Taxes on Labour and Unemployment in a Shirking Model with Union Bargaining

Lutz Altenburg \textsuperscript{a,*}, Martin Straub \textsuperscript{b}

\textsuperscript{a}FernUniversität Hagen, Hagen, Germany
\textsuperscript{b}Universität Bielefeld, Bielefeld, Germany

Abstract

This paper studies the impact of labour taxation in a shirking model with union bargaining. It is shown that if the ratio of unemployment compensation to the net-of-tax wage is kept fixed, a tax cut leads to higher unemployment. When unemployment benefits are kept fixed in real terms so that the benefits replacement ratio is allowed to change, the effect of a tax cut on unemployment is ambiguous. Adverse employment effects are ruled out if unions are powerless or the labor share is constant.

Keywords: Efficiency wages; Labour taxation; Unemployment; Union bargaining

JEL classification: E24; J32; J41; J51

* Corresponding author: Lutz Altenburg, E-mail: Lutz.Altenburg@FernUni-Hagen.de

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1 Introduction

It is often believed that the high tax burden on labour has been a major source of the high unemployment rates in Europe that have persisted over the past couple of decades. Correspondingly, in the economic policy debate a popular cure for Europe’s unemployment is to cut labour taxes (personal income taxes, employers’ and employees’ social security contributions). The basic argument is simply that lower taxes help to bring down the cost of labour and firms respond by increasing labour demand. Though there may be some truth in the argument, economists are typically more sceptical about the capability of tax cuts to lower unemployment. Labour taxes operate through the wedge they drive between the labour cost to an employer and the take-home pay of workers. And the empirical evidence on whether the tax wedge has significant effects on labour costs and unemployment is, at best, mixed.  

Nor do equilibrium models of unemployment predict that lower labour taxes are necessarily good for employment. 2 Recently, Pissarides (1998) has simulated the impact on equilibrium unemployment of a linear employment tax within four popular models of the labour market: competitive, union bargaining, job matching, and efficiency wages/shirking. Among the conclusions reached by Pissarides are the following. The response of equilibrium unemployment to a change in labour tax rates crucially depends on the system of unemployment compensation. If unemployment benefits are indexed to post-tax wages through a fixed replacement ratio, tax cuts are largely absorbed by increases in real wages and have only little (if any) beneficial impact on employment. If, however, unemployment benefits are fixed in real terms, all of the investigated models imply that the gain of employment can be sizeable.

Whereas Pissarides (1998) considered alternative models of equilibrium unemployment separately, in this paper we develop a framework that combines two explanations for unemployment: a modified version of the Shapiro-Stiglitz (1984) shirking model and decentralised union bargaining. 3 In this framework we study the effects of a cut in proportional labour taxes. In so doing we take up the issue addressed by Pissarides, namely that of the interaction between unemployment benefits and tax cuts.

A key feature of the Shapiro-Stiglitz model is that a worker has the choice between expending

1 See Layard et al. (1991) and, for a more recent report, Nickell and Layard (1997). Using regressions based on cross-sections for 20 OECD countries and controlling for a set of other variables, Nickell and Layard find that the overall tax burden on labour raises unemployment, though the effect is small.

2 As regards proportional labour taxes, apparently none of the existing models of equilibrium unemployment predicts that tax cuts are harmful to employment (though there are models in which they have no effect). However, things are different for progressive taxation (an issue not addressed in this paper). A cut in the marginal tax rate that leaves the average level unchanged (making the tax less progressive) can lead to higher unemployment. This is true in models where wages are determined by a bargain (e.g., Holmlund and Kolm, 1995) as well as in efficiency wage models (e.g., Hoel, 1990).

3 The model we use builds upon Altenburg and Straub (1998), who examined the impact of the unemployment benefits replacement ratio on unemployment, without considering labour taxes.
some fixed positive level of effort and not supplying any effort and, if he shirks, faces an exogenous probability of being caught and fired. We retain the assumption of a fixed detection probability but allow for firms to have discretion over the minimum effort level to require from its employees. Another feature of the Shapiro-Stiglitz model (like most other efficiency wage models) is that wages are unilaterally set by firms. This clearly contrasts with the fact that workers, particularly in Europe, typically have their wages determined by agreements between firms and unions. It has therefore been argued that it might be more natural to regard union bargaining and efficiency wage considerations as “complementary, mutually reinforcing explanations for unemployment” (Summers, 1988). Following this argument, our framework integrates employee shirking into a right-to-manage bargaining model. Firms and unions first bargain over wages, then firms choose employment as well as a minimum effort standard, taking wages as given. Workers whose performance does not come up to the required standard are fired if detected. As concerns production and factor supply, we use the standard assumptions of labour market models (which are also made by Pissarides, 1998). Labour is the only variable input, and there is a large fixed number of firms with a production function that exhibits diminishing returns; physical labour supply is treated as exogenous, with factor movements to or from the economy being ignored.

Our main finding is that when wages are determined by bargaining and firms choose both the effort standard and the employment level, a tax cut can lead to higher unemployment. In accordance with Pissarides (1998), we also find that the unemployment compensation system is critical to the results. If unemployment benefits are indexed to post-tax consumer wages, a tax cut always raises unemployment. In contrast, when unemployment benefits are held constant in real terms (implying that the benefit replacement ratio is allowed to change), lower labour taxes may either decrease or increase unemployment. The reason is that the effective labour input of firms can vary through a change in both the number of employed workers and the level of effort supplied by each worker. In particular, in response to a change in labour tax rates the employment level and the labour input measured in efficiency units can move in opposite directions. The way this happens is that when taxes are cut bargainers generally tend to agree on a higher take-home pay of workers. Despite this, the cost of labour per efficiency unit is likely to fall. For one thing, this has the commonly expected beneficial effects: firms increase their effective labour input, hence output, and make higher profits. But unemployment may rise rather than fall: firms may utilise their work force more effectively by asking greater effort of their employees; as a result they may end up employing a smaller number of workers than before though increasing their labour input in efficiency units. It turns out, however, that this is ruled out in the special cases where the labour share is constant or unions have no bargaining power.

Extensions of the Shapiro-Stiglitz model that allow for performance standards to be endogenously determined are not new. A variant close to our model is that by Simmons (1991). However, he maintains the assumption that wages are unilaterally set by firms and does not
consider labour taxation. 4 Our paper is also related to more recent research on the choice of working time (another dimension of “work intensity”) and the consequences of working time regulation. Moselle (1996) studies these issues in a shirking model, whereas Rocheteau (1999) uses a search-matching model with worker moral hazard. Models that combine efficiency wage considerations with union bargaining are not new either. They have been used to study a variety of issues (e.g., Hoel, 1989; Layard et al., 1991; Rødseth, 1993; Bulkley and Myles, 1996). But to our knowledge, there is still no attempt to analyse the effects of labour taxation within such a framework.

The paper is organised as follows. In Section 2 we develop the basic framework and show how a tax cut affects the bargaining outcome in an individual firm. Section 3 describes the labour market equilibrium for a fixed replacement ratio and examines the effects of a tax cut in this case. Section 4 studies the effects of a tax cut when benefits are held fixed in real terms. In order to assess the quantitative importance of our results, in Section 5 we solve the model numerically. Section 6 concludes.

2 The framework

We consider an economy with a large fixed number of identical workers. There are many identical firms, fixed in number too, producing a homogeneous good and each employing many workers. All firms are unionised, each bargaining with its own union. The decisions of each firm, its associated union and its employees are made in three stages. In stage one, the wage is determined by a Nash bargain between the firm and the union; in stage two the firm sets the level of employment; in stage three the firm chooses a minimum effort standard to be met by its employees, who then decide whether or not to comply with this standard. We confine attention to steady states.

2.1 The determination of effort

The model is set in continuous time. Workers live forever, and have an instantaneous utility $U(y, e) = (y^{\alpha}/\alpha) - (e^\theta/\theta)$, where $y$ is real income (measured in units of the single good produced) and $e$ is effort expended on a job. It is assumed that $0 < \alpha \leq 1$ and $\theta > 1$. 5 Let $w_y$ be the real pre-tax wage in a typical firm and $w \equiv (1 - \tau_w)w_y$ be the real net-of-tax wage (the consumer wage) received by its workers, where $0 < \tau_w < 1$ denotes the tax rate on labour income. Given

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4 In another variant of shirking models with effort taken as a continuous variable, pioneered by Sparks (1986), an employee’s probability of being caught shirking is assumed to vary inversely with the level of effort he chooses. A model of this type is used by Pisauro (1990) to analyse the effects of labour taxation on wages and unemployment, where the focus is on the case of unemployment benefits fixed in real terms.

