2024), and Terranova, Cozzarini, Reissl, and Tavoni (2024). If the price hikes are indeed caused by a rational bubble, the incentive created by the EU ETS for cost-effective emission reductions risks becoming ineffective. In this case, it would be advisable to redesign the EU ETS to minimize the likelihood of (rational) bubbles in future trading periods. Conversely, if the price increases are due to other factors, such as the anticipation of a future shortage of certificates, interventions in the design of the EU ETS would be unnecessary and might

[1] In accordance with Calel and Dechezleprêtre (2016), we use the term "firms subject to reporting obligations" to refer to companies that operate at least one installation regulated by the EU ETS.

undermine the efficiency of the trading system. Therefore, the primary objective of this paper is to provide an empirical analysis to determine whether the price increases during the third and fourth trading periods can genuinely be attributed to a rational bubble.

Under the conditions that fuel-switching, i.e., investing in transitioning the production process from environmentally detrimental energy sources to environmentally friendly alternatives, is a perfect substitute for buying emission permits, producers will shift their production from cheaper but environmentally harmful energy sources to more expensive but cleaner alternatives. This transition will continue as long as the cost of reducing CO_2 emissions is lower than the price of allowances. In a scenario of market equilibrium without a rational bubble, the marginal abatement costs – i.e., the price within the EU ETS – will on average align with the switching costs associated with transitioning from cheaper, polluting energy sources to pricier, cleaner alternatives, see Montgomery (1972), Rubin (1996) and Kling and Rubin (1997). This forms the rationale for the folk principle considering switching costs as the fundamental in the EU ETS.

The interpretation of rational bubble tests based on switching costs assumes that purchasing emission allowances is a perfect substitute for abatement solutions. This assumption does not hold if market actors make decisions under uncertainty. For example, there may be uncertainty about allowance price determinants, market demand for products and services provided by CO_2 emitting firms, or policy uncertainty (see Zhao 2003, Chesney and Taschini 2012, Taschini 2021). Additionally, the assumption fails if transaction costs, such as informational and contractual costs, have an impact (see Baudry, Faure, and Quemin 2021). Therefore, the empirical evidence suggesting a rational bubble in the EU ETS, derived from tests based on the fundamental, i.e., switching costs, could be due to either the presence of a rational bubble in the EU ETS or the misspecification of the empirical proxy used for the fundamental (see, among others, Phillips, Wu, and Yu 2011, Phillips and Yu 2011, Phillips, Shi, and Yu 2015a,b, Harvey, Leybourne, Sollis, and Taylor 2016, for tests against rational bubbles based on fundamentals).

We address this issue by employing a bubble testing approach that bypasses the need to specify the fundamental. Instead, the employed method in this paper relies on market expectations, following the approach of Pavlidis, Paya, and Peel (2017) and Pavlidis, Paya, and Peel (2018), utilizing futures prices as a proxy. Although the approach by Pavlidis, Paya, and Peel (2017, 2018) eliminates the need to specify the fundamental, it assumes risk neutral market participants. However, purchasers (sellers) of CO_2 allowances can hedge against anticipated price increases (decreases) on the futures market. Hence, it stands to reason that hedging pressure will arise, particularly in the event of strong price dynamics (see, among others, Bessembinder 1992, Bessembinder and Chan 1992, De Roon, Nijman, and Veld 2000, Dewally, Ederington, and Fernando 2013, for empirical evidence supporting the *Hedging Pressure Hypothesis* on futures markets).

We first demonstrate that an explosive price process has the potential to render the dollar risk premium explosive, even if the risk premium expressed in percentage terms remains stationary. This dynamic could lead to the erroneous conclusion that a rational bubble underlies the observed price surge employing tests based on market expectations. However, we further show that a slight modification of the methodology proposed by Pavlidis, Paya, and Peel (2017) is (asymptotically) robust to the presence of an explosive risk premium, provided that the underlying fundamental process has a unit root (is mildly explosive). Mildly explosive processes have been demonstrated to effectively capture the features of moderately explosive behavior observed in various economic and financial time series. It is worth noting that the assumption of the fundamental process being a random walk is common in the bubble testing literature (see, for instance, Diba and Grossman 1988a,b). In this context, allowing for mild explosiveness in the fundamental process represents a relaxation of this assumption. Such an extension is particularly relevant in the case of the EU ETS, where explosive price behavior has been documented (see, for example, Friedrich, Fries, Pahle, and Edenhofer 2020).

Utilizing the a slight modification of the framework of Pavlidis, Paya, and Peel (2017), under the assumption that the fundamental process is either integrated of order one or mildly explosive, we adopt an agnostic stance regarding the true behavior of the fundamental process and the risk premium's trend. Consequently, our approach accommodates the possibility of an explosive risk premium, a phenomenon that may hold substantial economic relevance given the rapidly growing price expectations observed in the EU ETS during the period under study. In alignment with Pavlidis, Paya, and Peel (2017), we propose a testing procedure that builds upon the predictive regression framework introduced by Fama (1984), denoted as Fama Predictive Regressions (FPRs). Specifically, we derive the slope coefficients for two predictive regressions under the null hypothesis of no rational bubble, while explicitly accounting for the presence of a (explosively trending) risk premium.

Furthermore, consistent with Pavlidis, Paya, and Peel (2017, 2018), we employ an endogenous instrumental variable-based (IVX) method (see Phillips and Magdalinos 2009, Kostakis, Magdalinos, and Stamatogiannis 2015, Yang, Long, Peng, and Cai 2020) to test within the FPRs. This approach enhances the robustness in the presence of a mildly explosive price process. Hence, estimating FPRs using the IVX methodology enables robust inference on the presence of a rational bubble, even when the fundamental process is unobserved and the risk premium exhibits explosive behavior.

Moreover, to examine whether the third trading phase (2013-2020) and the fourth trading phase (2021-ongoing) of the EU ETS have been free of rational bubbles (so far), we contribute with the following exercises to the empirical literature on carbon trading:

(i) We first investigate whether the EU ETS contains explosive episodes during the third and fourth trading period which is a necessary condition for a rational bubble in the EU ETS during this period. At this juncture, it seems acceptable to make a slight anticipation: We find pronounced explosiveness in EU ETS prices since 2018.

(ii) We estimate the predictive regression proposed by Fama (1984) employing the IVX technique by Kostakis, Magdalinos, and Stamatogiannis (2015) and using the IVX-AR approach by Yang, Long, Peng, and Cai (2020). This allows us to test against a rational bubble in the EU ETS in the presence of an (explosively) trending risk premium.

The structure of the paper is as follows: The next section examines pricing equations within the EU ETS, with a particular emphasis on switching costs and market expectations. Section 3 highlights the economic significance of an explosively trending risk premium and outlines the econometric testing procedure. Section 4 provides a comprehensive description of the data and presents the empirical findings. Finally, Section 5 concludes the paper by discussing policy implications and proposing potential directions for future research.

2. Pricing Equations for Emission Trading Systems

This section presents two fundamentally different approaches to price allowances in emissions trading systems: One is based on switching costs and the other on market expectations. The first approach relies on the implicit assumption that emission allowances are perfect substitutes for any technological abatement solution. Under this assumption, the ETS price should be determined by switching costs towards more CO_2-efficient energy sources (see Montgomery 1972, Rubin 1996, Kling and Rubin 1997, Carmona, Fehr, and Hinz 2009).

However, if market participants act under uncertainty (are faced with transaction costs), the assumption that allowances and fuel-switching are perfect substitutes is violated. Considering that investments in fuel-switching technologies often come with high costs, long-term durability, and irreversible investment requirements, they are typically not viewed as a perfect substitute to emission permits (see Chesney and Taschini 2012, Taschini 2021). Hence, the current spot price might reflect the expectation of scarcity of future emission allowances rather than current switching costs.

This section is structured in the following way: Initially, we explore an empirical modeling approach centered on switching costs, demonstrating that its foundational assumptions are overly restrictive and limit its reliability for empirical analysis. Subsequently, we shift to a more viable approach focused on market expectations.

2.1. Pricing with Switching Costs

A rational bubble can be characterized as a situation where the price of an asset becomes disconnected from its underlying fundamental value. Within a rational expectations framework, rational bubbles emerge solely from the expectations of market actors regarding future price increases (see Flood and Hodrick 1990). Since rational price expectations are positive, the price of the asset surges beyond its intrinsic value during a rational bubble (see Tirole 1985, Diba and Grossman 1988a,b).[2]

We denote the price in the ETS as P_t, viz.

$$P_t = U_t + B_t \qquad \text{with} \qquad t = 1, 2, \ldots, T \tag{1}$$

where $U_t \geq 0$ is the fundamental value and $B_t \geq 0$ is the bubble component. The decomposition is orthogonal and hence, U_t and B_t are uncorrelated components of P_t. Blanchard (1979) suggests to model the bubble component as

$$B_t = \begin{cases} \frac{1+\rho}{\pi} \times B_{t-1} + \epsilon_t & \text{with probability } \pi, \\ \epsilon_t & \text{with probability } 1 - \pi, \end{cases} \tag{2}$$

where $\rho > 0$ denotes the risk-free rate and ϵ_t is an independent and identically distributed (i.i.d.) random variable with zero mean and variance σ_ϵ^2, i.e., $\epsilon_t \sim \text{i.i.d.}\,(0, \sigma_\epsilon^2)$. We relax the i.i.d. assumption to allow for a general linear process, e.g., a stationary and invertible AutoRegressive Moving Average (ARMA) process. The bubble survives with probability π in period t. In this case, the bubble expands at an increased rate of $(1+\rho)/\pi$ to compensate investors for the potential bubble collapse. The bubble bursts with probability $1-\pi$ to a general linear process. We denote the conditional expectation given the information set $\mathcal{F}_t$ available at time t by $\mathbb{E}_t := \mathbb{E}\,[\cdot \mid \mathcal{F}_t]$. Since the bubble is a sub-martingale process, i.e.,

$$\mathbb{E}_t\,[B_{t+1}] - (1+\rho)\,B_t = 0, \tag{3}$$

the bubble component of the price process B_t is explosive and expecting that $B_{t+1} > B_t$ is rational, hence the term *rational bubble* (see Diba and Grossman 1988a). These characteristics are also fulfilled, for example, by the periodically collapsing bubble process according to Evans (1991).

[2] In line with the prevailing body of empirical research, our focus centers on rational extrinsic bubbles, as these bubbles correspond with explosive price trends. This entails that the decomposition of the price into fundamental and bubble components is orthogonal.

A time series with no deterministic component is said to be integrated of order d, denoted as $\mathrm{I}(d)$, if differencing the series d times results in a time series that has a stationary and invertible ARMA representation (see Engle and Granger 1987). No finite number of differencing of an explosive process has a stationary and invertible ARMA presentation, viz. an explosive process is $\mathrm{I}(\infty)$ and therefore, the bubble component is integrated of order infinity, i.e., $B_t \sim \mathrm{I}(\infty)$, see Diba and Grossman (1988a). We use the notion of integration and the $\mathrm{I}(d)$-notation in the following to analyze the empirical implications for the ETS price given the presence or absence of a rational bubble, respectively.

Next, we turn to modeling the fundamental U_t: Assuming that emission allowances can perfectly substitute fuel-switching – meaning a transition from cheap but polluting to expensive but clean energy sources – producers will adjust their production processes. This adjustment will occur as long as the marginal cost of avoiding CO_2 emissions does not surpass the price of allowances (see Montgomery 1972, Rubin 1996, Kling and Rubin 1997). Hence, in market equilibrium and assuming the absence of a rational price bubble, the price within the ETS aligns with the switching costs towards CO_2-efficient energy sources. Expanding upon the illustration provided in Carmona, Fehr, and Hinz (2009) regarding the transition from coal to gas for power generation (power generation is often seen as indicative for the whole industrial sector because it reflects the central short-term abatement measure within the EU ETS), the switching costs in the EU ETS are given as

$$S_t = \frac{\eta_{\text{gas}} \times P_t^{(\text{gas})} - \eta_{\text{coal}} \times P_t^{(\text{coal})}}{E_{\text{coal}} - E_{\text{gas}}}. \tag{4}$$

Here, $P_t^{(\text{gas})}$ and $P_t^{(\text{coal})}$ denote the prices of natural gas and coal at time t, while E_{gas} and E_{coal} represent their respective constant average CO_2 emissions. Similarly, η_{gas} and η_{coal} signify the heat input coefficients associated with gas and coal. The precise numerical values for these constants can be found, for instance, in Carmona, Fehr, and Hinz (2009).

Hence, the empirical pricing equation of emission allowances with $B_t = 0$ based on switching costs as the fundamental value reads as

$$P_t = U_t = \frac{\eta_{\text{gas}}}{E_{\text{coal}} - E_{\text{gas}}} \times P_t^{(\text{gas})} - \frac{\eta_{\text{coal}}}{E_{\text{coal}} - E_{\text{gas}}} \times P_t^{(\text{coal})} + \varepsilon_t, \tag{5}$$

where $P_t^{(\mathrm{gas})}$ and $P_t^{(\mathrm{coal})}$ are time series that are assumed to be integrated of order one, respectively. Further, let ε_t be a zero-mean innovation term integrated of order zero, i.e., $\varepsilon_t \sim \mathrm{I}(0)$. Thus, ε_t might follow a stationary and invertible ARMA process.

