Dynamic Effects of Offshoring

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by

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Abstract: We analyze the effects of offshoring in a multisector-growth model where sectors differ by TFP-growth and where capital accumulation takes place. Our paper's focus is on the dynamic effects in the country which reallocates a part of its intermediate production to foreign production sites. We show that offshoring induces structural changes and has effects on GDP-growth which are omitted by "standard trade theory". These effects arise only if capital accumulation and demand patterns associated with Baumol's "cost disease" are incorporated into the model. Our model predicts that offshoring slows down the transition from a manufacturing economy to a services economy, which takes place in modern societies, thus having impacts on GDP-growth. A further implication (which is not modeled explicitly in our paper) is that these structural changes may change the "role" of the economy in world trade. That is, the dynamic effects may have impacts on the (quantitative) results of standard trade theory.

Keywords: trade in intermediates, offshoring, outsourcing, intermediates, structural change, Baumol's cost disease, multi-sector growth models, neoclassical growth theory

JEL Classification Numbers: F43, O41, O14

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1. Introduction

Although a well known fact for a long time, offshoring (i.e. shift of intermediate production to foreign locations) has become one of the most prominent terms in political debate in last years. For example, fears about offshoring to the “new” EU-member countries in the Eastern Europe arose in Western Europe; see e.g. Marin, 2006, for data on European offshoring. In the United States of America fears about offshoring to India or China dominate the political and scientific debate. These fears arose due to opening of international borders, which has been caused by change in political regime/directives in (ex-)communist countries, and due to new progress in information, communication (but as well transportation) technologies, which made offshoring more profitable and feasible. As a result, offshoring is “one of the most rapidly growing components of trade” (Grossman and Helpman, 2005, p. 36) with the potential for being the “next industrial revolution” (Blinder, 2007b).  

In our paper we refer to the following definition of offshoring: offshoring means here that firms shift activities to foreign production facilities or service providers (to unaffiliated firms or to own affiliates). Especially, a part of the intermediate production or service provision is shifted to foreign locations. The case where the whole production process is shifted to foreign location abroad is not analysed here; for this case traditional (final-goods)trade theory may be more adequate.

General Literature on Offshoring

There is a lot of literature which is somehow related to offshoring (if we use the term “offshoring” in a relatively broad sense). Much of this literature uses different “terms” for describing the subject of analysis, e.g. there are theories of “multinational corporations”, “foreign direct investment”, “intermediate trade”, “international fragmentation”, “intra-firm trade” etc. However, more or less all these theories provide some sort of contribution to a theory of offshoring. Examples of this literature include: Jones and Kierzkowski (2001), Grossman and Helpman (2005), Markusen et al. (2005), Helpman (2006), Choi (2007) and Rodriguez-Clare (2007, 2010). Some overviews of literature and important topics

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**Topics Associated with Offshoring**

The two main discussion points in the literature are: (1) whether offshoring leads to an increase in welfare (in the long run in the industrialized countries) and (2) to what extent is offshoring associated with unemployment. As discussed by Rodriguez-Clare (2007, 2010) there are two opposing effects which are decisive for point (1): a) the positive productivity effect, which refers to the increase in productivity of the domestic economy that is caused by internalization of cross-country production-cost-differences; b) the negative terms-of-trade effect, which refers to a worsening of terms of trade due to an increase in the supply of the domestic good on the world market. Regarding point (2), e.g. Blinder (2005, 2007b) argues that offshoring can lead to a long transition period with high unemployment, where the unemployment may be the stronger, the more labor is reallocated across sectors that differ by skill type (not skill level; e.g., the services sector requires relatively well developed soft-skills whereas the manufacturing sector requires mathematical/engineering skills).

**Aim of our Paper**

The aim of our paper is to contribute to both discussion-points (welfare and unemployment) by studying (1) the productivity effect in detail (i.e. in addition to the direct productivity effects of offshoring, which are in general studied in the literature, we study the role of the productivity effects via structural change associated with Baumol’s cost disease and via capital accumulation), and (2) the cross-sector reallocations (i.e. structural change) associated with offshoring. The latter are not only important for assessing unemployment but are also relevant for nearly every long run policy. Furthermore, we provide a dynamic framework for studying the effects of offshoring (in contrast to the static frameworks studied by,

2 Structural change means here reallocation of labor across sectors such as manufacturing, services and agriculture, where sectors differ by technology (e.g. by the growth rate of total-factor-productivity (TFP)).