5 Note that $1 - \alpha$ is the coefficient of relative risk aversion while $\theta$ is the elasticity of disutility from expending effort. Additive separable utility functions of a constant elasticity form are standard in the literature on efficiency wages with effort taken as a continuous variable (e.g., Sparks, 1986; Simmons, 1991).
the negotiated wage (determined in stage one) and the tax rate, the firm chooses a minimum effort standard, \(e\). An employed worker is then faced with the decision whether to shirk or work. Employees who exert effort at a level less than \(e\) (shirkers) face a risk of being separated from their job at rate \(\delta + q\) per unit time, while employees who comply with the effort standard (nonshirkers) lose their job at rate \(\delta\) per unit time. The parameter \(\delta\) is interpreted as the rate at which jobs break up and applies to all employees. The parameter \(q\) is the rate at which shirkers are detected and fired. Both rates are exogenous. \(^6\) Thus a shirker’s best choice is to supply zero effort, while a nonshirker’s best choice is to exert effort exactly at the required minimum level.

Let \(V^S\) be the expected lifetime utility of an employed shirker, \(V^N\) be the expected lifetime utility of an employed nonshirker, and let \(V^U\) be the expected lifetime utility of a currently unemployed worker. When selecting an effort level, the firm and its employees take \(V^U\) as given. Finally, let \(r\) denote the discount rate. The value function of a nonshirker then satisfies the following asset pricing equation

\[
rv^N = \frac{w^\alpha}{\alpha} - \frac{e^\theta}{\theta} + \delta (v^U - v^N). \tag{1}
\]

Similarly, the value function of a shirker satisfies

\[
rv^S = \frac{w^\alpha}{\alpha} + (\delta + q)(v^U - v^S). \tag{2}
\]

For an employee not to shirk at given values of \(w\) and \(v^U\), the effort standard set by the firm must be such that \(v^N \geq v^S\), or equivalently, using (1) and (2),

\[
e \leq \hat{e}(w, v^U) \equiv \left[\left(\frac{w^\alpha}{\alpha} - rv^U\right)\left(\frac{\theta q}{q + r + \delta}\right)\right]^{1/\theta}. \tag{3}
\]

This is the no-shirking condition: \(\hat{e}(w, v^U)\) is the highest possible level of effort the firm can demand from its workers to get them to work. But asking workers for an effort level lower than \(\hat{e}\) will not pay because it reduces profits. Hence, at the negotiated wage, the firm will choose an effort standard equal to \(\hat{e}\) so that \(v^N = v^S \equiv v\). For \(w^\alpha/\alpha > rv^U\), \(\hat{e}\) is positive, increasing in \(w\), and decreasing in \(v^U\). From \(\theta > 1\) and \(0 < \alpha \leq 1\), \(\hat{e}\) is strictly concave in \(w\).

### 2.2 Employment decision

Let \(N\) and \(L\) denote the number of workers employed by the firm and its effective labour input, respectively. As it is standard in the literature, we assume that \(L = eN\). The firm’s production function \(F(L)\) is assumed to be twice continuously differentiable, increasing and strictly concave, with \(F(0) = 0\) and \(F'(0) = \infty\). The firm’s labour cost per employee (the producer wage) is \((1 + \tau_f)w_g\), where \(\tau_f > 0\) is the tax rate on labour falling on firms. Since the firm will choose
an effort standard equal to \( \tilde{e}(w, V^U) \), the firm’s problem at stage two is

\[
\max_{\tilde{N}} F(\tilde{e}\tilde{N}) - (1 + \tau_f)w_0\tilde{N}.
\]

The first-order condition can be written as

\[
\tau F'(\tilde{e}\tilde{N})\tilde{e} = w, \tag{4}
\]

where

\[
\tau \equiv (1 - \tau_w)/(1 + \tau_f), \quad 0 < \tau \leq 1
\]

is the ratio of the consumer wage to the producer wage, being a measure of the tax wedge between the two. \(^7\) Equation (4) implicitly defines an employment function \( \tilde{N}(w, \tau, V^U) \).

### 2.3 Wage determination

We assume that wages are determined by a generalised Nash bargain between each firm and its union. In the literature, there is still no consensus about which union objective function to use. The most popular one is the utilitarian objective function, according to which the union cares not only about wages but also about employment. Some authors, however, have argued that trade unions might rather be concerned with the well-being of its employed members (insiders), neglecting the interests of unemployed persons (e.g., Weitzman, 1987; Oswald, 1993). Here we follow the latter approach. \(^8\) Correspondingly, the union objective is taken to be an employed worker’s expected discounted lifetime utility, \( V \). This implies that the union takes full account of the workers’ disutility from providing the required level of effort. If agreement is not reached, the firm is assumed not to take up production, leaving all of its workers unemployed. In that case the value of the union objective is taken as the expected lifetime utility of an unemployed worker, \( V^U \). Upon substitution of (3) into (1) we find that the union contribution to the Nash bargain is given by \( V - V^U = [(w^{\alpha}/\alpha) - rV^U]/(q + r + \delta) \) or, equivalently, \( (w^{\alpha}/\alpha) - rV^U \).

Let us now turn to the firm’s contribution to the Nash bargain. Substitute the employment function \( \tilde{N}(w, \tau, V^U) \) into the expression for profits to obtain

\[
F(\tilde{e}\tilde{N}) - (1 + \tau_f)w_0\tilde{N} = F(\tilde{e}\tilde{N}) - (w/\tau)\tilde{N}
\]

\[
= (1/\tau)\{\tau F[\tilde{e}(w, V^U), \tilde{N}(w, \tau, V^U)] - w\tilde{N}(w, \tau, V^U)\},
\]

\(^7\) The smaller the tax rates are, the larger is the value of \( \tau \). For the comparative statics effects we will study it does not matter whether it is firms or workers who pay the labour tax as long as taxation leads to the same value of \( \tau \). To write the model’s equations in terms of \( \tau \) rather than the tax rates is therefore analytically more convenient.

\(^8\) It can be shown that the basic qualitative results of the paper continue to hold when a utilitarian objective function is used so long as the elasticity of demand for effective labour is not increasing in \( L \). For a proof see Altenburg and Straub (2000).
which gives the maximum value of profits per unit time if a bargain is struck. If agreement is not reached, profits are assumed to be zero. The multiplicative constant $1/\tau$ is irrelevant to the Nash solution, the firm’s contribution simplifies to $\Pi(w) \equiv \tau F\tilde{e}(w, \cdot)\tilde{N}(w, \cdot) - w\tilde{N}(w, \cdot)$. The wage rate can be found by maximising $\Omega(w) \equiv \beta \log[(w^\alpha/\alpha) - rV_U] + (1 - \beta) \log \Pi(w)$ with respect to $w$, where $0 \leq \beta < 1$ denotes the union bargaining power. The first-order condition is $\Omega_w = \beta w^{\alpha-1} - rV_U + (1 - \beta) \tilde{N}(w, \cdot)\left[\varrho(w, V_U) - 1\right] = 0$, \hspace{1cm} (5)

where $\varrho \equiv w(\partial\tilde{e}(w, \cdot)/\partial w)/\tilde{e}(w, \cdot)$ is the elasticity of effort supply with respect to the firm’s own net–of–tax wage. Note that for $\beta > 0$ we have $\varrho < 1$, while in the limit case where the union is powerless and the firm sets the wage unilaterally ($\beta = 0$), (5) reduces to the well-known Solow condition $\varrho = 1$.

So far there is no guarantee that (5) has a solution. In order to ensure the existence of a unique solution, we need to impose a restriction on the class of production functions. Let $s \equiv (1 + \tau_f)w_gN/F = wN/\tau F$ be the gross share of labour in revenue. In view of (4), this is given by $s = LF'(L)/F(L)$. Then the following condition is supposed to hold.

**Assumption 1.** If $0 < \beta < 1$, then either $s''(L) < 0$ for $L > 0$ or $s$ is constant.

Suppose that the underlying technology of the firm is given by a production function with two inputs, capital (here kept fixed) and labour, and exhibits constant returns to scale. Then Assumption 1 is equivalent to the elasticity of substitution being smaller than or equal to one. Together with a minimal restriction on the parameter values (which in an equilibrium with $0 < \mu < 1$ turns out to be always satisfied), Assumption 1 ensures that the Nash product has a unique local and global maximum (for a proof see Appendix). Equation (5) thus determines the negotiated net wage as a function $\tilde{w}(\tau, V_U)$ of the tax parameter and the value for a worker of being unemployed. Substituting $\tilde{w}(\tau, V_U)$ back into (3) and (4) gives the firm’s optimal choices of the effort standard and employment.

### 2.4 Partial equilibrium effects of a tax cut

Before considering the general equilibrium effects of a tax cut, let us first look how it affects the bargaining outcome in an individual firm, with $V_U$ taken as given. Apart from the response of the negotiated wage and the levels of effort and employment, it is of interest to know the change in the effective labour input. From (4) we see that $L$ is negatively related to the cost per efficiency unit of labour given by $(1 + \tau_f)w_g/N/e = w/\tau e$. The latter is the key variable with regard to the tax incidence on firms: the higher the cost per efficiency unit of labour, the lower are profits. The results are summarised in the following proposition.