Time series are said to be co-integrated (co-explosive) when they share a common (explosive) stochastic trend, meaning that there is a linear combination of these time series with a lower degree of integration than the underlying variables. Under the above assumptions about the underlying time series, the switching costs and the spot prices in the ETS are co-integrated (co-explosive) with the testable co-integration (co-explosiveness) vector

$$(1, \psi_{\mathrm{gas}}, \psi_{\mathrm{coal}})' := \left(1, -\frac{\eta_{\mathrm{gas}}}{E_{\mathrm{coal}} - E_{\mathrm{gas}}}, \frac{\eta_{\mathrm{coal}}}{E_{\mathrm{coal}} - E_{\mathrm{gas}}}\right)', \tag{6}$$

since $P_t \sim \mathrm{I}(1)$ $(P_t \sim \mathrm{I}(\infty))$, $P_t^{(\mathrm{gas})} \sim \mathrm{I}(1)$ $\left(P_t^{(\mathrm{gas})} \sim \mathrm{I}(\infty)\right)$, $P_t^{(\mathrm{coal})} \sim \mathrm{I}(1)$ $\left(P_t^{(\mathrm{coal})} \sim \mathrm{I}(\infty)\right)$ and $P_t + \psi_{\mathrm{gas}} \times P_t^{(\mathrm{gas})} + \psi_{\mathrm{coal}} \times P_t^{(\mathrm{coal})} = \varepsilon_t \sim \mathrm{I}(0)$. This leads to the following pricing equation.[3]

Pricing Equation 1 *With a potentially non-zero bubble component, it follows that the spot price process in the emission trading system, if emission certificates and switching costs are perfect substitutes, is given by*

$$\underbrace{P_t}_{\mathrm{I}(1)/\mathrm{I}(\infty)} = \underbrace{B_t}_{\mathrm{I}(\infty)} - \psi_{gas} \times \underbrace{P_t^{(gas)}}_{\mathrm{I}(1)/\mathrm{I}(\infty)} - \psi_{coal} \times \underbrace{P_t^{(coal)}}_{\mathrm{I}(1)/\mathrm{I}(\infty)} + \underbrace{\varepsilon_t}_{\mathrm{I}(0)}.$$

It becomes evident that when $B_t > 0$, the price dynamics P_t must exhibit explosiveness, and when $B_t = 0$, the price dynamics could potentially be explosive, contingent upon an explosive coal and/or gas price. Therefore, the presence of explosiveness in the price P_t alone does not provide adequate grounds to ascertain the existence of a rational bubble. Yet, when $B_t = 0$, the differential between the price and the switching costs is integrated to an order of zero, i.e., $P_t - S_t = \varepsilon_t \sim \mathrm{I}(0)$, since P_t and S_t are co-integrated or co-explosive, respectively. In contrast, when $B_t > 0$, the differential exhibits explosiveness, viz. $P_t - S_t = B_t + \varepsilon_t \sim \mathrm{I}(\infty)$. To examine the presence of a rational bubble

[3] There is the possibility of co-integration or co-explosiveness between gas and coal prices. Nevertheless, since these instances are not pertinent to the subsequent analysis, they will not be elaborated upon here.

within the ETS, one approach is to employ a right-sided unit root test on the differential between the ETS price and the switching costs. Empirical evidence in favor of the alternative, i.e., that $P_t - S_t$ is explosive, could argue for the existence of a rational bubble. However, rejecting the null hypothesis could stem from a misspecification of the fundamental, leading to inconclusive outcomes. For example, Creti and Joëts (2017) determine the fundamental of the EU ETS as $0.520 \times P_t^{(\mathrm{gas})} + 0.632 \times P_t^{(\mathrm{oil})} + 0.514 \times S_t - 0.260 \times P_t^{(\mathrm{stock})}$ by principle component analysis, whereas $P_t^{(\mathrm{oil})}$ denotes the price for crude oil and $P_t^{(\mathrm{stock})}$ is the value of an appropriate stock market index at time t. Specifying the fundamental in this way is in marked contrast to the specification based solely on switching costs from coal to gas as in Equation (5), highlighting the different interpretations of the *true* underlying fundamental in the ETS and the possibility of misspecification.

Consequently, discovering a method to identify a rational bubble without the need to explicitly define a fundamental is essential, particularly in the context of an ETS. It becomes even more crucial considering an additional limitation of the switching costs approach which assumes that investments in fuel-switching technologies and the acquisition of emission allowances are perfect substitutes. In situations where market participants are faced with transaction costs or if market actors anticipate alterations in future switching costs, varying penalties for non-compliance, or a potential adjustment in the quantity of certificates available, the perfect substitutes assumption might be violated and the validity of Pricing Equation 1 would be compromised (see Zhao 2003, Chesney and Taschini 2012, Taschini 2021, Baudry, Faure, and Quemin 2021). Especially due to dynamic political measures to reduce CO_2 emissions, the assumption that market actors decide under certainty appears unrealistic (see Pahle, Günther, Osorio, and Quemin 2023). The next section provides an approach which does not require the assumption that fuel-switching and buying certificates are perfect substitutes.[4]

[4] Given that the assumptions of an empirical study based on the switching costs approach seem too restrictive, we avoid analyzing Pricing Equation 1 using cointegration or coexplosivity methods in the subsequent empirical analysis. However, note that there are several studies that analyze a potential co-integration equilibrium between switching

2.2. Pricing with Market Expectations

Building on the findings by Hitzemann and Uhrig-Homburg (2018), this section introduces a pricing equation grounded in market expectations to analyze rational bubbles in the ETS. We adopt a perspective that incorporates banking, ensuring a realistic alignment with the context of the EU ETS. Banking in the context of this paper means that market actors are allowed to use certificates beyond the current compliance date $\mathcal{T}$ and they exercise this option when they anticipate an increase in allowance prices. In order to fix ideas, consider an ETS where banking is not allowed, and a forward contract is established at time t with a delivery date of $\mathcal{T}$: If the delivery date coincides with the compliance date $\mathcal{T}$, the speculative bubble component is removed from the forward price, since the delivered certificate can only be used for compliance purposes and not for speculation. Moreover, if it is possible to bank, speculation can continue beyond $\mathcal{T}$, and the bubble component does not necessarily disappear from the price expectations. Therefore, inferring a bubble based on market expectations appears challenging if banking is permitted. However, as noted by Pavlidis, Paya, and Peel (2017, 2018), under rational bubbles and in the absence of significant trading frictions, i.e., with banking allowed, the weighting of the bubble component in price expectations differs from its weighting in the actual spot price. As the fundamental components are weighted equally in the equations for prices and price expectations, explosiveness in the differential between price expectations and the actual prices indicates conclusively a rational bubble.

Specifically, Hitzemann and Uhrig-Homburg (2018) propose a stochastic equilibrium model that takes banking directly into account. The authors demonstrate that the equilibrium permit price can be represented as a sequence of European binary call options written on aggregate emissions across the entire economy. Therefore, assuming discrete time, the fundamental reads as

$$U_t = \sum_{p=k}^{N_{\mathcal{T}}} (1+\rho)^{-\left(\mathcal{T}_p - t\right)} \times \mathbb{E}_t\left[\varpi_{\mathcal{T}_p} \times \left(1 - \mathcal{P}_{\mathcal{T}_p}\right)\right] = \sum_{p=k}^{N_{\mathcal{T}}} (1+\rho)^{-\left(\mathcal{T}_p - t\right)} \times \mathbb{E}_t\left[U_{\mathcal{T}_p}\right], \tag{7}$$

costs and emission prices (see Creti, Jouvet, and Mignon 2012, Koch, Fuss, Grosjean, and Edenhofer 2014, Rickels, Görlich, and Peterson 2015). Further, see Hintermann, Peterson, and Rickels (2016) for a comprehensive review on the empirical literature about allowance price dynamics during phase II.

whereas $t \in (\mathcal{T}_{k-1}, \mathcal{T}_k]$, $\rho > 0$ is the risk-free rate, $N_{\mathcal{T}}$ denotes the number of compliance periods of the ETS, $\varpi_{\mathcal{T}_p}$ is the penalty for non-compliance and $1 - \mathcal{P}_{\mathcal{T}_p}$ represents the likelihood of economy-wide cumulative emissions surpassing the economy-wide permit holdings in trading period $\mathcal{T}_p$. It is important to emphasize that the standard cost-of-carry no-arbitrage condition remains intact here, as the futures price still includes the rational bubble component if banking is allowed. The orthogonal relationship between the fundamental and rational bubble components yields the following pricing equation.

Pricing Equation 2 *If banking is allowed in the emission trading system, the spot price in the ETS is determined as*

$$P_t = \sum_{p=k}^{N_{\mathcal{T}}} (1+\rho)^{-\left(\mathcal{T}_p - t\right)} \times \mathbb{E}_t\left[U_{\mathcal{T}_p}\right] + B_t.$$

Hence, the price with a bubble component at time $t+n$ is given by

$$\begin{aligned} P_{t+n} = U_{t+n} + B_{t+n} &= \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_{t+n}\left[U_{\mathcal{T}_p}\right] \right) + B_{t+n} \\ &= \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_{t+n}\left[U_{\mathcal{T}_p}\right] \right) + \left(\frac{1+\rho}{\pi}\right)^n B_t + \epsilon_{t+n} \end{aligned} \tag{8}$$

with $(t+n) \in (\mathcal{T}_{k-1}, \mathcal{T}_k]$. For the price expectations at time t about time $t+n$, we receive

$$\begin{aligned} \mathbb{E}_t\left[P_{t+n}\right] &= \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_t\left[U_{\mathcal{T}_p}\right] \right) + \mathbb{E}_t\left[B_{t+n}\right] \\ &= \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_t\left[U_{\mathcal{T}_p}\right] \right) + (1+\rho)^n \times B_t \end{aligned} \tag{9}$$

with $(t+n) \in (\mathcal{T}_{k-1}, \mathcal{T}_k]$. The differential between actual and expected ETS prices is given by

$$D_{t+n} := P_{t+n} - \mathbb{E}_t\left[P_{t+n}\right] = \vartheta_{t+n} + (1+\rho)^n \left(\frac{1}{\pi^n} - 1\right) \times B_t + \epsilon_{t+n}, \tag{10}$$

whereas the prediction error of the fundamental component reads as

$$\vartheta_{t+n} = \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_{t+n}\left[U_{\mathcal{T}_p}\right] \right) - \sum_{p=k}^{N_{\mathcal{T}}} \left((1+\rho)^{-\left(\mathcal{T}_p - t - n\right)} \times \mathbb{E}_t\left[U_{\mathcal{T}_p}\right] \right) \tag{11}$$

and ϵ_{t+n} is the prediction error for the bubble component. Note that ϑ_{t+n} when $B_t = 0$ and $\vartheta_{t+n} + \epsilon_{t+n}$ when $B_t > 0$ correspond to the prediction which minimizes the mean-squared prediction error (MSPE). This is because, by definition, conditional expectation minimizes the MSPE under quadratic loss, see Granger (1969) and Baumeister (2023).

With $\vartheta_{t+n} \sim \mathrm{I}(0)$ and $\vartheta_{t+n} + \epsilon_{t+n} \sim \mathrm{I}(0)$, respectively, we receive for $B_t = 0$,

$$\underbrace{D_{t+n}}_{\mathrm{I}(0)} = \underbrace{\vartheta_{t+n}}_{\mathrm{I}(0)} \tag{12}$$

and for $B_t > 0$, we obtain

$$\underbrace{D_{t+n}}_{\mathrm{I}(\infty)} = \underbrace{\vartheta_{t+n}}_{\mathrm{I}(0)} + (1+\rho)^n \left(\frac{1}{\pi^n} - 1\right) \times \underbrace{B_t}_{\mathrm{I}(\infty)} + \underbrace{\epsilon_{t+n}}_{\mathrm{I}(0)}. \tag{13}$$

Hence, the differential D_{t+n} does not depend on the market fundamental, see Pavlidis, Paya, and Peel (2017, 2018). This implies that the explosive dynamics of the differential between spot (see Equation 8) and expected spot prices (see Equation 9) are due solely to the presence of a rational bubble. As a result, one might consider using a right-sided unit root test to examine the potential explosiveness in D_{t+n}. However, it is important to notice that $\mathbb{E}_t\left[P_{t+n}\right]$ is not directly observable. Consequently, market expectations must be approximated using futures prices. This approach introduces the complication that the risk premium – defined as the difference between futures prices and spot price expectations – can distort the results due to its own explosively trending behavior in the absence of a rational bubble.[5]

3. Testing against Rational Bubbles in the Presence of a Risk Premium

The predominant empirical method for evaluating rational bubbles has been introduced by Phillips and Yu (2011) and Phillips, Shi, and Yu (2015a,b). Their proposed approach involves testing against explosive behavior in the differential between fundamental and price series. Moreover, these scholars present robust right-sided unit root tests, aiming to overcome concerns raised by Evans

[5] It is important to note that Hamilton and Wu (2014) propose a method for estimating market expectations. However, this approach inherently involves estimation errors which can propagate to right-sided unit root tests used for rational bubble inference, potentially leading to significant size distortions. To mitigate this issue, we refrain from estimating market expectations in the initial stage and subsequently using these estimates for bubble testing in the second stage.

(1991) regarding the limited power of standard unit root and co-integration tests in detecting periodically collapsing bubbles. One limitation of employing explosiveness testing in the differential between fundamental and price series to assess rational bubbles is its dependence on a proxy for the fundamental process. Although this concern might be manageable in the realm of stocks, where the dividend process is observable and can act as a substitute for the fundamental, the preceding chapter underscored the challenges associated with defining the fundamental in the context of an ETS.