3 Baumol (1967) shows that, if sectors differ by productivity growth and if demand is price inelastic, there is a GDP-growth slowdown in the economy, since factors are reallocated to low-productivity-growth-sectors. In the literature this slowdown is often referred to as the growth slowdown associated with Baumol’s “cost disease”. For some new evidence on the effects associated with Baumol’s cost disease see, e.g., Nordhaus (2008).
e.g., Bhagwati et al., 2004, and Samuelson et al. 2004). In this way we can analyze the effects of offshoring on GDP-growth and on dynamic structural-change-patterns between technologically distinct sectors. These dynamic effects are rather rarely analyzed in the literature\(^4\) (since they are difficult to study, i.e. there are no balanced growth paths\(^5\)).

**Modeling Framework**

Our framework is based on the growth model presented by Ngai and Pissarides (2007), i.e. we do not rely on a (static) trade model but work with a growth model.\(^6\) The aggregate structure of the model is similar to the Ramsey-model (for a discussion of the Ramsey-model, see e.g. Barro and Sala-i-Martin, 2004). A part of the intermediates production is taken over by the foreign country, i.e. offshoring takes place. Differences in prices across countries come from differences in technology (TFP-growth) across countries. Of course, since different technologies are used across countries, we assume that domestic and foreign intermediates are not perfect substitutes. Overall, we model an economy where capital is accumulated and where the sectors differ by TFP-growth.\(^7\) Note that our model may be regarded as a Ricardian model in some sense, since trade arises due to differences in relative sector-productivities across countries.

**Related Literature**

The importance of analyzing offshoring in a framework where capital is accumulated has been mentioned by Milberg et al. (2006), pp. 6/7. The importance of analyzing offshoring in a framework where technologically distinct sectors exist is implied by the previous literature as well:

1.) As noted by Blinder (2005, 2007b) offshoring of high-productivity-growth-activities (that became possible by progress in information and communication

\(^4\) There is some empirical literature on the productivity-growth-effects of offshoring at the industry and plant level, e.g. Amiti and Wei (2005, 2006), Mann (2004) and Girma and Gorg (2004). (A further overview of the literature can be found in Olsen, 2006.) However, no clear patterns as to how offshoring affects productivity can be concluded from this literature (see Olsen, 2006, p. 28). Furthermore, Rodriguez-Clare (2007) studies offshoring in a dynamic model. However, he omits cross sector differences in productivity growth and capital accumulation, which are crucial to our analysis.

\(^5\) On difficulties in the analysis of dynamic multi-sector models (no existence of balanced growth paths) see e.g. Kongsamut et al. (2001).

\(^6\) An overview of trade models dealing with offshoring can be found in Baldwin and Robert-Nicoud (2007).
technologies) could lead to a GDP-growth slowdown in the economy, if the redundant factors are reallocated to sectors with lower productivity growth (according to the framework of Baumol’s, 1967, “cost disease”).

2.) The findings by Fixler and Siegel (1999) imply that (domestic) outsourcing has impacts on sector-productivity and on structural change in a framework similar to that by Baumol (1967) (i.e. in a framework where productivity growth differs across sectors). Therefore, one can expect that offshoring (or international outsourcing) has similar effects, as well.

3.) Last not least there is literature, which constitutes some links between (final-goods) trade theory and structural change theory. This literature assesses the relevance of trade for structural change and tries to integrate trade-theories into structural change theories. Examples of this literature are: Rowthorn and Ramaswamy (1999), Fagerberg (2000), Hsieh and Klenow (2007), Barry and Walsh (2008), Matsuyama (2009) and Yi and Zhang (2011). The latter use the Ngai and Pissarides (2007)-model, like us. However, in contrast to us, Yi and Zhang (2011) focus on final goods trade (not offshoring) and on reproducing some stylized facts of structural change (“hump-shaped” pattern of manufacturing employment).

Our Results

Our model postulates the following growth determinants which are relevant for the impact of offshoring on GDP-growth:

(1) Offshoring influences the (implicit) total-factor-productivity-growth of intermediates-production. This effect implies that offshoring acts like a kind of technological progress in intermediates-production by integrating the foreign cost-advantages into domestic production. In fact, this effect is studied in classical trade theories.

(2) Effect (1) has an indirect effect on aggregate growth as well: the productivity increase affects the rate of capital-accumulation. This effect is similar to the effect of an increase in technological progress in neoclassical growth models, (e.g. in the Ramsey-model).

(3) The increase in capital accumulation leads to shift of production factors from consumption-goods-industries to capital-goods-industries. That is, some

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7 Evidence on different TFP-growth rates across sectors is presented by, e.g., Baumol et al. (1985).
production factors are reallocated to capital-producing industries to produce the additional capital required due to Effect (2). Since Uzawa (1964) has studied the reallocations between consumption-goods-industries and capital-goods-industries, we name Effect (3) “Uzawa’s structural change”.