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9 The assumptions that the fall-back value of profits is zero (or, equivalently, equal to the negative of fixed costs) and the fall-back point for the union is equal to the value for a worker of being unemployed are often made in wage bargaining models, particularly in those ones in which an intertemporal setting is used (e.g., Layard and Nickell, 1990). For a motivation of modeling the parties’ behaviour this way see Booth (1995).
Proposition 1. (i) If $0 < \beta < 1$ and $s' < 0$, a tax cut increases the firm’s net-of-tax wage and the level of effort; the effect on the employment level is ambiguous;
(ii) if $\beta = 0$ or $s$ is constant, it leaves unaffected the net-of-tax wage and the level of effort but increases employment;
(iii) a tax cut reduces the firm’s cost per efficiency unit of labour, and therefore raises its effective labour input, output and profits.

Proof. See Appendix.

The interpretation of these results is as follows. At a given $w$, a tax cut causes the firm’s cost per efficiency unit of labour to fall, giving it an incentive to increase its effective labour input, $L = eN$. With $w$ and hence $e$ constant, the only way to do so is by increasing $N$. How does a higher $L$ affect the negotiated wage when $0 < \beta < 1$ and $s' < 0$? With $s' < 0$ a higher $L$ tends to reduce the proportional marginal cost to the firm (in terms of diminished profits) from a wage increase while leaving unaffected the proportional marginal benefit to the union from increasing the wage. As a consequence, $w$ goes up. This allows the firm to let its employees work harder. Despite the higher wage, the effective labour cost falls implying that the overall effect on $L$ is positive. By contrast, the overall effect of a tax cut on the firm’s employment level is ambiguous: the reduced labour cost at a given $w$ tends to increase $N$ while the higher negotiated wage tends to reduce it, and it cannot be said which of the sub-effects dominates.

In the limit case where unions are powerless, $w$ is determined by the Solow condition $g(w, V_U) = 1$. With $V_U$ taken as given, the efficiency wage cannot be affected by the tax wedge. There is, however, a positive impact effect on the firm’s level of employment: as already known, by reducing effective unit labour costs, the tax cut gives the firm an incentive to increase $L$. As the net wage remains unaltered, so too does the level of effort. Hence, the firm is induced to hire additional workers. 10

Proposition 1 highlights the critical role of Assumption 1 in determining the response of the negotiated net wage to a tax change. If it did not hold, i.e., if the labour share were increasing in $L$, the present model would predict that a cut in labour taxes (working in the same way as does a rise in total factor productivity) lowers the net wage at the individual firm level. It would therefore fail “to capture the basic intuition that when the firm does well, it pays higher wages” (Nickell, 1999). 11 Assumption 1 is irrelevant in the limit case where the union has no bargaining strength.

10 In the other special case where the labour share is constant the negotiated wage remains unaltered because the proportional marginal benefit to the union from increasing the wage and the proportional marginal cost to the firm are both unaffected by a change in the tax wedge.

11 The critical role of a non-increasing labour share is also a characteristic of standard trade union models with a union objective that puts no weight to employment. See Weitzman (1987).
3 A fixed benefit replacement ratio

3.1 Market equilibrium

We now turn to the analysis of an aggregate labour market equilibrium. At the firm level, the agents take $V_U$ as fixed. For the economy as a whole, however, $V_U$ depends on the choices of wages and employment levels in all firms. A first step is therefore to calculate the equilibrium $V_U$.

When unemployed, a worker is assumed to receive real unemployment benefits, $B$, which are untaxed. He finds a new job at some firm with probability $a$ per unit time. Since all firm-union pairs are identical, in equilibrium each will agree on the same wage, and, as a consequence, each firm must set the same effort standard. Thus, if re-employed, a worker’s expected lifetime utility takes on the same value, $V \equiv V^N = V^S$, everywhere. Then, by analogy with (1) and (2), we have

$$rV_U = \frac{B^\alpha}{\alpha} + a(V - V_U).$$

(6)

In steady state movements into and out of unemployment must balance. Since effort standards are chosen at a level that induces employees to work, a proportion $\delta$ of workers per unit time enters unemployment. Denoting the unemployment rate by $u$, we have $a = \delta(1 - u)/u$. Using this together with (2) in (6) yields

$$rV_U = \left(\frac{q + r + \delta}{q + r + \delta/u}\right)\frac{B^\alpha}{\alpha} + \left(\frac{\delta/u - \delta}{q + r + \delta/u}\right)\frac{w^\alpha}{\alpha}.$$

(7)

In this section the focus is on equilibria where unemployment benefits are indexed to the take-home pay of workers, i.e., the net replacement ratio, $b \equiv B/w$, is held fixed. In this case (7) becomes

$$rV_U = \left[1 - \frac{q + r + \delta}{q + r + \delta/u}(1 - b^\alpha)\right]\frac{w^\alpha}{\alpha}.$$

(8)

Substituting (8) into $\tilde{e}(w, V_U)$ given by (3), we obtain the aggregate effort supply function 12

$$e = e(w, u; b) \equiv \left[\frac{w^\alpha}{\alpha}(1 - b^\alpha)\frac{\theta q}{q + r + \delta/u}\right]^{1/\theta}.$$

(9)

It is defined for $0 \leq u \leq 1$ and $w \geq 0$, increasing in $w$ and $u$, and decreasing in $b$. It is also strictly concave in $w$, the elasticity with respect to $w$ being always smaller than one.

Since both the labour force size and the number of firms are fixed, physical labour units can, without loss of generality, be normalized so that the aggregate labour force divided by the number of firms is one. 13 Then a firm’s employment level $N$ is related to the unemployment

12 It corresponds to the aggregate no-shirking condition (holding with equality) in the original Shapiro-Stiglitz framework.

13 Notice that this normalization is made only for notational convenience. It does not mean that there are as many workers as firms. Rather, as noticed above, each firm should be thought of as employing many workers (i.e., many physical labour units).
率为 $u$ 由 $N = 1 - u$，其有效劳动输入 $L$ 给定为

$$L = (1 - u)e.$$  \hspace{1cm} (10)

一个常见的劳动市场均衡描述方式是，当失业率和工资由两个曲线在 $(u, w)$ 空间上的交点确定时，即一个总劳动需求曲线和一个工资设定曲线。在本文框架下，然而，更方便的描述均衡是通过在 $(u, L)$ 空间的两个曲线的交点。其中一个通过将 (9) 和 (10) 代入 (4)。这给出

$$\tau F'(L)L^{-(\theta - \alpha)/\alpha} = (1 - u)^{(q + r + \delta/u)\alpha/q\theta(1 - b^\alpha)}.$$

(11)

方程 (11) 定义了函数 $L(u)|_{LD}$ 对于 $0 < u < 1$，给出了有效劳动输入和失业率的组合，这些组合是与均衡劳动需求一致的。考虑到 $\lim_{L \to 0} F'(L) = \infty$，$L(u)|_{LD}$ 随着 $u$ 趋近于零或一趋于零。而且，$L(u)|_{LD}$ 在 $0 < \bar{u} < 1$ 定义的唯一最大值由 $(\theta - \alpha) = \delta(1 - \bar{u})/\bar{u}^2(q + r + \delta/\bar{u})$。注意 $\bar{u}$ 不依赖于 $b$ 和 $\tau$。函数 $L(u)|_{LD}$ 如图 1 中的 LD 曲线所示。

第二个方程在 $L$ 和 $u$ 通过使用 (3), (4) 和 (8) 在第一阶条件 (5)。这给出

$$\tilde{\varrho}(u; b) = \frac{(1 - \beta)s(L)}{(1 - \beta - \beta\theta)s(L) + \beta\theta},$$

(12)

其中

$$\tilde{\varrho}(u; b) \equiv \frac{(q + r + \delta/u)}{q + r + \delta} \frac{\alpha}{\theta(1 - b^\alpha)}$$

(13)

是努力供给弹性（与 $e(w, V^U)$ 相关）对一个企业的自身净税工资的弹性，在均衡（当工资处处相同时）。方程 (12) 是 $L$ 和 $u$ 之间的一个关系，必须满足均衡工资设定。让我们首先考虑 $0 < \beta < 1$ 和 $s'(L) < 0$ 的情况。在最小条件的情况下...
the parameter values (given in the Appendix) equation (12) defines a function $L(u)|_{WS}$ with domain $0 < u_0(b) \leq u < \pi \leq 1$ and $0 \leq b < \bar{b} < 1$. Since the left-hand side of (12) decreases with $u$ and its right-hand side decreases with $L$, $L(u)|_{WS}$ is strictly increasing in $u$. In Fig. 1 this function is depicted by the curve labeled WS.

The equilibrium values of the unemployment rate and the effective labour input $(u^*, L^*)$ are given by the intersection $E$ of the LD and WS curves. With $L$ and $u$ determined, (10) determines the level of effort; and given $e$ and $u$, (9) determines the net-of-tax wage, $w$.

In the two special cases where either unions are powerless ($\beta = 0$) or $s$ is constant, (12) reduces to an equation in $u$ alone, implying that WS becomes vertical and the equilibrium is unique. By contrast, when $0 < \beta < 1$ and $s' < 0$, our assumptions are not strong enough to rule out multiple equilibria. However, in what follows we ignore such cases and assume that the equilibrium is unique in the neighbourhood of the considered $\tau$.