Pavlidis, Paya, and Peel (2017, 2018) address this challenge by investigating market expectations instead of opting for a proxy for the fundamental. Consequently, Pavlidis, Paya, and Peel (2017) utilize the continuous futures price at time t with a delivery date of $t+n$, represented as $F_{n,t}$, as a proxy of market expectations. However, this approach comes with a limitation, as the risk premium which accounts for the difference between the futures price and the market expectation, remains unobservable. The method outlined below avoids the need for observing market expectations directly and does not impose restrictive assumptions on the trend behavior of the differential between market expectations and its proxy, i.e., the futures price.

3.1. The Role of the Risk Premium

As stated previously in Section 2.2, Equation (10) predicts a stationary equilibrium between actual prices and price expectations in the absence of a rational bubble. Should the researcher opt for assuming a stationary risk premium, denoted as $RP_{n,t} \sim \mathrm{I}(0)$ where

$$RP_{n,t} := F_{n,t} - \mathbb{E}_t \left[P_{t+n} \right], \tag{14}$$

she can apply a stationarity test to

$$P_{t+n} - F_{n,t} = \vartheta_{t+n} + (1+\rho)^n \left(\frac{1}{\pi^n} - 1 \right) \times B_t - RP_{n,t} + \epsilon_{t+n} \tag{15}$$

to test against the presence of a rational bubble. However, rejecting the null hypothesis of stationarity for $P_{t+n} - F_{n,t}$ could result from either the presence of a bubble price component or a violation of the assumption of a stationary risk premium. Furthermore, if the researcher assumes a risk premium integrated of order one, i.e., $RP_{n,t} \sim \mathrm{I}(1)$, a right-sided unit root test may be applied. In this context, rejecting the null hypothesis of no rational bubble could indicate either the existence of a rational bubble or a violation of the assumed risk premium dynamics, specifically if the risk premium is not integrated of order one but instead exhibits explosive behavior, specifically $RP_{n,t} \sim \mathrm{I}(\infty)$.

3.1.1. Is an explosive risk premium a phenomenon that holds economic relevance?

To answer this question, we define the risk premium as in Equation (14), a negative (positive) risk premium indicates *normal backwardation* (*normal contango*) in the futures market, i.e., $F_{n,t} < \mathbb{E}_t[P_{t+n}]$ $(F_{n,t} > \mathbb{E}_t[P_{t+n}])$. Originally proposed by Keynes (1930) and Hicks (1939), the notion of normal backwardation posits that hedgers typically maintain short positions as an insurance against the market price risk. The *Insurance Hypothesis* implies that the futures price should be lower than the expected future spot price, acting as compensation to the speculator for furnishing insurance to the producer. More generally, the *Hedging Pressure Hypothesis* states that, in normal backwardation (normal contango), the risk premium is driven by sellers (buyers) engaged in future contracts to insure against anticipated price decreases (increases), see, among others, Bessembinder (1992), Bessembinder and Chan (1992), De Roon, Nijman, and Veld (2000), and Dewally, Ederington, and Fernando (2013) for empirical support in favor of the Hedging Pressure Hypothesis.

In the context of an ETS, firms regulated and holding surplus emission certificates may choose to sacrifice a premium to shift the price risk to the long position, viz. they are willing to receive the certainty equivalent $\mathbb{E}_t[\mathcal{U}(P_{t+n})]$ whereas $\mathbb{E}_t[\mathcal{U}(P_{t+n})] < \mathbb{E}_t[P_{t+n}]$. In this case $\mathcal{U}(\cdot)$ denotes the utility function of a representative agent holding the short position. Conversely, firms facing a shortage of certificates might be inclined to pay a premium to transfer the price risk to the short position, viz. these agents are willing to pay the certainty equivalent $\mathbb{E}_t[\mathcal{U}(P_{t+n})]$ whereas $\mathbb{E}_t[\mathcal{U}(P_{t+n})] > \mathbb{E}_t[P_{t+n}]$. In this case $\mathcal{U}(\cdot)$ denotes the utility function of a representative agent holding the long position.

To account for the possibility that a non-zero risk premium, $\mathbb{E}_t\left[\mathcal{U}\left(P_{t+n}\right)\right] - \mathbb{E}_t\left[P_{t+n}\right]$, may depend on the price process, and recognizing that the price process could be potentially explosive, the risk premium is also potentially explosive. We explicitly model the risk premium using the framework proposed by Baumeister (2023) below.

3.1.2. Modeling risk premia. Building on the framework established by Baumeister (2023), we model the final payoff of a long position in an n-period futures contract initiated at time t as

$$\frac{P_{t+n} - F_{n,t}}{F_{n,t}} = \lambda_{0n} + \lambda_{1n}' \mathbf{x}_t + \upsilon_{t+n}. \tag{16}$$

Here, $\mathbf{x}_t$ is defined as a $(K \times 1)$ vector comprising proxies or latent risk factors and υ_{t+n} is a prediction error with mean zero. Further, λ_{0n} represents a horizon-specific intercept, while λ_{1n} denotes a vector of horizon-specific slope coefficients. Note that we refer to that as the risk premium in percentage terms.

Re-writing equation (16) yields

$$P_{t+n} = F_{n,t}\left(1 + \lambda_{0n} + \lambda_{1n}' \mathbf{x}_t + \upsilon_{t+n}\right),$$

and applying conditional expectations leads to

$$\mathbb{E}_t\left[P_{t+n}\right] = F_{n,t}\left(1 + \lambda_{0n} + \lambda_{1n}' \mathbf{x}_t\right).$$

Furthermore, substituting $RP_{n,t} + \mathbb{E}_t\left[P_{t+n}\right]$ for $F_{n,t}$ and simplifying gives

$$-RP_{n,t} = RP_{n,t}\left(\lambda_{0n} + \lambda_{1n}' \mathbf{x}_t\right) + \mathbb{E}_t\left[P_{t+n}\right]\left(\lambda_{0n} + \lambda_{1n}' \mathbf{x}_t\right).$$

Finally, solving for $RP_{n,t}$ yields

$$RP_{n,t} = \mathbb{E}_t\left[P_{t+n}\right] \times -\frac{\left(\lambda_{0n} + \lambda_{1n}' \mathbf{x}_t\right)}{\left(1 + \lambda_{0n} + \lambda_{1n}' \mathbf{x}_t\right)} =: \varrho_{n,t} \mathbb{E}_t\left[P_{t+n}\right], \tag{17}$$

which implies that the certainty equivalent, i.e., $F_{n,t}$, is given by $(1 + \varrho_{n,t})\mathbb{E}_t\left[P_{t+n}\right]$. In the context of the EU ETS, the median regulated firm may be willing to incur $\varrho_{n,t}$ as a premium to hedge against the risk of increasing certificate prices. Consequently, assuming a stationary process $\varrho_{n,t}$ with a positive mean appears reasonable in this context, suggesting that normal contango is likely to be the prevailing market condition. Hence, this modeling approach implies that the risk premium would proportionally scale with the potentially explosive price expectations, i.e.,

$$RP_{n,t} = \mathbb{E}_t\left[\mathcal{U}\left(P_{t+n}\right)\right] - \mathbb{E}_t\left[P_{t+n}\right] = (1+\varrho_{n,t})\mathbb{E}_t\left[P_{t+n}\right] - \mathbb{E}_t\left[P_{t+n}\right] = \varrho_{n,t}\mathbb{E}_t\left[P_{t+n}\right]. \tag{18}$$

To simplify the exposition, we treat $\varrho_{n,t}$ as a non-zero constant, specifically assuming $\varrho_{n,t} = \varrho_n$ for all t. However, it is important to note that this assumption can be relaxed without difficulty. Under the constant ϱ_n assumption, we obtain

$$RP_{n,t} = \varrho_n(1+\rho)^n B_t + \varrho_n \theta^n U_t. \tag{19}$$

This result implies the presence of normal contango when $\varrho_n > 0$ and normal backwardation when $\varrho_n < 0$. Consequently,

$$\operatorname{Cov}\left(B_t, RP_{n,t}\right) = \varrho_n(1+\rho)^n \operatorname{Var}\left(B_t\right) \quad \text{and} \quad \operatorname{Cov}\left(U_t, RP_{n,t}\right) = \varrho_n \theta^n \operatorname{Var}\left(U_t\right) \tag{20}$$

since $\operatorname{Cov}\left(U_t, B_t\right) = 0$ (extrinsic bubble). Further, $\operatorname{Var}(\cdot)$ and $\operatorname{Cov}(\cdot,\cdot)$ denote the population variance and covariance, respectively.

3.2. Fama Predictive Regressions

Fama (1984) presents two predictive regression models (hereafter referred to as FPRs) to analyze the time-varying premiums of forward exchange rates. In this section, we contend that these FPRs have the capability to infer rational bubbles and are robust against the time-trending patterns of the risk premium.

Fama (1984) splits the futures price at time t and delivery date $t+n$ into the expected price at time t with respect to time $t+n$ and a risk premium $RP_{n,t}$, i.e.,

$$F_{n,t} = \mathbb{E}_t\left[P_{t+n}\right] + RP_{n,t} \tag{21}$$

to receive the differential between the futures price and the current price, i.e.,

$$F_{n,t} - P_t = \mathbb{E}_t\left[P_{t+n}\right] - P_t + RP_{n,t}, \tag{22}$$

and to obtain the differential between the futures price and future spot price, i.e.,

$$F_{n,t} - P_{t+n} = \mathbb{E}_t\left[P_{t+n}\right] - P_{t+n} + RP_{n,t}. \tag{23}$$

Bearing this in mind, Fama (1984) considers the two regressions

$$\begin{aligned} &\text{(FPR 1)} \qquad F_{n,t} - P_{t+n} &&= \mu_{1,n} + \beta_{1,n}\left(F_{n,t} - P_t\right) + e_{1,t+n}, \\ &\text{(FPR 2)} \qquad P_{t+n} - P_t &&= \mu_{2,n} + \beta_{2,n}\left(F_{n,t} - P_t\right) + e_{2,t+n}, \end{aligned}$$

whereas $\mu_{1,n}$ and $\mu_{2,n}$ are constants, $\beta_{1,n}$ and $\beta_{2,n}$ are slope coefficients and $e_{1,t+n}$ and $e_{2,t+n}$ are errors with mean zero. Given that there is no rational bubble, we obtain from Equation (10), Equation (22) and Equation (23) the slope coefficient of FPR 1 as

$$\beta_{1,n} := \frac{\operatorname{Cov}\left(F_{n,t} - P_{t+n}, F_{n,t} - P_t\right)}{\operatorname{Var}\left(F_{n,t} - P_t\right)} = \frac{\operatorname{Cov}\left(-\vartheta_{t+n} + RP_{n,t}, \mathbb{E}_t\left[P_{t+n}\right] - P_t + RP_{n,t}\right)}{\operatorname{Var}\left(F_{n,t} - P_t\right)}, \tag{24}$$

For $\operatorname{Cov}\left(\vartheta_{t+n}, RP_{n,t}\right) = 0$ and $\operatorname{Cov}\left(\vartheta_{t+n}, P_t\right) = 0$, we receive

$$\beta_{1,n} = \frac{\operatorname{Var}\left(RP_{n,t}\right) + \operatorname{Cov}\left(RP_{n,t}, \mathbb{E}_t\left[P_{t+n}\right] - P_t\right)}{\operatorname{Var}\left(RP_{n,t}\right) + \operatorname{Var}\left(\mathbb{E}_t\left[P_{t+n}\right] - P_t\right) + 2\operatorname{Cov}\left(RP_{n,t}, \mathbb{E}_t\left[P_{t+n}\right] - P_t\right)}. \tag{25}$$

Apparently, the risk premium may introduce an upward bias in the estimator of $\beta_{1,n}$ when testing the null hypothesis of $\mathcal{H}_0 : \beta_{1,n} = 0$ to infer the presence of a rational bubble (see Maynard 2003).

For $\beta_{2,n}$, i.e., the slope coefficient of FPR 2, we obtain

$$\beta_{2,n} := \frac{\operatorname{Cov}\left(P_{t+n} - P_t, F_{n,t} - P_t\right)}{\operatorname{Var}\left(F_{n,t} - P_t\right)} = \frac{\operatorname{Cov}\left(\mathbb{E}_t\left[P_{t+n}\right] - P_t + \vartheta_{t+n}, \mathbb{E}_t\left[P_{t+n}\right] - P_t + RP_{n,t}\right)}{\operatorname{Var}\left(F_{n,t} - P_t\right)} \tag{26}$$

and with the above argument, i.e., $\operatorname{Cov}\left(\vartheta_{t+n}, RP_{n,t}\right) = 0$ and $\operatorname{Cov}\left(\vartheta_{t+n}, P_t\right) = 0$, we receive

$$\beta_{2,n} = \frac{\operatorname{Var}\left(\mathbb{E}_t\left[P_{t+n}\right] - P_t\right) + \operatorname{Cov}\left(RP_{n,t}, \mathbb{E}_t\left[P_{t+n}\right] - P_t\right)}{\operatorname{Var}\left(RP_{n,t}\right) + \operatorname{Var}\left(\mathbb{E}_t\left[P_{t+n}\right] - P_t\right) + 2\operatorname{Cov}\left(RP_{n,t}, \mathbb{E}_t\left[P_{t+n}\right] - P_t\right)}. \tag{27}$$

Fama (1984) notes that both FPRs convey identical information, resulting in $\beta_{1,n} + \beta_{2,n} = 1$, $\mu_{1,n} + \mu_{2,n} = 0$ and the disturbances sum up to zero for each time period t.