### 3.2 Cutting labour taxes

We begin by considering the impact of an increase in $\tau$ on the unemployment rate and the effective labour input, focusing on the case where $0 < \beta < 1$ and $s' < 0$. Using the LD and WS schedules (equations (11) and (12)) makes this quite simple an exercise which can be done without any tedious calculations. An increase in $\tau$ (a cut in labour taxes) shifts the LD curve upward, as depicted by the dotted line in Fig. 2. The WS curve is not affected. And since it slopes upward, the upshot is an increase in both unemployment and the effective labour input (hence, output). Recognising that along WS workers’ effort increases with $u$, we also find that an increase in $\tau$ results in a higher level of effort. Finally, from $F'(L) = w/\tau e$ and $F''(L) < 0$ it then follows that a higher $L$ comes about with lower effective unit labour costs, $(1+\tau_f)w_g/e = w/\tau e$, and thus higher profits.

To ascertain the impact of a tax cut on the net-of-tax wage it suffices to know how $w$ varies with $u$ along the WS curve. Consider the case where $s' < 0$ and $0 < \beta < 1$. From (10) we have $e = L/(1-u)$. Combining this with (9) and substituting $L(u)|_{WS}$ for $L$ we obtain

$$w(u)|_{WS} = h(u)[L(u)|_{WS}]^{\theta/\alpha},$$  \hspace{1cm} (14)

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14 Fig. 1 shows the case where $L(u)|_{WS}$ is defined for $u_0(b) \leq u < \pi = 1$ and tends to a finite positive value as $u$ tends to one. Another possible case is one where $L(u)|_{WS}$ is defined for $u_0(b) \leq u < \pi \leq 1$ and goes to infinity as $u$ tends to $\pi$. For a proof of existence of equilibrium see Appendix.

15 Uniqueness of equilibrium will clearly be ensured if there is no intersection of the LD and WS schedules for $u_0(b) < u < \bar{u}$. Conditions sufficient for this to be the case are: (i) $\bar{u} \leq u_0(b)$; or (ii) if $\bar{u} > u_0(b)$, then $L(\bar{u})|_{WS} \leq L(u_0)|_{LD}$.

16 If there were multiple equilibria, there would be at least three intersections between the LD and the WS curve. The equilibrium associated with the lowest unemployment rate has the property that the WS curve intersects the LD curve from below. Accordingly, its comparative-statics properties would be precisely those we are going to analyse below.
where \( h(u) \equiv (1-u)^{-\theta/\alpha}[\alpha(q+r+\delta/u)/\theta q(1-b^\alpha)]^{1/\alpha} \). It is easy to check that \( h'(u) \leq (>) 0 \) as \( u \leq (>) \tilde{u} \), with \( \tilde{u} \) defined by \( \theta = \delta(1-\tilde{u})/\tilde{u}^2(q+r+\delta/\tilde{u}) \). Since \( L(u)|_{WS} \) is strictly increasing in \( u \), we immediately find that the following condition is sufficient for \( w(u)|_{WS} \) to be increasing with \( u \) and thus the consumer wage to increase with \( \tau \).

**Assumption 2.** *Equilibrium unemployment \( u^* \) is such that \( \tilde{u} < u^* \).*

Noting that \( \delta(1-u)/u^2(q+r+\delta/u) \) is strictly decreasing in \( u \), we see that \( \tilde{u} < \tilde{\tilde{u}} \). Assumption 2 can be shown to be equivalent to assuming that, as seems plausible, firms get a higher amount of effective labour from their work-force as each of them increases employment at a given net wage. 17 But then the only way a smaller number of workers can be led to provide a higher amount of effective labour is by increasing the net wage (thus making them work harder than they already do, given the higher unemployment rate).

Let us now take a glance at the special cases where either unions are powerless (\( \beta = 0 \)) or the gross labour share is constant. Clearly, a change in \( \tau \) leaves unemployment unaffected because in either case the WS schedule is vertical (and independent of \( \tau \)). The effects on \( L, e, \) and \( (1 + \tau_f)w_g/e \) have the same signs as in the case where \( 0 < \beta < 1 \) and \( s' < 0 \). 18 As now the level of effort goes up while the unemployment rate remains constant, the net-of-tax wage unambiguously rises. We sum up the previous findings in the following proposition.

---

17 Substituting (9) into (10) and differentiating with respect to \( u \) gives

\[
\frac{\partial(1-u)e(w, u; b)}{\partial u} = -e + (1-u)e_u = e \left[ \frac{\delta(1-u)}{\theta u^2(q+r+\delta/u)} - 1 \right] \geq (>) 0
\]

as \( u \leq (>) \tilde{u} \).

18 Since employment remains constant, \( e \) increases in the same proportion as \( L \).
Proposition 2. Suppose the benefit replacement ratio is kept fixed. Then a cut in ad valorem labour taxes
(i) raises the effective labour input and output, reduces the cost per efficiency unit of labour, therefore raising profits, and increases the level of effort, regardless of whether or not unions have bargaining strength and whether the gross labour share decreases with \( L \) or is constant;
(ii) if \( 0 < \beta < 1 \) and \( s' < 0 \), it increases unemployment and, if Assumption 2 holds, raises the net–of–tax wage;
(iii) if \( \beta = 0 \) or \( s \) is constant, it leaves unemployment unaffected but increases the net–of–tax wage.

The most striking result is that, when the replacement ratio is fixed, a cut in labour taxes unambiguously raises unemployment provided that unions have bargaining strength. The apparent paradox is due to the fact that in response to a change in tax rates the number of workers and the labour input in efficiency units move in opposite directions. At a given \( w \), lower taxes reduce the costs of labour per efficiency unit, which makes it profitable for firms to let a given number of employees work harder than before so as to raise \( L \) (as indicated by the upward shift of the LD curve). Clearly, for this to occur, net wages must rise at a given employment level. To see how to reconcile this with equilibrium wage setting, rewrite equation (12) as

\[
\beta \theta \tilde{\rho}(u; b) = (1 - \beta)(1 - \tilde{\rho}(u; b))s(L)/(1 - s(L)).
\]

The terms \( \theta \tilde{\rho} \) and \( (1 - \tilde{\rho})s/(1 - s) \) represent, respectively, the elasticity of an employed worker’s excess utility and the (absolute) elasticity of profits with respect to a firm’s own net wage (evaluated in equilibrium). The former is decreasing in \( u \), while the latter is increasing in \( u \) and, by \( s' < 0 \), decreasing in \( L \). Since \( L \) must rise at a given \( N \), equilibrium wage setting cannot be maintained at the initial employment level. A higher \( L \) lowers the marginal proportional cost to a firm from increasing \( w \) while leaving unchanged the marginal proportional benefit to the union. Negotiated wages tend to rise, which in turn works towards lower employment and higher effort. To restore equilibrium, unemployment must in fact rise, while workers end up providing extra effort to such an extent that despite their reduced number they provide more labour in efficiency units than before. Eventually, this may (or may not) come about with an increase in equilibrium net wages. It certainly does if Assumption 2 is satisfied. Otherwise net wages might fall if the effort-enhancing effect of unemployment is sufficiently large. 19

4 A fixed level of unemployment benefits

So far we have assumed that when taxes are cut, the ratio of unemployment benefits to post-tax wages is held fixed. In this section we assume that unemployment benefits are held fixed in real

---

19 Note that the general equilibrium effects on unemployment of a tax cut contrast sharply with the effects at the firm level when \( V^U \) is taken as fixed. According to Proposition 1, in the case where \( s' < 0 \) and \( \beta > 0 \) a tax cut has an ambiguous effect on the firm’s employment level, and when \( s \) is constant or unions are powerless it unambiguously raises employment.
terms, which implies that the benefit replacement ratio will change as taxes are cut. This case can most easily be analysed by utilising a link that exists between the equilibria for the two policy regimes. Holding \( B \) constant means that in equilibrium the now variable replacement ratio and the consumer wage must satisfy

\[
 bw = B = \text{const.}
\]

The system describing an equilibrium for fixed real benefits thus comprises (9), (10), (11), (12), and (15) with the unknowns \( u, w, L, e, \) and \( b \). Uniqueness of equilibrium for a given \( b \) is no guarantee for uniqueness of equilibrium with \( B \) taken as a parameter. The latter is ensured as well if the following conditions are supposed to hold (for a proof see Appendix).

**Assumption 3.** (i) In a neighbourhood of the considered \( \tau \) the equilibrium for a fixed \( b \) is unique for all \( 0 \leq b < \bar{b} \); and (ii) \( bw^*(\tau, b) \) is strictly monotonically increasing in \( b \) on \( [0, \bar{b}) \), where \( w^*(\tau, b) \) is the equilibrium consumer wage with \( b \) taken as a parameter.

Condition (ii) is a plausible one. It requires that the higher the level of the replacement ratio is chosen, the higher is the implied level of real unemployment benefits. In the limit cases where unions are powerless or the gross labour share is constant, Assumption 3 is always satisfied because then \( \partial w^*/\partial b > 0 \) (see Lemma 1 below).

We now turn to the impact of a cut in wage taxes on the unemployment rate, the effective labour input, and the consumer wage. Let \( u^*(\tau, b), L^*(\tau, b), \) and \( w^*(\tau, b) \) represent an equilibrium with \( b \) taken as a parameter, and let \( u^{**}(\tau, B), L^{**}(\tau, B), w^{**}(\tau, B), \) and \( b^{**}(\tau, B) \) denote an equilibrium for fixed real benefits \( B \), with \( b^{**}(\tau, B) \) defined by \( b^{**}w^*(\tau, b^{**}) = B \). Then we have the identities

\[
 x^{**}(\tau, B) = x^*(\tau, b^{**}(\tau, B)), \quad x = u, L, w,
\]

\[
 b^{**}(\tau, B)w^*(\tau, b^{**}(\tau, B)) = B.
\]

Differentiating (16) and (17) with respect to \( \tau \) gives, respectively,

\[
 \frac{\partial x^{**}}{\partial \tau} = \frac{\partial x^*}{\partial \tau} + (\frac{\partial x^*}{\partial b})(\frac{\partial b^{**}}{\partial \tau}), \quad x = u, L, w,
\]

\[
 \frac{\partial b^{**}}{\partial \tau} = -\frac{b(\frac{\partial w^*}{\partial \tau})}{w^* + b(\frac{\partial w^*}{\partial b})}.
\]

Equations (18) tell us that with unemployment benefits kept fixed the impact of a tax cut on the variables considered can be thought of as being decomposed into two sub-effects. In addition to the already known effects that would occur at a constant replacement ratio there are the effects due to the ensuing change in the replacement ratio. Assumption 3 implies that \( w^* + b(\frac{\partial w^*}{\partial b}) > 0 \) so that, by (19), \( \text{sign}(\partial b^{**}/\partial \tau) = (-1)\text{sign}(\partial w^*/\partial \tau) \), and, from (18) and

\[20\] A change in the benefit replacement ratio also comes about when unemployment benefits are linked to wage taxes through a government budget constraint. It can be shown that if the whole revenue from wage taxes is used to finance unemployment benefits, the qualitative effects of a tax cut are similar to those obtained when unemployment benefits are held constant in real terms; see Altenburg and Straub (2000).
The effects of \( \tau \) on \( L \) and \( u \) depend on the sign (and possibly the magnitude) of \( \partial L^*/\partial b \) and \( \partial u^*/\partial b \), respectively. Before examining the impact of a tax cut we therefore need to establish the following facts concerning the impact of an increase in \( b \). \(^{22}\)

**Lemma 1.** Suppose that \( b \) is taken as a parameter. Then (i) \( \partial L^*/\partial b < 0 \); (ii) if \( 0 < \beta < 1 \) and \( s' < 0 \), both \( \partial u^*/\partial b \) and \( \partial w^*/\partial b \) can be of either sign; (iii) if \( \beta = 0 \) or \( s \) is constant, \( \partial u^*/\partial b > 0 \) and \( \partial w^*/\partial b > 0 \).

**Proof.** See Appendix.

Making use of Lemma 1 and Proposition 2, together with the numerical results presented below, we get from (18) the following:

**Proposition 3.** When unemployment benefits are held fixed in real terms, the impact of a cut in ad valorem labour taxes is as follows:

(i) If \( 0 < \beta < 1 \) and \( s' < 0 \), then there are cases where \( \partial u^{**}/\partial \tau < 0 \) as well as cases where \( \partial u^{**}/\partial \tau > 0 \); if Assumption 2 holds (implying \( \partial w^*/\partial \tau > 0 \)), then \( \partial L^{**}/\partial \tau > 0 \) and \( \partial w^{**}/\partial \tau > 0 \);

(ii) if \( \beta = 0 \) or \( s \) is constant, then \( \partial u^{**}/\partial \tau < 0 \), \( \partial L^{**}/\partial \tau > 0 \), and \( \partial w^{**}/\partial \tau > 0 \).

The effects of an increase in \( \tau \) on \( L \) and \( u \) may again be illustrated by shifts in the LD and WS curves. We already know that an increase in \( \tau \) shifts the LD curve upward while leaving the WS curve unaffected. In addition there are the indirect effects due to the induced change in \( b \). Suppose that the replacement ratio falls as \( \tau \) increases. For one thing, the smaller \( b \) raises the level of effort at a given employment level and so causes the LD curve to shift up further. A smaller \( b \) also lowers \( \tilde{\varrho}(u, b) \) at a given \( u \) and thus shifts the WS curve to the left. This explains at once why in the limit cases \( (\beta = 0 \text{ or } s = \text{const.}) \) where WS is vertical the unemployment rate falls as \( \tau \) increases. Even in these special cases the curve shifts do not give a clear answer about the change in \( L \). In the case of \( 0 < \beta < 1 \) and \( s' < 0 \) the effects on both \( L \) and \( u \) appear ambiguous. However, as Lemma 1 shows, \( L \) decreases with \( b \) in any case, which implies that the indirect effect on \( L \) (working through a decrease in \( b \)) tends to reinforce the positive direct effect. By contrast, the ambiguous response of the unemployment rate is in fact proven by the numerical examples presented below. If \( \partial u^*/\partial b > 0 \) and \( \partial b^{**}/\partial \tau < 0 \), the indirect effect of a tax cut tends to make unemployment lower. This effect may or may not be powerful enough to dominate the deleterious direct effect. If \( \partial u^*/\partial b < 0 \) and \( \partial b^{**}/\partial \tau < 0 \), then the indirect effect reinforces the direct effect, giving an unambiguous increase in unemployment.

\(^{21}\) Inserting (19) into (18) gives

\[
\frac{\partial w^{**}}{\partial \tau} = \frac{w^*}{w^* + b(\partial w^*/\partial b)} \frac{\partial w^*}{\partial \tau}.
\]

\(^{22}\) For an interpretation of these effects, see Altenburg and Straub (1998).
5 Numerical results

In this section we present the results from simulating the model for each of the two cases considered above: a fixed replacement ratio and fixed real benefits. One aim is to assess how much of an adverse impact on employment tax cuts might have when the replacement ratio is fixed. Another is to show that there are indeed circumstances in which lower labour taxes lead to higher unemployment when unemployment benefits are kept constant in real terms. The union objective function is taken to be the insider dominated one. As in Pissarides (1998), the production function is assumed to be CES, with two inputs, labour and capital, the fixed capital input being normalized to one. Hence,

\[ F(L) = A \left[ c + (1 - c)L^\psi \right]^{1/\psi} \]

with \( A > 0, 0 < c < 1, \) and \( \psi < 0 \). Note that the assumption \( s'(L) < 0 \) is equivalent to \( \psi < 0 \), which implies an elasticity of substitution between labour and capital, \( \sigma \), smaller than one.

In the case of a constant replacement ratio we set \( b = 0.6 \), while in the case of a constant \( B (= bw^*(\tau, b)) \) the latter is fixed at a level where \( b = 0.6 \) at \( \tau = 0.6 \). We report the results for two sets of parameter values in detail. In our first example the remaining parameters are specified as: \( A = 1, \ c = 0.3, \ \psi = -0.4286 \) (\( \sigma = 0.7 \)), \( \alpha = 0.2, \ \theta = 8, \ r = 0.05, \ \delta = 0.1 \), \( q = 0.7 \), and \( \beta = 0.2096 \). The parameter values of the CES production function and the coefficient of relative risk aversion, \( 1 - \alpha = 0.8 \), are identical with those assumed by Pissarides (1998). The inspection rate \( q \) (about which there is no evidence) is close to that used by him in his shirking model. Nor is there direct evidence about the elasticity of disutility of effort, \( \theta \). If effort is viewed as a dimension of labour supply, \( \theta \) might be considered as being related to the intertemporal elasticity of substitution in labour supply; \( \theta = 8 \) may then be taken as a reasonable value, implying an intertemporal substitution elasticity equal to \( 1/(\theta - 1) \approx 0.14 \). If a period of unit length is assumed to be a year, the value of the separation rate \( \delta \) implies that the expected duration of employment for a nonshirker is 10 years, while that for a shirker, \( 1/(\delta + q) \), is 15 months. The value of the union power \( \beta \) is calibrated to give 10% unemployment in the absence of taxes (\( \tau = 1 \)) for \( b = 0.6 \).

Table 1 reports the results for our first parameter set. They show that there are only tiny adverse employment effects of tax cuts at a constant replacement ratio. The effects on the effective labour input are pretty small too. This is because the macro elasticity of effort with respect to \( w, \alpha/\theta \), is only 0.025, implying little variability in the level of effort. As a consequence, tax cuts are nearly entirely absorbed by increases in the take-home pay of workers. By contrast, when benefits are fixed in real terms, tax cuts lead to considerable gains of employment. These are due to the accompanying decreases in \( b \), which tend to lower unemployment because in this

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23 We also simulated the case where unions have a utilitarian objective. The results showed quantitative effects of tax policy quite similar to those obtained under an insider dominated objective function.