3.2.1. Slope Coefficients $\beta_{1,n}$ and $\beta_{2,n}$ under an Ongoing Rational Bubble. Note that in the following we assume that

(i) $\mathrm{Cov}\,(U_t, B_t) = 0$, i.e., the bubble is extrinsic;

(ii) the fundamental evolves according to a mildly explosive process; specifically $U_t = \theta U_{t-1} + \vartheta_t$, where $\theta = 1 + c \times T^{-\alpha}$ with $c > 0$ and $\alpha \in (0,1)$, as detailed in Phillips and Magdalinos (2007), or $c = 0$, representing a random walk;

(iii) the risk premium is generated by Equation (19);

(iv) that the variance of the bubble dominates the variance of the fundamental. Specifically, $\mathrm{Var}\,(U_t) \to \infty$ (since the fundamental follows a random walk or is mildly explosive), $\mathrm{Var}\,(B_t) \to \infty$ (since the bubble is a submartingale and explosive), and $\mathrm{Var}\,(U_t)\,/\,\mathrm{Var}\,(B_t) \to 0$ as $T \to 0$.

Building upon these assumptions, we conduct an analysis of $\beta_{1,n}$ and $\beta_{2,n}$ both in the absence and presence of a rational bubble, taking into account the influence of a trending risk premium. Hence, we obtain in the absence of a bubble, i.e., $B_t = 0$,

$$\beta_{1,n} \to 1 \quad \text{and} \quad \beta_{2,n} \to 0 \quad \text{for} \quad c > 0 \quad \text{as} \quad T \to \infty,$$

$$\beta_{1,n} = 1 \quad \text{and} \quad \beta_{2,n} = 0 \quad \text{for} \quad c = 0.$$

In contrary, we obtain in the presence of a bubble, i.e., $B_t > 0$,

$$\beta_{1,n} \to C \quad \text{and} \quad \beta_{2,n} \to 1 - C \quad \text{for} \quad c > 0 \quad \text{as} \quad T \to \infty,$$

$$\beta_{1,n} = C \quad \text{and} \quad \beta_{2,n} = 1 - C \quad \text{for} \quad c = 0,$$

whereas $C < 1$ is a constant that only depends on the autoregressive coefficient of the bubble $\left(\frac{1+\rho}{\pi}\right)^n$, on $(1+\rho)^n$, and on ϱ_n. Refer to Figure 2 for an illustration of $1 - C$. A detailed derivation of C is provided in Appendix A.

Insert Figure 2 here.

To summarize our theoretical findings on the FPRs: In the absence of a risk premium, the efficient market hypothesis is characterized by $\mathcal{H}_0 : \beta_{1,n} = 0$ and $\mathcal{H}_0 : \beta_{2,n} = 1$. Conversely, when a risk premium is present, and under assumptions (i) to (iii), the efficient market hypothesis corresponds to $\mathcal{H}_0 : \beta_{1,n} = 1$ and $\mathcal{H}_0 : \beta_{2,n} = 0$.

3.2.2. Drawing Inference on $\beta_{1,n}$ and $\beta_{2,n}$. Based on the findings above, we can use a statistical testing procedure for testing the hypotheses

$$\mathcal{H}_0 : \beta_{1,n} = 1 \quad \text{(no rational bubble)} \qquad \text{vs.} \quad \mathcal{H}_A : \beta_{1,n} \neq 1 \quad \text{(rational bubble)},$$

$$\mathcal{H}_0 : \beta_{2,n} = 0 \quad \text{(no rational bubble)} \qquad \text{vs.} \quad \mathcal{H}_A : \beta_{2,n} \neq 0 \quad \text{(rational bubble)}$$

using the conventional t-test based on the OLS method without the need to specify the fundamental or impose assumptions on the trending behavior of the risk premium. However, with a significant level of persistence in the explanatory variable, the conventional t-test results derived from the OLS method lose their validity. Stambaugh (1999) has demonstrated convincingly that this issue becomes more pronounced when the disturbances in the predictive regression are strongly correlated with the regressor's innovations.

Kostakis, Magdalinos, and Stamatogiannis (2015) present the IVX procedure that strengthens the robustness of inference concerning the degree of persistence of the explanatory variable, encompassing mildly explosive behavior. More explicitly, employing an instrument $z_{n,t}$ for the regressor, denoted here as $x_{n,t} := F_{n,t} - P_t$, where $z_{n,t}$ represents a transformed version of $x_{n,t}$ designed to achieve a controllable level of persistence, results in a robust inference procedure that effectively accounts for the impacts of non-stationarity. We focus in the following on FPR 2 and denote the regressand by $y_{n,t}$, i.e., $y_{n,t} := P_{t+n} - P_t$. The IVX estimator of the slope coefficient of FPR 2 is given by

$$\widehat{\beta}_{2,n} = \frac{\sum_{t=1}^{T} z_{n,t} \widetilde{y}_{n,t}}{\sum_{t=1}^{T} z_{n,t} \widetilde{x}_{n,t}}, \tag{28}$$

whereas $\widetilde{x}_{n,t}$ and $\widetilde{y}_{n,t}$ denote demeaned counterparts of $x_{n,t}$ and $y_{n,t}$, respectively. Kostakis, Magdalinos, and Stamatogiannis (2015) demonstrate the convergence of $\widehat{\beta}_{2,n}$ to a mixed Gaussian limiting distribution, a result that remains valid irrespective of the level of persistence exhibited by the regressor in the model. Consequently, this feature facilitates the development of a Wald-type statistic to test $\beta_{2,n} = 0$, denoted as $W_{\widehat{\beta}_2}$ which converges to a standard χ^2-distribution, i.e.,

$$W_{\widehat{\beta}_{2,n}} = \frac{\left(\widehat{\beta}_{2,n} - \beta_{2,n}\right)^2}{\widehat{Var}\left(\widehat{\beta}_{2,n}\right)} = \frac{\widehat{\beta}_{2,n}^2}{\widehat{Var}\left(\widehat{\beta}_{2,n}\right)} \Rightarrow \chi^2_{(1)} \qquad \text{as} \qquad T \to \infty, \tag{29}$$

whereas $\Rightarrow$ denotes convergence in distribution and $\widehat{Var}\left(\widehat{\beta}_{2,n}\right)$ is the estimated variance of $\widehat{\beta}_{2,n}$ (see Kostakis, Magdalinos, and Stamatogiannis 2015, for details).

3.2.3. Monte Carlo Simulation. In finite samples and under normal contango, $\widehat{\beta}_{2,n}$ tends to be above zero in the absence of a rational bubble, as the variance of the fundamental does not completely vanish from the numerator of $\beta_{2,n}$ (see Appendix A). This may result in small-sample size distortions when using the Wald test to test $\beta_{2,n} = 0$ against $\beta_{2,n} \neq 0$. To investigate the implications of this bias, we conduct a Monte Carlo simulation with 10,000 replications, comparing the size performance of the FPR approach to that of the KPSS method.

For this size analysis, we evaluate the rejection rates of the Wald statistic, as proposed by Kostakis, Magdalinos, and Stamatogiannis (2015), under the null hypothesis $\mathcal{H}_0 : \beta_{2,n} = 0$. In this scenario, the price process is defined as $P_t = U_t$, where the fundamental U_t evolves according to the autoregressive model

$$U_t = \theta U_{t-1} + \vartheta_t, \quad \theta = 1 + c \times T^{-\alpha}, \quad \vartheta_t \sim \text{i.i.d. } N(0,1), \tag{30}$$

with $c = 0.1$, $\alpha \in \{0.7, 0.75, 0.8, 0.85\}$, and $U_0 = 100$. We examine two sample sizes, $T = 150$ and $T = 300$. Figure 3 provides a representative example, demonstrating that the process exhibits long-term increasing behavior.

Insert Figure 3 here.

As a natural competitor, we apply the KPSS test to $P_{t+1} - F_{1,t}$, following the methodology of Kwiatkowski, Phillips, Schmidt, and Shin (1992). Additionally, the risk premium $RP_{1,t}$ is modeled under two configurations. If $\varrho_n > 0$, it follows $RP_{1,t} = \varrho_n \theta U_t$; otherwise, it is generated as $RP_{1,t} = 0.5 RP_{1,t-1} + \varphi_t$, where $\varphi_t \sim$ i.i.d. $N(0,1)$.

Insert Table 1 here.

The results, summarized in Table 1, indicate that the IVX-based Wald statistic achieves nominal size under all configurations. Thus, in an empirically relevant set-up, the small-sample bias is negligible. In contrast, the KPSS test exhibits significant size distortions, particularly for larger values of ϱ_n and higher persistence parameters, highlighting the advantages of the FPR approach in finite samples.[6]

4. Data and Empirical Results

Prior to exploring the empirical findings, we begin this section by introducing the EU ETS (futures) price data used in our empirical analysis. Next, we examine whether market and futures prices exhibit explosive behavior using the GSADF test (see Phillips, Shi, and Yu 2015a,b). Subsequently, we shift our focus to the core of our analysis, testing $\mathcal{H}_0 : \beta_{2,n} = 0$ against $\mathcal{H}_1 : \beta_{2,n} \neq 0$. To this end, we employ the approach by Yang, Long, Peng, and Cai (2020), applied within a rolling window framework. The latter approach addresses potential size distortions in the testing procedure of Kostakis, Magdalinos, and Stamatogiannis (2015) which can arise when serial correlation and heteroskedasticity are present in the error term of the predictive regression.

4.1. Data

We utilize weekly price data from the Bloomberg database spanning from January 4, 2013, to October 11, 2023, encompassing a total of $T = 563$ observations. The Bloomberg abbreviation for the spot price is `ICEDEU3 Index`. All fundamental contracts were actively traded and monitored on the Intercontinental Currency Exchange (ICE). Based on these contracts, our analysis focuses on generic continuous futures prices (as required by Pricing Equation 2) with a delivery date of one, two, three and four months, respectively. Since we consider weekly data, we receive $n \in \{4, 8, 12, 16\}$, i.e., delivery in 4, 8, 12 and 16 weeks.[7]

4.2. Is there an Explosive Episode in EU ETS Spot and Futures Rates?

We start our empirical exercise with testing against explosive episodes in the EU ETS price and futures price series over the full sample from 2013 to 2023. Explosiveness in spot prices is a necessary

[6] For a detailed power analysis of the FPR approach against the GSADF test applied to D_{t+n}, we refer the reader to Pavlidis, Paya, and Peel (2017).

[7] Note that contracts with $n > 16$ (4 months) are traded far less frequently than those with $n \leq 16$. Therefore, we omit consideration of $n > 16$ due to this liquidity constraint.

but, as described above, not a sufficient condition for a rational bubble. Nevertheless, the timing of when the spot price becomes explosive holds significance for subsample analysis, such as predictive regression and co-explosiveness analysis, in the subsequent sections.

We employ the GSADF test on the spot and futures price series in its original levels, and we find evidence for explosive behavior at the 5% significance level indicated by superscript r, as reported in Table 2.

Insert Table 2 here.

The critical values for the GSADF test are determined by wild bootstrapping with 999 repetitions accounting for multiplicity and heteroskedasticity (see Phillips and Shi 2020). Further, the test uses $T = 563$ observations and a minimum window size of $r_0 := \left\lfloor 0.01 + 1.8\sqrt{T} \right\rfloor = 42$ observations, following the rule proposed by Phillips, Shi, and Yu (2015a,b). Furthermore, as highlighted by Vasilopoulos, Pavlidis, and Martínez-García (2022), simulations provide evidence suggesting the effective performance of the GSADF test when a limited fixed number of lags is employed. In contrast, utilizing information criteria for lag selection may lead to notable distortions in size. As a result, we adopt the approach proposed by Pavlidis, Yusupova, Paya, Peel, Martínez-García, Mack, and Grossman (2016), employing two variations of a fixed number of lags. Specifically, we integrate one and four lags within the augmented Dickey-Fuller regression.

Subsequently, our next objective is to date-stamp the beginning and the end of the explosive periods. Hence, we use the Backward Supremum Augemented Dickey Fuller (BSADF) sequence with critical values obtained by wild bootstrap (see Phillips, Shi, and Yu 2015a,b, Phillips and Shi 2020). Overall, our findings for the spot and futures price series reveal (i) consistent indications of explosiveness across all series, and (ii) a notably similar timing pattern of the explosive phases in spot and futures prices during the third and fourth trading phase. The results are summarized by Figure 4 and Figure 5, indicating episodes of explosive behavior during the end of the third and during the fourth trading period.

Insert Figures 4 and 5 here.

4.3. Is there a Rational Bubble in the EU ETS?

As the date stamping indicates periods of explosive price behavior during the third trading period from 2018 to its conclusion in 2021, as well as at the onset of the fourth trading period from 2022 to 2023, we conclude that there is empirical evidence supporting the fulfillment of the first-order condition for a rational bubble.