24 For adult males the available estimates of the intertemporal elasticity of substitution in labour supply are within the -0.07 to 0.45 range. See Pencavel (1986, Table 1.22).
### Table 1
**Numerical solution: first parameter set**

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( u(%) )</th>
<th>( L )</th>
<th>( e )</th>
<th>( w )</th>
<th>( \text{Constant } b )</th>
<th>( u(%) )</th>
<th>( L )</th>
<th>( e )</th>
<th>( w )</th>
<th>( \text{Constant } B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9.94</td>
<td>0.928</td>
<td>1.030</td>
<td>0.372</td>
<td>18.27</td>
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<td>1.029</td>
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<td>0.694</td>
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<td>1.035</td>
<td>0.448</td>
<td>9.96</td>
<td>0.932</td>
<td>1.035</td>
<td>0.448</td>
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<td></td>
</tr>
<tr>
<td>0.7</td>
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<td>0.936</td>
<td>1.040</td>
<td>0.524</td>
<td>6.91</td>
<td>0.968</td>
<td>1.040</td>
<td>0.517</td>
<td>0.520</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>9.98</td>
<td>0.939</td>
<td>1.043</td>
<td>0.600</td>
<td>5.44</td>
<td>0.987</td>
<td>1.044</td>
<td>0.588</td>
<td>0.457</td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>9.99</td>
<td>0.940</td>
<td>1.045</td>
<td>0.638</td>
<td>4.97</td>
<td>0.994</td>
<td>1.046</td>
<td>0.624</td>
<td>0.431</td>
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</tr>
<tr>
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<td>0.942</td>
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<td>0.676</td>
<td>4.59</td>
<td>0.999</td>
<td>1.047</td>
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<td>0.407</td>
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</tr>
<tr>
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<td>0.714</td>
<td>4.28</td>
<td>1.004</td>
<td>1.049</td>
<td>0.696</td>
<td>0.386</td>
<td></td>
</tr>
</tbody>
</table>

Parameter values: \( A = 1, \ c = 0.3, \ \psi = -0.4286 (\sigma = 0.7), \ \alpha = 0.2, \ \theta = 8, \ r = 0.05, \ \delta = 0.1, \ q = 0.7, \ \beta = 0.2096. \)

The expected duration of employment for a nonshirker is now 20 years, while that for a shirker is slightly less than 6 months (the same values of \( \delta \) and \( q \) are used in a recent paper by Albrecht and Vroman, 1999). Again \( \beta \) is calibrated to give 10% unemployment at a zero-tax equilibrium for \( b = 0.6. \)

In our second example the parameters are specified as follows: \( A = 1, \ c = 0.3, \ \psi = -9 (\sigma = 0.1), \ \alpha = 1, \ \theta = 5, \ r = 0.05, \ \delta = 0.05, \ q = 2, \) and \( \beta = 0.3214. \) As compared to the first example, the characteristic points are a low elasticity of factor substitution \( \sigma, \) a smaller separation rate \( \delta, \) a larger inspection rate \( q, \) and a more elastic response of effort to changes in \( w. \)

The results of this simulation set are reported in Table 2. With a fixed replacement ratio tax cuts have now comparatively large adverse employment effects. For example, increasing \( \tau \) from 0.7 to 0.8 raises unemployment by a considerable 1.4 percentage points. A further peculiar feature is that when unemployment benefits are fixed in real terms, the relationship between the unemployment rate and the tax wedge is U-shaped: for \( 0 < \tau < 0.68 \) unemployment decreases with \( \tau \) but for higher values of \( \tau \) tax cuts cause unemployment to rise. Remarkably, this happens to be the case despite the fact that \( u^*(\tau, b) \) is again monotonically increasing in \( b \) so that the indirect effect of a tax cut (working through a decrease in \( b \)) always tends to lower unemployment. The reason is that for low tax rates (high values of \( \tau \)) the indirect effect is outweighed by the large adverse direct effect.

\[25\] The expected duration of employment for a nonshirker is now 20 years, while that for a shirker is slightly less than 6 months (the same values of \( \delta \) and \( q \) are used in a recent paper by Albrecht and Vroman, 1999). Again \( \beta \) is calibrated to give 10% unemployment at a zero-tax equilibrium for \( b = 0.6. \)
Table 2
Numerical solution: second parameter set

<table>
<thead>
<tr>
<th>τ</th>
<th>u(%)</th>
<th>L</th>
<th>e</th>
<th>w</th>
<th>u(%)</th>
<th>L</th>
<th>e</th>
<th>w</th>
<th>b</th>
</tr>
</thead>
<tbody>
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<td>0.829</td>
<td>0.857</td>
<td>0.410</td>
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<td>0.786</td>
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<td>4.11</td>
<td>0.869</td>
<td>0.906</td>
<td>0.498</td>
<td>4.11</td>
<td>0.869</td>
<td>0.906</td>
<td>0.498</td>
<td>0.600</td>
</tr>
<tr>
<td>0.7</td>
<td>5.22</td>
<td>0.901</td>
<td>0.950</td>
<td>0.583</td>
<td>3.81</td>
<td>0.920</td>
<td>0.956</td>
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<td>0.533</td>
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<tr>
<td>0.8</td>
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<td>0.960</td>
<td>1.003</td>
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</tr>
<tr>
<td>0.85</td>
<td>7.46</td>
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<td>1.008</td>
<td>0.709</td>
<td>4.67</td>
<td>0.977</td>
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<td>0.651</td>
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<td>1.045</td>
<td>0.676</td>
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<tr>
<td>0.95</td>
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<td>1.042</td>
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<td>5.68</td>
<td>1.004</td>
<td>1.065</td>
<td>0.699</td>
<td>0.428</td>
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</table>

Parameter values: \( A = 1, \ c = 0.3, \ \psi = -9 \ (\sigma = 0.1), \ \alpha = 1, \ \theta = 5, \ r = 0.05, \ \delta = 0.05, \ q = 2, \ \beta = 0.3214. \)

Finally, it should be noted that it is easy to find examples where the relationship between unemployment and the replacement ratio at a given \( \tau \) is U-shaped: up to some critical value of \( b \) unemployment decreases with \( b \) but thereafter increases with it. Therefore, when real unemployment benefits are constant, in those cases, too, there may exist a possibly wide range of sufficiently small tax rates over which unemployment will rise as taxes are cut (it certainly does if \( b \) is monotonically decreasing with \( \tau \)). Now the reason is that over that range the indirect effect of a tax cut (operating via a decrease in \( b \)) tends to increase unemployment, thus adding to the deleterious direct effect. \(^{26}\)

6 Conclusions and remarks

The paper has studied the impact of a cut in proportional wage taxes in a model which combines two explanations for unemployment: employee shirking and union bargaining. In contrast to Shapiro and Stiglitz (1984), we have allowed for an endogenously determined effort standard. Combined with wage bargaining, this leads to results that depart from those obtained in both the standard right-to-manage bargaining model and the original framework of Shapiro and Stiglitz. A tax cut may now be absorbed by a rise not only in net wages but also in the level of effort, without boosting employment. At worst, a tax cut may be damaging to employment. The main conclusions are the following. First, when the benefit replacement ratio is kept fixed, \(^{26}\)

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\(^{26}\) An appropriate example is provided by the following set of parameter values: \( A = 1, \ c = 0.3, \ \psi = -9 \ (\sigma = 0.1), \ \alpha = 0.2, \ \theta = 5, \ r = 0.05, \ \delta = 0.2, \ q = 0.7, \) and \( \beta = 0.6. \) They were chosen to give functions \( u^*(\tau, b) \) and \( w^*(\tau, b) \) that are both non-monotonic in \( b \): if \( \tau = 0.6, \ u^* \) is minimised at \( b \approx 0.36 \) and \( w^* \) attains a maximum at \( b \approx 0.57 \) (note that this proves statement (ii) in Lemma 1). Again, when unemployment benefits are fixed in real terms, the relationship between \( u \) and \( \tau \) turns out to be U-shaped.
a cut in wage taxes leads to higher unemployment (by contrast, the Shapiro-Stiglitz model predicts a beneficial effect on employment while in the standard union model there is no effect). Second, when unemployment benefits are held constant in real terms, a tax cut may either decrease or increase unemployment (rather than decreasing it unambiguously). Third, if either the technology is Cobb-Douglas or firms set efficiency wages, a harmful impact of a tax cut on employment is ruled out: with a fixed replacement ratio, unemployment remains unchanged (while it would fall in the Shapiro-Stiglitz model); when unemployment compensation is fixed in real terms, unemployment falls. It is worth noting that even in cases where unemployment remains unaffected, a tax cut reduces the costs per efficiency unit of labour and increases profits. This too contrasts with the results obtained in the standard right-to-manage model. The latter predicts that if the replacement ratio is fixed and worker utility is isoelastic, a change in labour taxes affects neither employment nor wage costs, implying that these taxes are borne entirely by labour.

Our numerical computations, using a CES production function, have shown that a key parameter in determining the impact of labour taxation on employment in a unionised economy is the elasticity of factor substitution. If it has a not too small magnitude, the adverse employment effect of a tax cut that results in the case of a fixed replacement ratio is small (if not negligible); moreover, in cases where real benefits are held constant, tax cuts always boost employment, just as they do in standard labour market models. However, should the elasticity of factor substitution be small, the model’s predictions with regard to tax policy are not that bright. Not only can the negative employment effect of a tax cut be sizeable when unemployment benefits are indexed to post-tax wages. If initial wage taxes are low, further reductions in tax rates can also raise unemployment in cases where unemployment benefits are held fixed in real terms.