We conduct a rolling window analysis using the Fama predictive regressions estimated via the approach outlined in Yang, Long, Peng, and Cai (2020) for the period from January 2013 to October 2023. Specifically, we employ a window size of $w_0 := T_2 - T_1 = 90$ observations, and the lag length in the method proposed by Yang, Long, Peng, and Cai (2020) is selected based on the Bayesian Information Criterion (BIC). To account for the issue of multiple hypothesis testing, we follow Pavlidis, Paya, and Peel (2017) and apply a Bonferroni correction. Specifically, we adjust the nominal significance level by dividing it by the number of hypotheses tested. Hence, we test the null hypothesis $\mathcal{H}_0 : \beta_{2,n} = 0$ against the alternative $\beta_{2,n} \neq 0$ using the sequence of test statistics

$$\mathcal{W}_n := \left\{ W_{\widehat{\beta}_{2,n}} \right\}_{t=T_1}^{T_2}.$$

The null hypothesis $\mathcal{H}_0 : \beta_{2,n} = 0$ is rejected if $\sup\{\mathcal{W}_n\} > F^{-1}_{\chi^2_1}(1-\delta/w_0)$, where $F^{-1}_{\chi^2_1}(1-\delta/w_0)$ represents the $(1-\delta/w_0)$-quantile of the chi-squared distribution with 1 degree of freedom. Here, δ denotes the significance level. Specifically, rejection occurs if $\sup\{\mathcal{W}_n\} > F^{-1}_{\chi^2_1}(1-0.05/90) = 11.92$. Table 3 summarizes the supremum of the sequences. We fail to reject the null hypothesis in all cases considered, as shown in Table 3.

Insert Table 3 here.

Further, the upper panel of Figures 6 to 9 illustrate the sequences of test statistics along with their corresponding critical values.

Insert Figures 6 to 9 here.

Furthermore, we highlight the importance of incorporating additional lags in predictive regressions, specifically by adopting the approach of Yang, Long, Peng, and Cai (2020) as an augmentation to the method developed by Kostakis, Magdalinos, and Stamatogiannis (2015). To address the bias introduced by correlated regression errors in the Kostakis, Magdalinos, and Stamatogiannis (2015) framework, Yang, Long, Peng, and Cai (2020) use autoregressive prewhitening. To evaluate this adjustment, we analyze the sequence of a Wald statistic to test the joint hypothesis that all included lags of the error term are equal to zero. The results, presented in the lower panel of Figures 6 to 9, indicate that the null hypothesis is rejected whenever the test statistic exceeds $F^{-1}_{\chi^2_5}(1-0.05) = 11.07$, given the inclusion of up to 5 lags. In the majority of cases, the null hypothesis is clearly rejected, underscoring the necessity of this augmented approach.

5. Conclusion, Discussion and Future Research

The importance of tackling climate change underscores the critical need for effective regulation, specifically through optimally functioning carbon taxation or emission trading systems (refer to Stroebel and Wurgler 2021, for a survey of the prevailing views among academics, professionals, and regulators regarding the primary risks anticipated for typical businesses and investors in the next five years). Therefore, it is crucial to leverage the insights acquired from previous trading phases for shaping the design of future emission trading systems and for implementation of best possible regulations. As emission trading schemes play a central role in climate policy, understanding their market dynamics and potential for improvement is of paramount importance.

The empirical investigation, in particular, aims to enhance our understanding of whether the notable rise in EU ETS prices since 2018 can be linked to a rational bubble. However, the analysis reveals no evidence of a rational bubble being the driving force behind the surge in allowance prices in 2018. Instead, it implies that the surge in EU ETS prices could be attributed to the expectation of impending scarcity, stemming from significant policy changes affecting emission caps. This is supported by the parallel (explosive) trend behavior inferred in future spot and futures rates.

Although we refrain from asserting the superiority of a Pigouvian tax or a trading system in mitigating carbon emissions, a significant drawback of a trading system would be (rational) bubble formation. After all, the price signal originating from the emission allowance market stands as a critical factor influencing the abatement decisions of regulated companies. These decisions may not be optimal if the market prices are somehow distorted. We alleviate concerns regarding rational bubble formation in previous trading phases. Therefore, in our perspective, there is no necessity for policy intervention to prevent potential rational bubbles in the EU ETS architecture, given the analysis of historical data at our disposal.

Our research opens up various possibilities for future research. First, we have demonstrated that FPRs can be used to test against rational bubbles also in the presence of an explosive risk premium. Hence, FPRs can be employed across various asset classes where futures prices are observable, without the necessity of assuming the trend behavior of the risk premium or establishing a fundamental. This encompasses bonds, stocks, and notably commodities and currencies, where the presence of a risk premium in the futures price holds significant sway (see Bessembinder 1992).

Second, Quemin and Pahle (2023) developed a diagnostic toolkit to assess the extent and impact of speculation in emission trading systems, applying it to the EU ETS. However, increased speculation does not necessarily indicate market inefficiency. Incorporating FPRs with a rolling window approach could enhance this toolkit, offering a real-time view of inefficiencies in future trading phases of the EU ETS and other emission trading systems globally.

References

Baudry M, Faure A, Quemin S (2021) Emissions trading with transaction costs. *Journal of Environmental Economics and Management* 108:102468.

Baumeister C (2023) Measuring market expectations. *Handbook of Economic Expectations* 413–441.

Bessembinder H (1992) Systematic risk, hedging pressure, and risk premiums in futures markets. *The Review of Financial Studies* 5(4):637–667.

Bessembinder H, Chan K (1992) Time-varying risk premia and forecastable returns in futures markets. *Journal of Financial Economics* 32(2):169–193.

Blanchard OJ (1979) Speculative bubbles, crashes and rational expectations. *Economics Letters* 3(4):387–389.

Calel R, Dechezleprêtre A (2016) Environmental policy and directed technological change: evidence from the european carbon market. *Review of Economics and Statistics* 98(1):173–191.

Carmona R, Fehr M, Hinz J (2009) Optimal stochastic control and carbon price formation. *SIAM Journal on Control and Optimization* 48(4):2168–2190.

Chesney M, Taschini L (2012) The endogenous price dynamics of emission allowances and an application to CO2 option pricing. *Applied Mathematical Finance* 19(5):447–475.

Creti A, Joëts M (2017) Multiple bubbles in the European Union Emission Trading Scheme. *Energy Policy* 107:119–130.

Creti A, Jouvet PA, Mignon V (2012) Carbon price drivers: Phase I versus Phase II equilibrium? *Energy Economics* 34(1):327–334.

De Roon FA, Nijman TE, Veld C (2000) Hedging pressure effects in futures markets. *The Journal of Finance* 55(3):1437–1456.

Dewally M, Ederington LH, Fernando CS (2013) Determinants of trader profits in commodity futures markets. *The Review of Financial Studies* 26(10):2648–2683.

Diba BT, Grossman HI (1988a) Explosive rational bubbles in stock prices? *The American Economic Review* 78(3):520–530.

Diba BT, Grossman HI (1988b) Rational inflationary bubbles. *Journal of Monetary Economics* 21(1):35–46.

Engle RF, Granger CW (1987) Co-integration and error correction: Representation, estimation, and testing. *Econometrica* 251–276.

Evans GW (1991) Pitfalls in testing for explosive bubbles in asset prices. *The American Economic Review* 81(4):922–930.

Fama EF (1984) Forward and spot exchange rates. *Journal of Monetary Economics* 14(3):319–338.

Flood RP, Hodrick RJ (1990) On testing for speculative bubbles. *Journal of Economic Perspectives* 4(2):85–101.

Friedrich M, Fries S, Pahle M, Edenhofer O (2020) Rules vs. discretion in cap-and-trade programs: Evidence from the EU emission trading system. *CESifo Working Paper* 8637.

Granger CW (1969) Prediction with a generalized cost of error function. *Journal of the Operational Research Society* 20:199–207.

Hamilton JD, Wu JC (2014) Risk premia in crude oil futures prices. *Journal of International Money and Finance* 42:9–37.

Harvey DI, Leybourne SJ, Sollis R, Taylor AR (2016) Tests for explosive financial bubbles in the presence of non-stationary volatility. *Journal of Empirical Finance* 38:548–574.

Hicks JR (1939) *Value and capital* (Oxford: Oxford University Press).

Hintermann B, Peterson S, Rickels W (2016) Price and Market Behavior in Phase II of the EU ETS: A Review of the Literature. *Review of Environmental Economics and Policy* 10(1):108–128.

Hitzemann S, Uhrig-Homburg M (2018) Equilibrium price dynamics of emission permits. *Journal of Financial and Quantitative Analysis* 53(4):1653–1678.

Huang W, Wang Y (2024) Identifying price bubbles in global carbon markets: Evidence from the SADF test, GSADF test and LPPLS method. *Energy Economics* 107626.

Keynes JM (1930) *A treatise on money, vol II* (London: MacMillan).

Kling C, Rubin J (1997) Bankable permits for the control of environmental pollution. *Journal of Public Economics* 64(1):101–115.

Koch N, Fuss S, Grosjean G, Edenhofer O (2014) Causes of the EU ETS price drop: Recession, CDM, renewable policies or a bit of everything? – New evidence. *Energy Policy* 73:676–685.

Kostakis A, Magdalinos T, Stamatogiannis MP (2015) Robust econometric inference for stock return predictability. *The Review of Financial Studies* 28(5):1506–1553.

Kwiatkowski D, Phillips PC, Schmidt P, Shin Y (1992) Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics* 54(1-3):159–178.

Maynard A (2003) Testing for forward-rate unbiasedness: On regression in levels and in returns. *Review of Economics and Statistics* 85(2):313–327.

Montgomery WD (1972) Markets in licenses and efficient pollution control programs. *Journal of Eonomic Theory* 5(3):395–418.

Pahle M, Günther C, Osorio S, Quemin S (2023) The emerging endgame: The EU ETS on the road towards climate neutrality. *Available at SSRN* .

Pavlidis E, Yusupova A, Paya I, Peel D, Martínez-García E, Mack A, Grossman V (2016) Episodes of exuberance in housing markets: in search of the smoking gun. *The Journal of Real Estate Finance and Economics* 53:419–449.

Pavlidis EG, Paya I, Peel DA (2017) Testing for speculative bubbles using spot and forward prices. *International Economic Review* 58(4):1191–1226.

Pavlidis EG, Paya I, Peel DA (2018) Using market expectations to test for speculative bubbles in the crude oil market. *Journal of Money, Credit and Banking* 50(5):833–856.

Phillips PC, Magdalinos T (2007) Limit theory for moderate deviations from a unit root. *Journal of Econometrics* 136(1):115–130.

Phillips PC, Magdalinos T (2009) Econometric inference in the vicinity of unity. *Singapore Management University, CoFie Working Paper* 7.

Phillips PC, Shi S (2020) Real time monitoring of asset markets: Bubbles and crises. *Handbook of Statistics*, volume 42, 61–80 (Elsevier).

Phillips PC, Shi S, Yu J (2015a) Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review* 56(4):1043–1078.

Phillips PC, Shi S, Yu J (2015b) Testing for multiple bubbles: Limit theory of real-time detectors. *International Economic Review* 56(4):1079–1134.

Phillips PC, Wu Y, Yu J (2011) Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values? *International Economic Review* 52(1):201–226.

Phillips PC, Yu J (2011) Dating the timeline of financial bubbles during the subprime crisis. *Quantitative Economics* 2(3):455–491.

Quemin S, Pahle M (2023) Financials threaten to undermine the functioning of emissions markets. *Nature Climate Change* 13(1):22–31.

Rickels W, Görlich D, Peterson S (2015) Explaining European emission allowance price dynamics: Evidence from Phase II. *German Economic Review* 16(2):181–202.

Rubin JD (1996) A model of intertemporal emission trading, banking, and borrowing. *Journal of Environmental Economics and Management* 31(3):269–286.

Stambaugh RF (1999) Predictive regressions. *Journal of Financial Economics* 54(3):375–421.

Stroebel J, Wurgler J (2021) What do you think about climate finance? *Journal of Financial Economics* 142(2):487–498.

Taschini L (2021) Flexibility premium of emissions permits. *Journal of Economic Dynamics and Control* 126:104013.

Terranova R, Cozzarini C, Reissl S, Tavoni M (2024) Detecting speculation in the market for EU emission allowances. *Available at SSRN 5046581* .

Tirole J (1985) Asset bubbles and overlapping generations. *Econometrica* 53(6):1499–1528.

Vasilopoulos K, Pavlidis E, Martínez-García E (2022) exuber: Recursive right-tailed unit root testing with R. *Journal of Statistical Software* 103:1–26.

Wei Y, Li Y, Wang Z (2022) Multiple price bubbles in global major emission trading schemes: Evidence from European Union, New Zealand, South Korea and China. *Energy Economics* 113:106232.

Yang B, Long W, Peng L, Cai Z (2020) Testing the predictability of us housing price index returns based on an IVX-AR model. *Journal of the American Statistical Association* 115(532):1598–1619.

Zhao J (2003) Irreversible abatement investment under cost uncertainties: tradable emission permits and emissions charges. *Journal of Public Economics* 87(12):2765–2789.

Appendix

A.1: Derivation of $\beta_{1,n}$ and $\beta_{2,n}$ for a mildly explosive process

In this section we present the derivation of $\beta_{1,n}$ and $\beta_{2,n}$ under a non-zero risk premium and in the presence or absence of an ongoing rational bubble.

Consider the slope coefficients of FPR 1 and FPR 2, i.e.,

$$\beta_{1,n} := \frac{\mathrm{Cov}\left(F_{n,t} - P_{t+n}, F_{n,t} - P_t\right)}{\mathrm{Var}\left(F_{n,t} - P_t\right)} \qquad \text{and} \qquad \beta_{2,n} := \frac{\mathrm{Cov}\left(P_{t+n} - P_t, F_{n,t} - P_t\right)}{\mathrm{Var}\left(F_{n,t} - P_t\right)}.$$

Hence, under the above assumption, we obtain the numerator of $\beta_{1,n}$ as

$$\begin{aligned}
&\mathrm{Cov}\left(F_{n,t} - P_{t+n}, F_{n,t} - P_t\right) \\
&= \mathrm{Cov}\left(-\vartheta_{t+n} - (1+\rho)^n\left(\frac{1}{\pi^n} - 1\right)B_t - \epsilon_{t+n} + RP_{n,t}, (\theta^n - 1)U_t + ((1+\rho)^n - 1)B_t + RP_{n,t}\right) \\
&= \mathrm{Var}\left(RP_{n,t}\right) - \underbrace{(1+\rho)^n\left(\frac{1}{\pi^n} - 1\right)((1+\rho)^n - 1)}_{=:\gamma_{1,n}}\mathrm{Var}\left(B_t\right) - (1+\rho)^n\left(\frac{1}{\pi^n} - 1\right)(\theta^n - 1)\underbrace{\mathrm{Cov}\left(B_t, U_t\right)}_{=0} \\
&+ \underbrace{\left(2(1+\rho)^n - \left(\frac{1+\rho}{\pi}\right)^n - 1\right)}_{=:\gamma_{2,n}}\mathrm{Cov}\left(B_t, RP_{n,t}\right) + (\theta^n - 1)\,\mathrm{Cov}\left(U_t, RP_{n,t}\right) \\
&- (\theta^n - 1)\underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, U_t\right)}_{=0} - ((1+\rho)^n - 1)\underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, B_t\right)}_{=0} - \underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, RP_{n,t}\right)}_{=0} \\
&- (\theta^n - 1)\underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, U_t\right)}_{=0} - ((1+\rho)^n - 1)\underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, B_t\right)}_{=0} - \underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, RP_{n,t}\right)}_{=0},
\end{aligned}$$

and the numerator of $\beta_{2,n}$ as

$$\begin{aligned}
&\mathrm{Cov}\left(P_{t+n} - P_t, F_{n,t} - P_t\right) \\
&= \mathrm{Cov}\left((\theta^n - 1)U_t + \vartheta_{t+n} + \left(\left(\frac{1+\rho}{\pi}\right)^n - 1\right)B_t + \epsilon_{t+n}, (\theta^n - 1)U_t + ((1+\rho)^n - 1)B_t + RP_{n,t}\right) \\
&= \underbrace{\left(\left(\frac{1+\rho}{\pi}\right)^n - 1\right)((1+\rho)^n - 1)}_{=:\gamma_{3,n}}\mathrm{Var}\left(B_t\right) + (\theta^n - 1)^2\,\mathrm{Var}\left(U_t\right) \\
&+ \underbrace{\left(\left(\frac{1+\rho}{\pi}\right)^n - 1\right)}_{=:\gamma_{4,n}}\mathrm{Cov}\left(B_t, RP_{n,t}\right) + (\theta^n - 1)\,\mathrm{Cov}\left(U_t, RP_{n,t}\right) \\
&+ (\theta^n - 1)\underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, U_t\right)}_{=0} + ((1+\rho)^n - 1)\underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, B_t\right)}_{=0} + \underbrace{\mathrm{Cov}\left(\vartheta_{t+n}, RP_{n,t}\right)}_{=0} \\
&+ (\theta^n - 1)\underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, U_t\right)}_{=0} + ((1+\rho)^n - 1)\underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, B_t\right)}_{=0} + \underbrace{\mathrm{Cov}\left(\epsilon_{t+n}, RP_{n,t}\right)}_{=0}.
\end{aligned}$$

For the denominator of $\beta_{1,n}$ and of $\beta_{2,n}$, we receive

$$\operatorname{Var}(F_{n,t}-P_t)=\operatorname{Var}\left(RP_{n,t}+(\theta^n-1)U_t+((1+\rho)^n-1)B_t\right)$$

$$=\operatorname{Var}(RP_{n,t})+\underbrace{((1+\rho)^n-1)^2}_{=:\gamma_n^2}\operatorname{Var}(B_t)+(\theta^n-1)^2\operatorname{Var}(U_t)$$

$$+2\underbrace{((1+\rho)^n-1)}_{=:\gamma_n}\operatorname{Cov}(B_t,RP_{n,t})+2(\theta^n-1)\operatorname{Cov}(U_t,RP_{n,t}).$$

Null hypothesis of no rational bubble. Under the null, the population variances and covariances are multiples of the variance of the fundamental. Hence, we have

$$\beta_{1,n}=\frac{\operatorname{Var}(RP_{n,t})+(\theta^n-1)\operatorname{Cov}(U_t,RP_{n,t})}{\operatorname{Var}(RP_{n,t})+(\theta^n-1)^2\operatorname{Var}(U_t)+2(\theta^n-1)\operatorname{Cov}(U_t,RP_{n,t})}=\frac{\varrho_n^2\theta^{2n}+(\theta^n-1)\varrho_n\theta^n}{\varrho_n^2\theta^{2n}+(\theta^n-1)^2+2(\theta^n-1)\varrho_n\theta^n}.$$

Substitution of the definition of a mildly explosive process yields

$$\frac{\varrho_n^2(1+cT^{-\alpha})^{2n}+((1+cT^{-\alpha})^n-1)\varrho_n(1+cT^{-\alpha})^n}{\varrho_n^2(1+cT^{-\alpha})^{2n}+((1+cT^{-\alpha})^n-1)^2+2((1+cT^{-\alpha})^n-1)\varrho_n(1+cT^{-\alpha})^n}.$$

Further, the binomial expansion $(1+cT^{-\alpha})^\kappa\approx 1+\kappa cT^{-\alpha}$ is applied for small $cT^{-\alpha}$, i.e., valid as $T\to\infty$. In the numerator, $(1+cT^{-\alpha})^{2n}\approx 1+2ncT^{-\alpha}$ and $(1+cT^{-\alpha})^n-1\approx ncT^{-\alpha}$ are substituted. This leads to

$$\varrho_n^2(1+2ncT^{-\alpha})+(ncT^{-\alpha})\varrho_n(1+ncT^{-\alpha}).$$

In the denominator, the same approximations are used. The terms become

$$\varrho_n^2(1+2ncT^{-\alpha})+(ncT^{-\alpha})^2+2(ncT^{-\alpha})\varrho_n(1+ncT^{-\alpha}).$$

The dominant term in both numerator and denominator as $T\to\infty$ is ϱ_n^2. This reduces the fraction to

$$\frac{\varrho_n^2}{\varrho_n^2}=1 \quad \text{as} \quad T\to\infty.$$

For $\beta_{2,n}$, we receive

$$\beta_{2,n}=\frac{(\theta^n-1)^2\operatorname{Var}(U_t)+(\theta^n-1)\operatorname{Cov}(U_t,RP_{n,t})}{\operatorname{Var}(RP_{n,t})+(\theta^n-1)^2\operatorname{Var}(U_t)+2(\theta^n-1)\operatorname{Cov}(U_t,RP_{n,t})}=\frac{(\theta^n-1)^2+(\theta^n-1)\varrho_n\theta^n}{\varrho_n\theta^n+(\theta^n-1)^2+2(\theta^n-1)\varrho_n\theta^n}.$$

Substitution of the definition of a mildly explosive process yields

$$\frac{\left((1+c\times T^{-\alpha})^n-1\right)^2+\left((1+c\times T^{-\alpha})^n-1\right)\varrho_n(1+c\times T^{-\alpha})^n}{\varrho_n^2(1+c\times T^{-\alpha})^{2n}+\left((1+c\times T^{-\alpha})^n-1\right)^2+2\left((1+c\times T^{-\alpha})^n-1\right)\varrho_n(1+c\times T^{-\alpha})^n},$$

whereas the binomial expansion $(1+cT^{-\alpha})^\kappa \approx 1+\kappa cT^{-\alpha}$ is applied for small $cT^{-\alpha}$, i.e., valid as $T\to\infty$. In the numerator, $(1+cT^{-\alpha})^n-1\approx ncT^{-\alpha}$ and $(1+cT^{-\alpha})^n\approx 1+ncT^{-\alpha}$ are substituted. This leads to

$$\frac{\left(ncT^{-\alpha}\right)^2+\left(ncT^{-\alpha}\right)\varrho_n\left(1+ncT^{-\alpha}\right)}{\varrho_n^2\left(1+2ncT^{-\alpha}\right)+\left(ncT^{-\alpha}\right)^2+2\left(ncT^{-\alpha}\right)\varrho_n\left(1+ncT^{-\alpha}\right)}.$$

Simplifying further gives

$$\frac{ncT^{-2\alpha}+ncT^{-\alpha}\varrho_n}{\varrho_n^2+2ncT^{-\alpha}\varrho_n^2+n^2c^2T^{-2\alpha}+2ncT^{-\alpha}\varrho_n}.$$

As $T\to\infty$, the dominant term in the numerator is $ncT^{-2\alpha}$ which vanishes faster than the leading term ϱ_n^2 in the denominator. This reduces the fraction to

$$\frac{0}{\varrho_n^2}=0 \quad \text{as} \quad T\to\infty.$$

Hence, we receive $\beta_{1,n}+\beta_{2,n}=1$ under the null hypothesis of no bubble.

Alternative hypothesis of a rational bubble. Under the alternative, the population variances and covariances are multiples of the variance of the fundamental. Hence, we have

$$\beta_{1,n}=\frac{\operatorname{Var}\left(RP_{n,t}\right)-\gamma_{1,n}\operatorname{Var}\left(B_t\right)+\gamma_{2,n}\operatorname{Cov}\left(B_t,RP_{n,t}\right)+\left(\theta^n-1\right)\operatorname{Cov}\left(U_t,RP_{n,t}\right)}{\operatorname{Var}\left(RP_{n,t}\right)+\gamma_n^2\operatorname{Var}\left(B_t\right)+\left(\theta^n-1\right)^2\operatorname{Var}\left(U_t\right)+2\gamma_n\operatorname{Cov}\left(B_t,RP_{n,t}\right)+2\left(\theta^n-1\right)\operatorname{Cov}\left(U_t,RP_{n,t}\right)}$$

with the numerator

$$\varrho_n^2(1+\rho)^{2n}\operatorname{Var}\left(B_t\right)+\varrho_n^2\theta^{2n}\operatorname{Var}\left(U_t\right)-\gamma_{1,n}\operatorname{Var}\left(B_t\right)+\gamma_{2,n}\varrho_n(1+\rho)^n\operatorname{Var}\left(B_t\right)+\left(\theta^n-1\right)\varrho_n\theta^n\operatorname{Var}\left(U_t\right)$$

and the denominator

$$\begin{aligned}&\varrho_n^2(1+\rho)^{2n}\operatorname{Var}\left(B_t\right)+\varrho_n^2\theta^{2n}\operatorname{Var}\left(U_t\right)+\gamma_n^2\operatorname{Var}\left(B_t\right)+\left(\theta^n-1\right)^2\operatorname{Var}\left(U_t\right)\\&+2\gamma_n\varrho_n(1+\rho)^n\operatorname{Var}\left(B_t\right)+2\left(\theta^n-1\right)\varrho_n\theta^n\operatorname{Var}\left(U_t\right).\end{aligned}$$

Using the assumption $\operatorname{Var}(U_t) / \operatorname{Var}(B_t) \to 0$ as $T \to \infty$, the dominant terms are those involving $\operatorname{Var}(B_t)$, so the numerator becomes

$$\operatorname{Var}(B_t)\left[\varrho_n^2(1+\rho)^{2n} - \gamma_{1,n} + \gamma_{2,n}\varrho_n(1+\rho)^n\right].$$

The denominator simplifies to

$$\operatorname{Var}(B_t)\left[\varrho_n^2(1+\rho)^{2n} + \gamma_n^2 + 2\gamma_n\varrho_n(1+\rho)^n\right].$$

Hence, taking the limit as $T \to \infty$ whereas $\operatorname{Var}(U_t) / \operatorname{Var}(B_t) \to 0$ as $T \to \infty$, $\beta_{1,n}$ becomes

$$\beta_{1,n} = \frac{\varrho_n^2(1+\rho)^{2n} - \gamma_{1,n} + \gamma_{2,n}\varrho_n(1+\rho)^n}{\left(\varrho_n(1+\rho)^n + \gamma_n\right)^2} =: C \qquad \text{for} \qquad T \to \infty.$$

Finally, we have

$$\beta_{2,n} = \frac{\gamma_{3,n}\operatorname{Var}(B_t) + (\theta^n - 1)^2\operatorname{Var}(U_t) + \gamma_{4,n}\operatorname{Cov}(B_t, RP_{n,t}) + (\theta^n - 1)\operatorname{Cov}(U_t, RP_{n,t})}{\operatorname{Var}(RP_{n,t}) + \gamma_n^2\operatorname{Var}(B_t) + (\theta^n - 1)^2\operatorname{Var}(U_t) + 2\gamma_n\operatorname{Cov}(B_t, RP_{n,t}) + 2(\theta^n - 1)\operatorname{Cov}(U_t, RP_{n,t})}$$

with the numerator

$$\gamma_{3,n}\operatorname{Var}(B_t) + (\theta^n - 1)^2\operatorname{Var}(U_t) + \gamma_{4,n}\varrho_n(1+\rho)^n\operatorname{Var}(B_t) + (\theta^n - 1)\varrho_n\theta^n\operatorname{Var}(U_t)$$

and the denominator as for $\beta_{1,n}$. Using the assumption $\operatorname{Var}(U_t) / \operatorname{Var}(B_t) \to 0$, the dominant terms are those involving $\operatorname{Var}(B_t)$, so the numerator becomes

$$\operatorname{Var}(B_t)\left[\gamma_{3,n} + \gamma_{4,n}\varrho_n(1+\rho)^n\right].$$

Hence, taking the limit as $T \to \infty$ whereas $\operatorname{Var}(U_t) / \operatorname{Var}(B_t) \to 0$, $\beta_{2,n}$ becomes

$$\beta_{2,n} = \frac{\gamma_{3,n} + \gamma_{4,n}\varrho_n(1+\rho)^n}{\left(\varrho_n(1+\rho)^n + \gamma_n\right)^2} \qquad \text{for} \qquad T \to \infty.$$

Adding $\beta_{1,n}$ and $\beta_{2,n}$, their combined numerator becomes

$$\varrho_n^2(1+\rho)^{2n} - \gamma_{1,n} + \gamma_{2,n}\varrho_n(1+\rho)^n + \gamma_{3,n} + \gamma_{4,n}\varrho_n(1+\rho)^n.$$

Substituting the definitions of γ_n, $\gamma_{1,n}, \gamma_{2,n}, \gamma_{3,n}, \gamma_{4,n}$, we simplify the combined numerator to

$$\varrho_n^2(1+\rho)^{2n} + \gamma_n^2 + 2\gamma_n\varrho_n(1+\rho)^n = \left(\varrho_n(1+\rho)^n + \gamma_n\right)^2.$$

The denominator of $\beta_{1,n}$ and $\beta_{2,n}$ is identical. Hence, we conclude $\beta_{1,n} + \beta_{2,n} = 1$, also under the alternative of a rational bubble.