The model presented in this paper offers a tractable framework for analysing the interaction between union bargaining and employee shirking. However, as some of its basic assumptions are controversial, a few remarks on these should be in order. An assumption essential to the results is that wages are determined by the Nash bargaining solution, with the disagreement point for the union equal to the value for a worker of being unemployed. Though being widely used, this approach has been criticised for the reason that the choice of the disagreement point does not seem very realistic. An alternative way of modeling wage bargaining would be to account for the fact that in the event of a conflict workers usually remain attached to the firm. In that case the parties may have options different from entering unemployment and not taking up production, respectively. These include hold-outs, strikes and lock-outs all of which can be expected to have different implications for the disagreement point and, therefore, the bargaining outcome (e.g., Moene, 1988).

These reservations aside, one could also adopt a model in which the firm and the union bargain over the minimum effort standard as well as wages. In such a framework we expect two types of equilibria to arise, depending on whether the no-shirking condition is binding or not. If it is binding, bargaining over wages and effort should be equivalent to bargaining over wages alone, which would imply that our findings remain unaltered in that case. However, the results
might be significantly affected when the no-shirking condition is not binding. In that case the bargaining outcome allows workers to expend less effort than the firm could require without inducing workers to shirk. Equilibria of a similar type have recently been analysed by Rocheteau (1999), though in a different framework.

A further distinguishing feature of our model is the timing of decisions, namely the assumption that the employment level is chosen by the firm after the wage is set. One might wonder about the changes induced by assuming that firms set employment prior to wage bargaining (e.g., Holden, 1988; Moene, 1988), while maintaining the assumption that the effort standard is chosen in the final stage.
Appendix

Maximum of the Nash product

Let $\bar{\pi} \equiv \sup s(L) \leq 1$. Then under Assumption 1

(i) the second-order condition for a maximum of the Nash product is satisfied; and

(ii) there exists a solution to (5) for any $V^U > 0$ if

$$\alpha \beta \theta < [\alpha \beta \theta + (\theta - \alpha)(1 - \beta)]\bar{\pi}. \quad (A1)$$

Proof. (i) Suppose that $\tilde{w}$ is a solution to (5). Using $\eta = -F'/LF''$, $s = LF'/F$, $\Pi_w = N(\rho - 1)$, and $\tilde{N}_w = -(N/w)[\eta + \rho/(1 - \eta)]$, we get from (5) for $w = \tilde{w}$:

$$\Omega_{ww}(\tilde{w}) = \frac{1}{w} \left\{ \frac{\beta \alpha w^{\alpha - 1} r V^U}{(\alpha / \omega) - r V^U} + \frac{(1 - \beta) s w}{1 - s} + (1 - \beta) (\rho - 1)^2 N[\eta(1 - s) - 1] \right\}. \quad (A2)$$

From $\rho = w^\alpha / \theta (w^\alpha / \alpha) - r V^U$ we have $\varrho_w < 0$. In view of $1 - \rho \geq 0$ and $s'(L) \leq 0 \Leftrightarrow \eta(1 - s) \leq 1$, it follows that $\Omega_{ww}(\tilde{w}) < 0$.

(ii) By using the expression for $\rho$, (5) can be rewritten as

$$LHS(w) := \alpha \beta \theta w^\alpha = \{(\alpha \beta \theta + (\theta - \alpha)(1 - \beta))w^\alpha - \alpha \theta (1 - \beta) r V^U\} s =: RHS(w).$$

Using (3) in (4) gives

$$\tau F'(L) = \frac{w}{(w^\alpha / \alpha) - r V^U} \left[ \frac{q + r + \delta}{\rho \theta} \right].$$

Hence $F' \rightarrow \infty$ and, therefore, $L \rightarrow 0$ as $w \rightarrow \infty$. Then from Assumption 1, $s \rightarrow \bar{\pi} \leq 1$ as $w \rightarrow \infty$. Thus, if (A1) holds, $LHS(w) - RHS(w) < 0$ as $w \rightarrow \infty$. Note that there is a lower bound $w$ defined by $w^\alpha / \alpha = r V^U$. This yields $LHS(w) - RHS(w) = \alpha^2(1 - \beta) s + \beta \theta (1 - s)]r V^U > 0$.

Remark. As can easily be checked, the condition for existence of an equilibrium with the replacement ratio taken as a parameter (see below) implies that

$$\alpha \beta \theta < [\theta(1 - b^\alpha)(1 - \beta) - \alpha(1 - \beta - \beta \theta)]\bar{\pi} < [\alpha \beta \theta + (\theta - \alpha)(1 - \beta)]\bar{\pi}.$$

Hence together with Assumption 1 that condition guarantees that (5) has a unique solution.

Proof of Proposition 1

(i) From $\Omega_{ww} < 0$ we have $\text{sign}(\partial \tilde{w} / \partial \tau) = \text{sign} \Omega_{w\tau}$ with

$$\Omega_{w\tau} = (1 - \beta)(\rho - 1) NF[\eta(1 - s) - 1] / \Pi^2, \quad (A3)$$

where use has been made of $\Pi_r = F$ and $\tilde{N}_r = -F'/\tau e F''$. Since $\beta > 0$ implies that $\rho < 1$, and $s' < 0$ is equivalent to $\eta(1 - s) - 1 < 0$, we have $\partial \tilde{w} / \partial \tau > 0$. By (3) this implies that $\tilde{e}$ too increases with $\tau$. 

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From $N(\tau, V^U) \equiv \tilde{N}(\bar{w}(\tau, V^U), \tau, V^U)$ we get
\[
\frac{\partial N}{\partial \tau} = \tilde{N}_w(\partial \bar{w}/\partial \tau) + \tilde{N}_\tau,
\] (A4)
where $\tilde{N}_w < 0$, and $\tilde{N}_\tau > 0$. From $\partial \bar{w}/\partial \tau > 0$ for $\beta > 0$ and $s' < 0$, it follows that $\partial N/\partial \tau$ has ambiguous sign in that case.

(ii) From (5), $\varrho = 1$ if $\beta = 0$. We also have $s' = 0$ iff $\eta(1 - s) = 1$. Therefore, as (A3) reveals, in either case $\partial \bar{w}/\partial \tau = 0$. Hence, from (A4), $\partial N/\partial \tau > 0$.

(iii) Define $c(\tau, V^U) \equiv \tilde{w}(\tau, V^U)/\tau \tilde{e}(\bar{w}(\tau, V^U), V^U)$. Then we get
\[
\frac{\partial c}{\partial \tau} = \frac{1 - \varrho}{\tau e} \frac{\partial \tilde{w}}{\partial \tau} - \frac{w}{\tau^2 e}.
\]
Using (A2), (A3), and $\partial \tilde{w}/\partial \tau = -\Omega_{\omega r}/\Omega_{\omega w}$ yields after some rearrangement
\[
\frac{\partial c}{\partial \tau} = \frac{1}{\tau^2 e \Omega_{\omega w}} \left\{ (\varrho - 1)(\tau - 1)\Omega_{\omega r} + \frac{\alpha \beta w^{\alpha - 1} r V^U}{[(w^\alpha)/\alpha - r V^U]^2} - \frac{(1 - \beta) s \varrho_w}{1 - s} \right\}.
\]
From $0 < \varrho \leq 1$, $0 < \tau \leq 1$, $\varrho_w < 0$, $\Omega_{\omega r} > 0$, and $\Omega_{\omega w} < 0$ it follows that $\partial c/\partial \tau < 0$. By (4) this implies that $L$ increases with $\tau$.

\[\square\]

Existence of equilibrium for a fixed $b$

Let $\bar{s} \equiv \sup s(L) \leq 1$. Then under Assumption 1 there exists an equilibrium for a fixed $b$ if and only if
\[
\tilde{\varrho}(1, b) \equiv \frac{\alpha}{\theta(1 - b^\alpha)} < \frac{(1 - \beta)\bar{s}}{(1 - \beta - \beta \theta)\bar{s} + \beta \theta}.
\] (A5)

Proof. The case where $0 < \beta < 1$ and $s'(L) < 0$. First we show that (A5) is equivalent to the existence of the relationship (12) as a function $L(u)|_{WS}$. Note that $\tilde{\varrho}$ is decreasing in $u$ and $\alpha/\theta(1 - b^\alpha)$ is the lowest possible value of $\tilde{\varrho}$, attained at $u = 1$ (implying that $L = 0$). Given that $s'(L) < 0$, $(1 - \beta)\bar{s}/[(1 - \beta - \beta \theta)\bar{s} + \beta \theta]$ is the value to which the RHS of (12) tends as $L$ goes to zero. So if (A5) did not hold, (12) would not be satisfied for any $0 \leq u < 1$. Next we show that (A5) is also sufficient for $L(u)|_{WS}$ to exist. From $\tilde{\varrho}(0, b) = \infty$ and $\tilde{\varrho}(1, b) = \alpha/\theta(1 - b^\alpha)$, (A5) implies that it is possible to find some (unique) $0 < u_0(b) < 1$ satisfying
\[
[\tilde{\varrho}(u_0, b)(1 - \beta - \beta \theta) - (1 - \beta)\bar{s} + \beta \theta \tilde{\varrho}(u_0, b)] = 0.
\] (A6)

Also note that $u_0$ is increasing in $b$ and there exists a supremum $\bar{b} < 1$, given by
\[
[\tilde{\varrho}(1, \bar{b})(1 - \beta - \beta \theta) - (1 - \beta)\bar{s} + \beta \theta \tilde{\varrho}(1, \bar{b})] = 0
\] (A7)
such that WS exists for $0 \leq b < \bar{b}$. Hence, there exists some $u_0(b) < 1$ if and only if $b < \bar{b}$. As for the domain of $L(u)|_{WS}$, two cases must be distinguished.