A.2: Tables and Figures

Table 1: Monte Carlo Simulation of the size of the FPR approach.

Fama Predictive Regressions						
ϱ_n	T	α	0.7	0.75	0.8	0.85
0	150		0.05	0.05	0.05	0.05
	300		0.05	0.05	0.04	0.05
0.01	150		0.06	0.05	0.05	0.05
	300		0.05	0.05	0.05	0.05
0.1	150		0.04	0.04	0.05	0.06
	300		0.04	0.04	0.06	0.06

KPSS						
ϱ_n	T	α	0.7	0.75	0.8	0.85
0	150		0.04	0.05	0.04	0.06
	300		0.04	0.06	0.05	0.05
0.01	150		0.30	0.20	0.14	0.12
	300		0.67	0.46	0.30	0.22
0.1	150		1.00	0.98	0.94	0.87
	300		1.00	0.98	0.96	0.92

Explanations: The table reports the rejection rates for testing $\mathcal{H}_0 : \beta_{2,n} = 0$ against $\mathcal{H}_1 : \beta_{2,n} \neq 0$ using the IVX methodology applied within the FPR framework (left panel) and the rejection rates of the KPSS test applied to the difference $P_{t+1} - F_{1,t}$ (right panel).

Table 2: Testing explosiveness of EU ETS spot and futures rates

	Lags	P_t	$F_{4,t}$	$F_{8,t}$	$F_{12,t}$	$F_{16,t}$
GSADF	1	4.96^r	4.97^r	4.98^r	4.97^r	4.97^r
	4	5.43^r	5.45^r	5.46^r	5.45^r	5.43^r

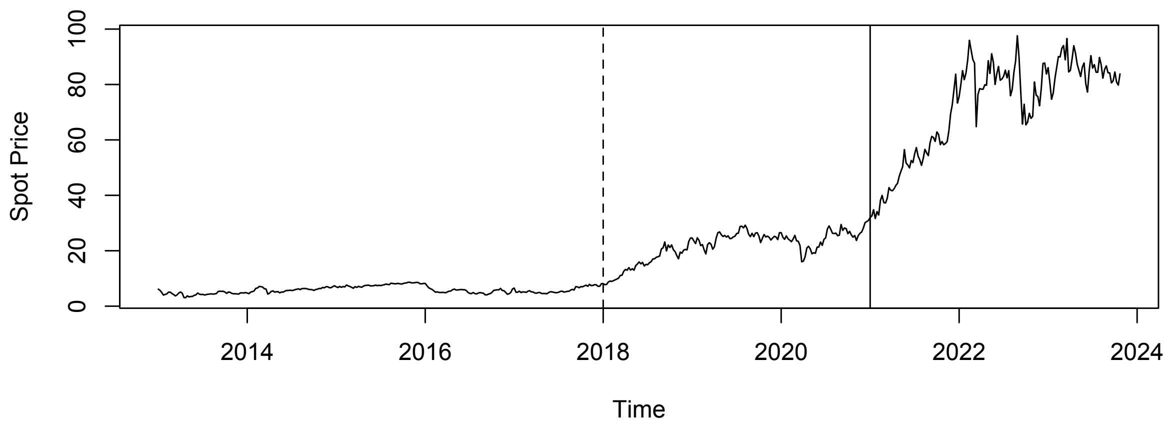
Explanations: The table presents the test statistics of the GSADF test applied to the spot and futures rates. Superscript r indicates rejection at the 5% significance level whereas critical values are obtained by bootstrapping with 999 repetitions.

Table 3: Testing rational bubbles in the EU ETS by the FPR approach

n	4	8	12	16
$\sup\{\mathcal{W}_n\}$	10.41	10.95	11.31	10.41

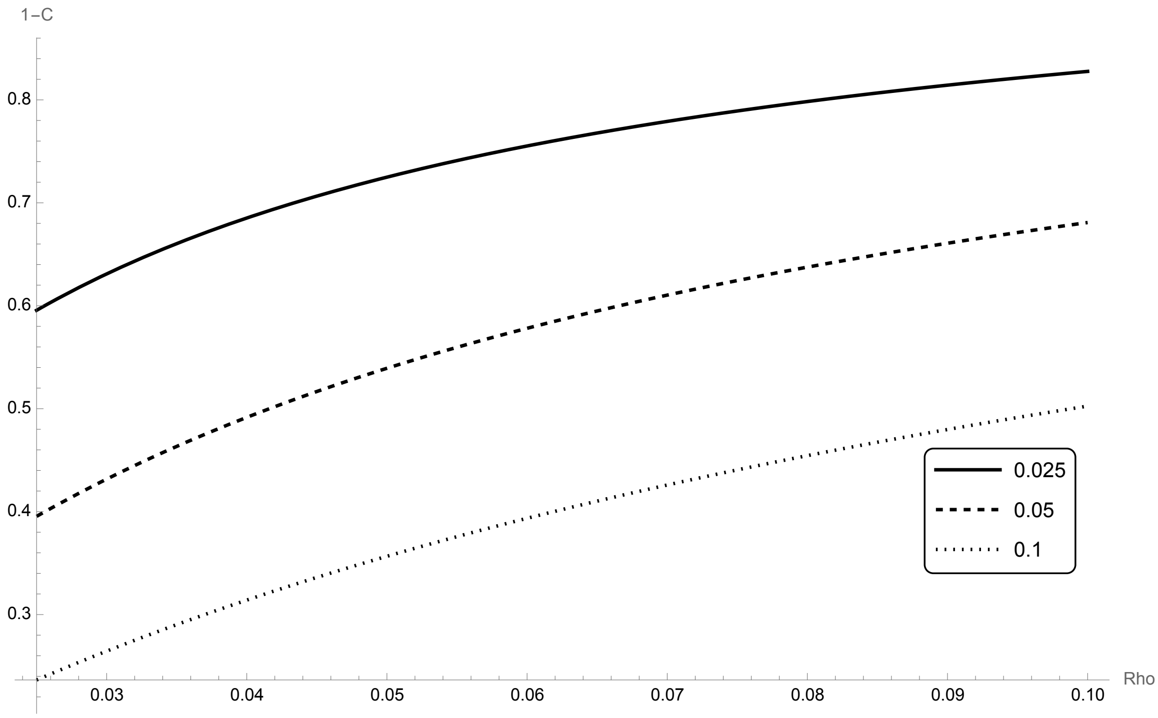
Explanations: The table presents test statistics derived as the supremum of the sequences $\mathcal{W}_n$. The null hypothesis is rejected at the nominal significance level of five percent when $\sup\{\mathcal{W}_n\} > 11.92$.

Figure 1: Spot price series of emission allowances in the EU ETS



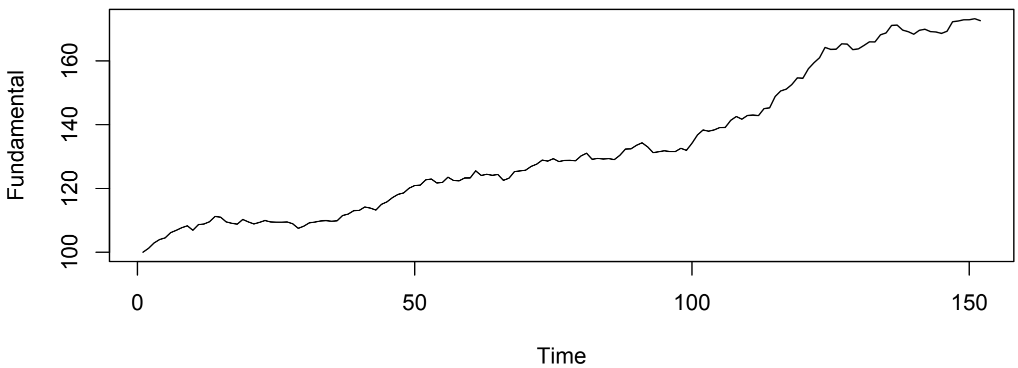
Explanations: The dotted vertical line indicates the (supposed) start of the price surge (2018). The solid vertical line indicates the start of the fourth trading period (2021).

Figure 2: Illustration of $1-C$ as a function of ρ



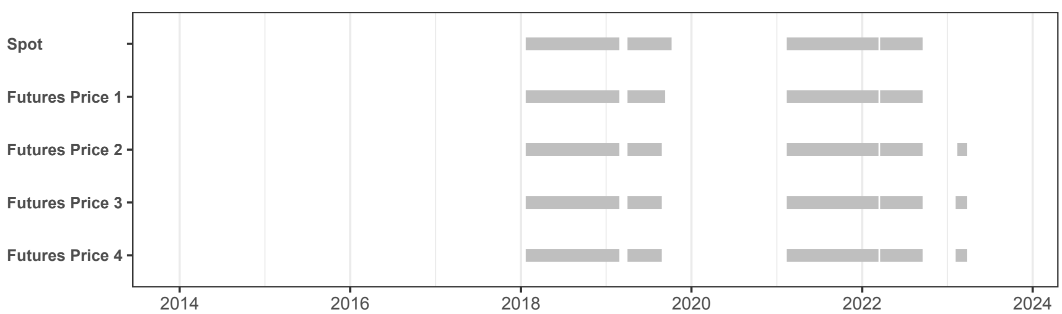
Explanations: This illustrates $1-C$ for varying $\rho \in [0.025, 0.1]$ with fixed values of $\pi = 0.99$, $\varrho_n \in \{0.025, 0.05, 0.1\}$, and $n = 1$.

Figure 3: Illustration of a mildly explosive process.



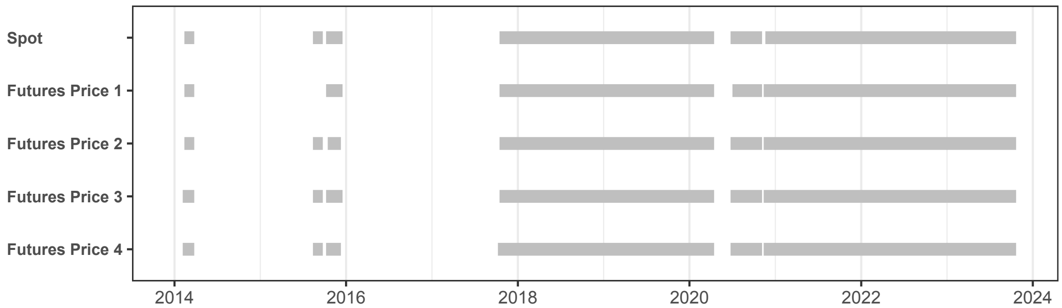
Explanations: The figure illustrates the behavior of sightly explosive process (refer to the text for details on the specification).

Figure 4: Timing of explosiveness in EU ETS spot and futures prices



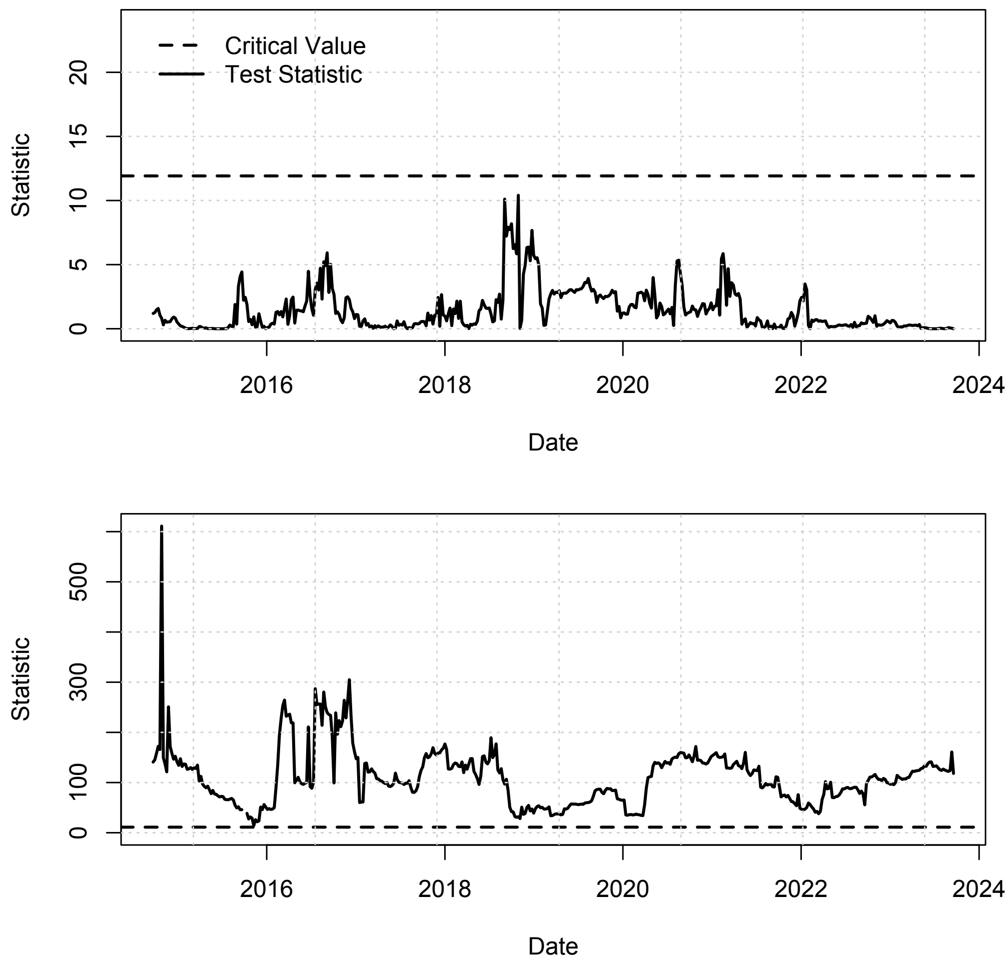
Explanations: The regions shaded in gray signify explosive periods, as determined by the BSADF procedure at a 5% significance level, employing one lag and a minimum window size of 36 observations. "Spot" denotes the date stamping for the spot price, while "Futures Price 1" corresponds to the futures price with delivery in one month (4 weeks), and so forth.

Figure 5: Timing of explosiveness in EU ETS spot and futures prices (cont'd)



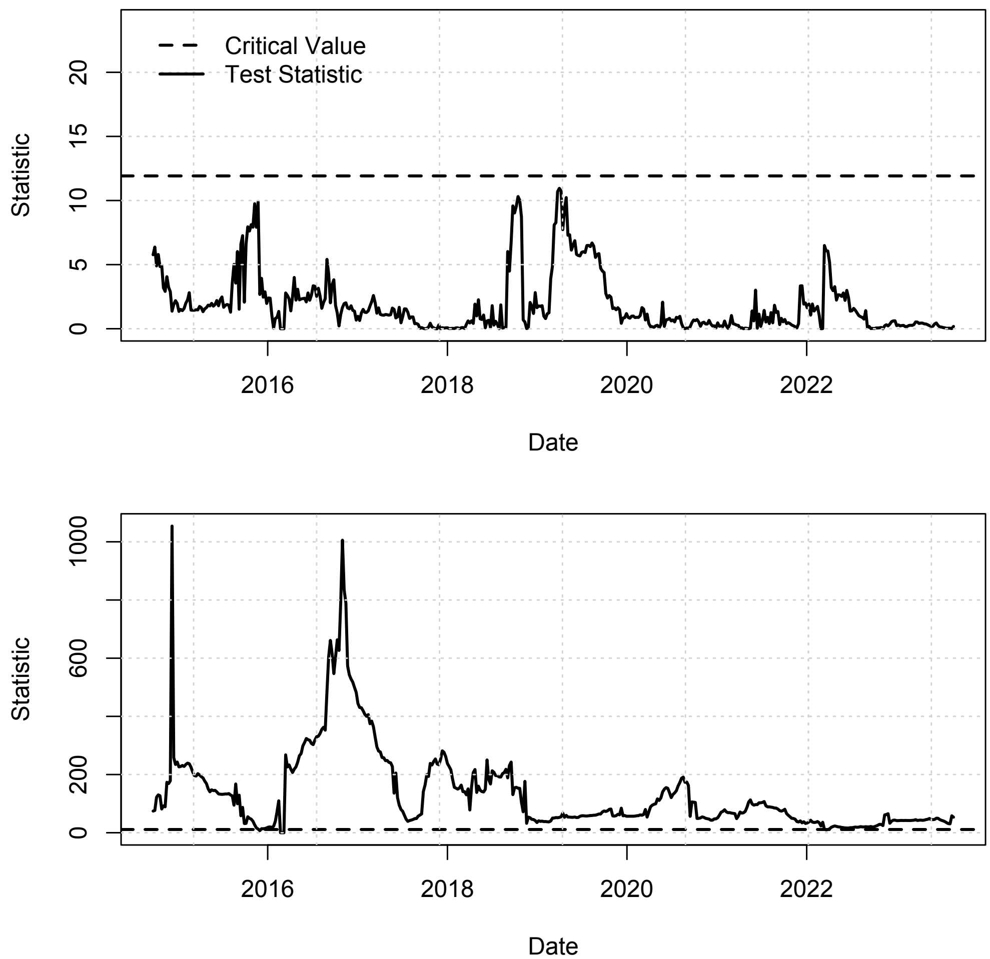
Explanations: The regions shaded in gray signify explosive periods, as determined by the BSADF procedure at a 5% significance level, employing four lag and a minimum window size of 36 observations. "Spot" denotes the date stamping for the spot price, while "Futures Price 1" corresponds to the futures price with delivery in one month (4 weeks), and so forth.

Figure 6: Test statistic sequences for $n = 4$



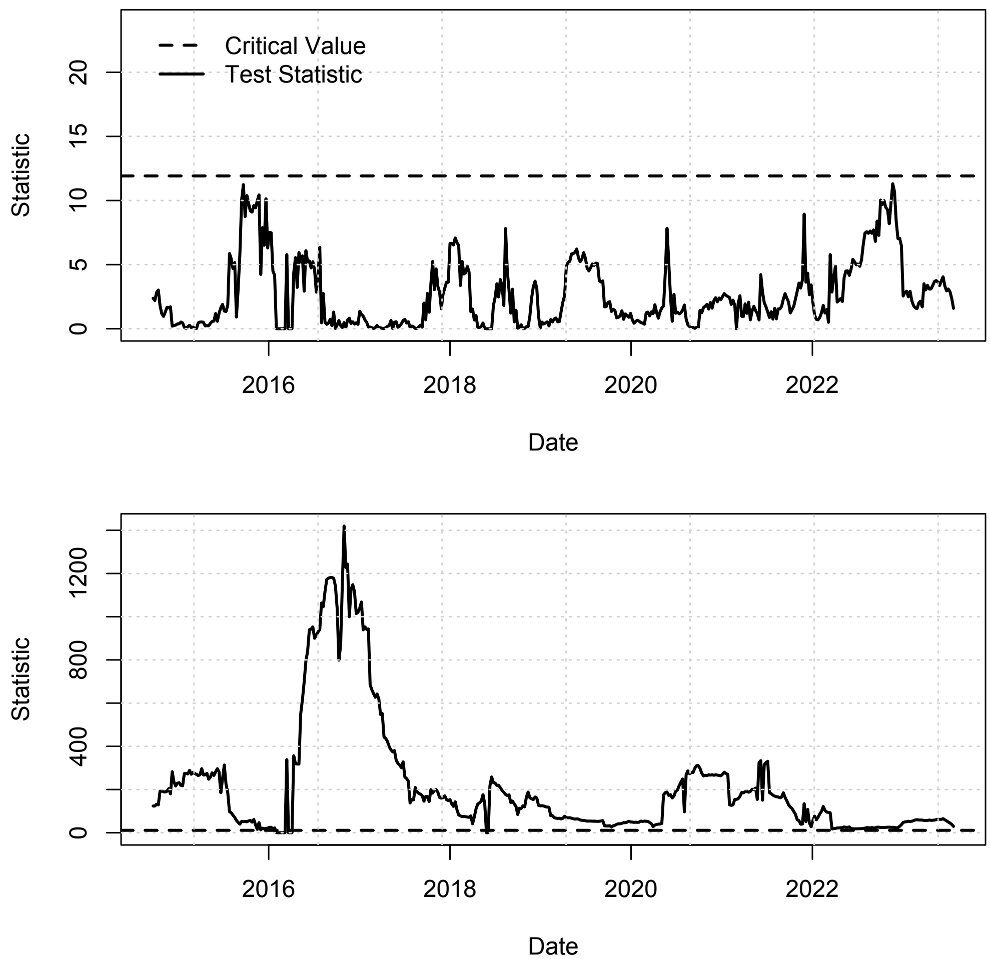
Explanations: The upper panel shows the the sequence $\mathcal{W}_n$. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_1}(1-0.05/90) = 11.92$. The lower panel shows the sequence of a Wald statistic sequence to test the joint hypothesis that all coefficients attached to the included lags of the error term are equal to zero. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_5}(1-0.05) = 11.07$.

Figure 7: Test statistic sequences for $n = 8$



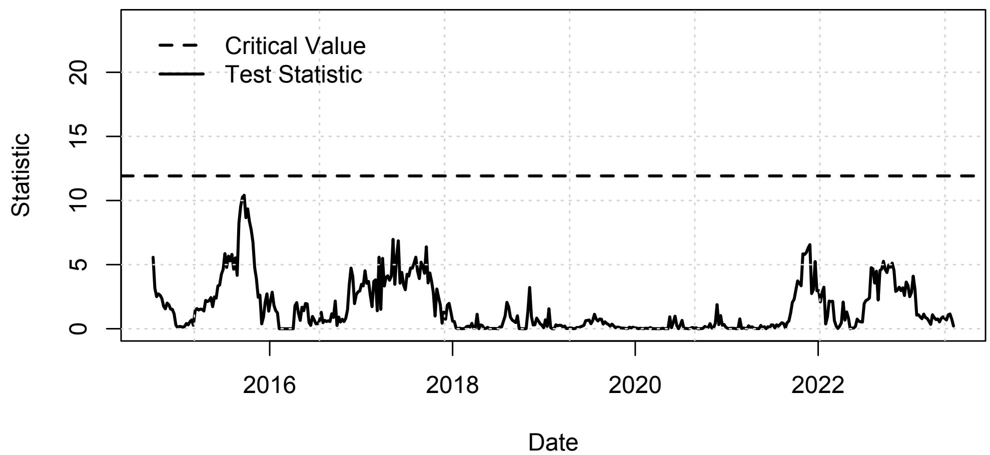
Explanations: The upper panel shows the the sequence $\mathcal{W}_n$. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_1}(1-0.05/90) = 11.92$. The lower panel shows the sequence of a Wald statistic sequence to test the joint hypothesis that all coefficients attached to the included lags of the error term are equal to zero. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_5}(1-0.05) = 11.07$.

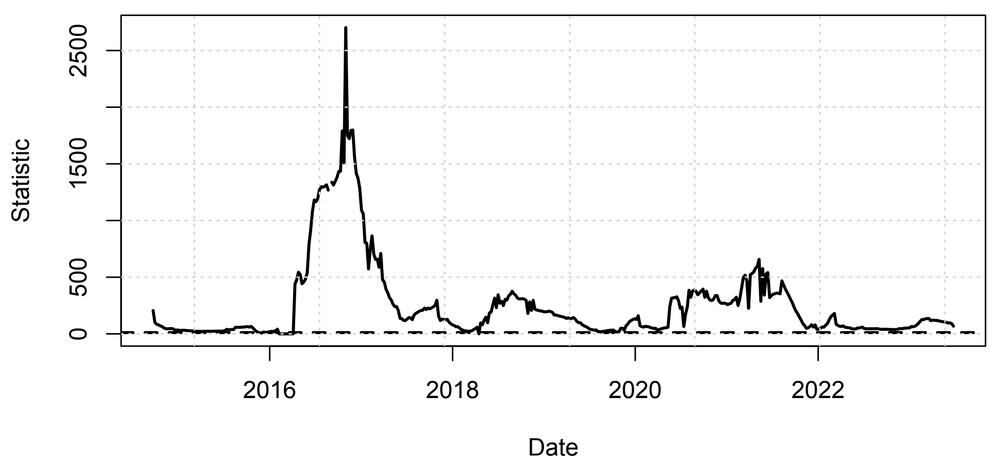
Figure 8: Test statistic sequences for $n = 12$



Explanations: The upper panel shows the the sequence $\mathcal{W}_n$. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_1}(1-0.05/90) = 11.92$. The lower panel shows the sequence of a Wald statistic sequence to test the joint hypothesis that all coefficients attached to the included lags of the error term are equal to zero. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_5}(1-0.05) = 11.07$.

Figure 9: Test statistic sequences for $n = 16$





Explanations: The upper panel shows the the sequence $\mathcal{W}_n$. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_1}(1-0.05/90) = 11.92$. The lower panel shows the sequence of a Wald statistic sequence to test the joint hypothesis that all coefficients attached to the included lags of the error term are equal to zero. The dashed line corresponds to the critical value $F^{-1}_{\chi^2_5}(1-0.05) = 11.07$.

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