Case (i). Let $\underline{s} \equiv \inf s(L) \geq 0$ and suppose that
\[
\frac{\alpha}{\theta(1 - b^\alpha)} > \frac{(1 - \beta)\underline{s}}{(1 - \beta - \beta \theta)\underline{s} + \beta \theta}.
\]
Then \( L(u) \mid _{WS} \) is defined for \( u_0(b) \leq u < 1 \) and tends to a finite positive value as \( u \) tends to one.

**Case (ii).** Suppose that
\[
\frac{\alpha}{\theta(1 - b^\alpha)} \leq \frac{(1 - \beta)S}{(1 - \beta - \beta\theta)S + \beta\theta}.
\]
Then there exists some \( u_0(b) < \bar{u} \leq 1 \) such that \( L(u) \mid _{WS} \) is defined for \( u_0(b) \leq u < \bar{u} \). Now \( L(u) \mid _{WS} \) goes to infinity as \( u \) tends to \( \bar{u} \).

From (12) and (A6) it follows that \( s \rightarrow \bar{s} \) as \( u \rightarrow u_0 \). From \( s'(L) < 0 \) and (11) we have \( L(u) \mid _{WS} \rightarrow 0 < L(u) \mid _{LD} \) as \( u \rightarrow u_0 \). Now consider the behaviour of \( L(u) \mid _{LD} \) and \( L(u) \mid _{WS} \) when \( u \) tends to one and to its upper bound, respectively. The right-hand side of (11) goes to \( \infty \) as \( u \) tends to one. Since \( F'(L) \rightarrow \infty \) as \( L \rightarrow 0 \), it follows that \( L(u) \mid _{LD} \rightarrow 0 = L(1) \mid _{LD} \) as \( u \rightarrow 1 \).

With regard to \( L(u) \mid _{WS} \), we have to distinguish between the two above cases. In the first case (where \( u \rightarrow 1 \)) we have \( L(1) \mid _{WS} > 0 = L(1) \mid _{LD} \). In the second case (where \( u \rightarrow \bar{u} \leq 1 \)) we have \( L(u) \mid _{WS} \rightarrow \infty \) as \( u \rightarrow \bar{u} \), hence \( L(u) \mid _{WS} > L(u) \mid _{LD} \) for any \( u < \bar{u} \) sufficiently close to \( \bar{u} \).

**The cases where \( \beta = 0 \) or \( s = \text{const.} \).** When \( \beta = 0 \), (12) reduces to \( \tilde{q}(u, b) = 1 \) while the condition (A5) reduces to \( \tilde{q}(1, b) < 1 \). There exists a (unique) solution \( 0 < u^* < 1 \) if and only if \( \alpha < \theta(1 - b^\alpha) \). A similar argument applies to the case where \( s \) is constant. \( \square \)

**Existence and uniqueness of equilibrium for a fixed \( B \)**

**Under Assumption 3 there exists a unique equilibrium for fixed real benefits.**

**Proof.** From \( 0 < w^*(\tau, 0) < \infty \), we have \( bw^*(\tau, b) = 0 \) for \( b = 0 \). To see the behaviour of \( bw^*(\tau, b) \) when \( b \) goes to \( \bar{b} \), we combine (9), (10) and (11) to get
\[
w^*(\theta - \alpha)/\theta = \tau F'(L^*) \left[ \frac{\theta(1 - b^\alpha)q}{\alpha(q + r + \delta/u)} \right]^{1/\theta}.
\]
(\( A8 \))

From (A6) and (A7), we have \( u_0(b) \rightarrow 1 \), hence \( u^*(\tau, b) \rightarrow 1 \) as \( b \rightarrow \bar{b} \), which, by (11), implies that \( L^*(\tau, b) \rightarrow 0 \) and \( F' \rightarrow \infty \) as \( b \rightarrow \bar{b} \). Therefore, since in (A8) the term in square brackets tends to a finite positive constant, \( bw^*(\tau, b) \rightarrow \infty \) as \( b \rightarrow \bar{b} \). From condition (ii) in Assumption 3 it follows that there exists a unique \( 0 < b^* < \bar{b} \) such that \( b^*w^*(\tau, b^*) = B \). \( \square \)

**Proof of Lemma 1**

(i) Use (13) in (11) to obtain
\[
\phi(u, L) \equiv \tau F'(L)L^{-(\theta - \alpha)/\alpha} - C(1 - u)^{-(\theta - \alpha)/\alpha}[\tilde{q}(u, b)]^{1/\alpha} = 0,
\]
where \( C \equiv [q/(q + r + \delta)]^{-1/\alpha} \). (12) can be rewritten as
\[
\psi(u, L) \equiv [(1 - \beta - \beta\theta)s(L) + \beta\theta]\tilde{q}(u, b) - (1 - \beta)s(L) = 0.
\]
Differentiating with respect to $b$ yields

\[
\begin{bmatrix}
\phi_u & \phi_L \\
\psi_u & \psi_L
\end{bmatrix}
\begin{bmatrix}
\partial u^*/\partial b \\
\partial L^*/\partial b
\end{bmatrix}
= \begin{bmatrix}
-\phi_b \\
-\psi_b
\end{bmatrix}.
\]

From this we get $\partial L^*/\partial b = \Delta L/\Delta$, where $\Delta = \phi_u \psi - \phi_L \psi_u$ and $\Delta_L = - (\phi_u \psi_b - \psi_u \phi_b)$ with

\[
\phi_u = -(1/\alpha)C(1-u)^{-\theta/\alpha} \bar{\varrho}^{1/\alpha}[(\theta - \alpha) + (1 - u)\bar{\varrho}^{-1} \tilde{\varrho}_u],
\]
\[
\phi_L = \tau[F''L^{-\theta/\alpha} - F'(\theta/\alpha - 1)L^{-\theta/\alpha}],
\]
\[
\phi_b = -(1/\alpha)C(1-u)^{-(\theta-\alpha)/\alpha} \bar{\varrho}^{(1-\alpha)/\alpha} \tilde{\varrho}_b,
\]
\[
\psi_u = [(1-\beta-\beta \theta)s + \beta \theta] \tilde{\varrho}_u,
\]
\[
\psi_L = -[(1-\beta)(1-\tilde{\varrho}) + \beta \theta \tilde{\varrho}]' s',
\]
\[
\psi_b = [(1-\beta-\beta \theta)s + \beta \theta] \tilde{\varrho}_b.
\]

If $0 < \beta < 1$ and $s' < 0$, uniqueness of an equilibrium for a fixed $b$ is equivalent to $-\phi_u/\phi_L = dL/du|_{LD} < dL/du|_{WS} = -\psi_u/\psi_L$. Since $\phi_L < 0$ and $\psi_L > 0$, this is equivalent to $\Delta < 0$. If $\beta = 0$ or $s' \equiv 0$, then $\psi_L = 0$, hence $\Delta = -\phi_L \psi_u < 0$. Using the above expressions we get

\[
\Delta_L = C(\theta - \alpha)(1-u)^{-\theta/\alpha} \bar{\varrho}^{1/\alpha}[(1-\beta-\beta \theta)s + \beta \theta] \tilde{\varrho}_b > 0.
\]

Hence, $\partial L^*/\partial b < 0$.

(ii) That in the case $0 < \beta < 1$ and $s' < 0$ both $\partial u^*/\partial b$ and $\partial w^*/\partial b$ can be of either sign is proven by a numerical example presented in section 5.

(iii) If $\beta = 0$ or $s = \text{const.}$, by (12), $\tilde{\varrho}(u^*, b) = \text{const}$. As $\tilde{\varrho}$ is decreasing in $u$ and increasing in $b$, it follows that $\partial u^*/\partial b > 0$. Moreover, with $\tilde{\varrho}(u^*, b) = \text{const.}$, (A8) simplifies to $w^* \theta - \alpha/\theta = F'(L^*) \text{const}$. Differentiating with respect to $b$ gives

\[
(1-\alpha/\theta)w^* - \alpha/\theta (\partial w^*/\partial b) = F''(L^*) (\partial L^*/\partial b) \cdot \text{const}.
\]

From $F'' < 0$ and $\partial L^*/\partial b < 0$ it follows that $\partial w^*/\partial b > 0$. \qed
References