(4) It has been shown by Baumol (1967) and generalized by Ngai and Pissarides (2007) that a sort of “degenerative” structural change arises in consumption-goods-industries (in closed economies), i.e. factors are reallocated to “stagnant” sectors, which results in a slow-down of average factor-productivity-growth. Thus, the growth rate of real GDP is declining over time. This aspect is well-known and associated with the term Baumol’s cost disease (see also footnote 3). We name it Baumol’s structural change. The following effect describes how offshoring affects Baumol’s structural change.

(5) Effect (3) implies that production factors are withdrawn from consumption-goods-industries. Hence, Effect (3) yields that Baumol’s structural change becomes less relevant from the viewpoint of the whole economy (consumption and capital): since less labor is employed in consumption-industries due to Effect (3), less labor is exposed to Baumol’s structural change. Therefore, the negative impact of Baumol’s structural change on average productivity growth and thus on real GDP-growth is weakened.

To sum up: Offshoring induces the following chain of impacts:

Offshoring $\rightarrow$ Increase in TFP of intermediates production (traditional trade theory) $\rightarrow$ increase in capital-accumulation (neoclassical growth theory) $\rightarrow$ reallocation of factors from consumption-goods-production to capital-production (Uzawa’s structural change) $\rightarrow$ decline in importance of “degenerative” consumption-industries-structural-change-patterns (Baumol’s structural change) for real GDP-growth $\rightarrow$ faster GDP-growth.

We can see that the traditional impact channel (Effect 1) induces a chain of dynamic effects, which affect the real GDP-growth over long run.

In general, models that exclude capital accumulation and structural change associated with Baumol’s cost disease do not take channels (2)-(5) into account. Our model implies that therefore (in some cases) these models omit the quantitatively more important part of the productivity effect: We show that the growth effects via channel (1) are smaller than the other effects, provided that the
economy-wide output-elasticity of capital is higher than the economy-wide output-elasticity of labor.

Furthermore, our results imply that offshoring of high-productivity-growth (hpg) activities and offshoring of low-productivity-growth (lpg) activities are different regarding the terms of trade development: Offshoring of lpg activities can be beneficial for the home country even when the terms of trade worsen in the long run. However, offshoring of hpg activities can be beneficial only if the terms of trade improve in the long run. The reason for this fact is that lpg activities feature increasing prices due to “Baumol’s cost disease”. Thus, even when the terms of trade worsen in the long run, it can be “cheaper” importing intermediates than producing them at home. This result may be of interest for the debate about (future) hpg services offshoring.  

Our results imply that structural change patterns are slowed down by offshoring (and thus less unemployment due to inter-sectoral barriers may exist in reality) in the long run equilibrium. (As explained above, the structural change slowdown comes from the shift from consumption production to capital production.) These results imply that offshoring can have different effects in comparison to domestic outsourcing: While our results imply that offshoring leads to a structural change slowdown, the results by Fixler and Siegel (1999) imply that domestic outsourcing leads to an acceleration of structural change.

Last but not least, our results imply that the economy, that offshores, first goes through a turbulent phase before reaching the smooth phase described above (i.e. the long-run equilibrium where structural-change-smoothing occurs and higher growth rates are achieved). This result supports the argumentation by Blinder (2005, 2007b). During the turbulent phase there are strong reallocations across sectors (thus, in reality high unemployment may be the case) in order to adapt the production-structures to the effects of offshoring; in fact, the factor reallocations during the turbulent phase are more or less the same as predicted by “static” standard trade models (represented by e.g. the models by Bhagwati et al. 2004): there are changes in exports and demand for domestic intermediates (and investment goods production). All these reallocations lead to a “manufacturing-sector renaissance” in our model. That is, the manufacturing employment share
(which is normally decreasing in industrialized countries) increases during this phase due to increased capital demand (and exports). This result may explain the fact that offshoring is not associated with higher unemployment in the manufacturing sector in empirical studies (e.g. by Amiti and Wei, 2005).

Overall, our model implies that the effects which are discussed in static trade theory (turbulent phase) induce secondary effects (dynamic effects during the smooth phase). Since the latter effects have an impact on structural change and productivity growth, they may have an impact on the relative role of the economy in international trade, in general. For example, slower structural change and faster GDP growth may be associated with changes in terms of trade or with changes in comparative advantage; thus, at least the secondary effects may have a quantitative impact on trade flows. Thus, it seems that modeling capital accumulation and Baumol’s structural change is not only important for structural change theory by also for trade theory.

The rest of the paper is set up as follows: In the next section (section 2) we present the model assumptions. Then, we calculate the model-optimum and equilibrium (section 3). Subsequently, we analyze the effects of offshoring on growth (section 4) and structural change (section 5). In section 6 we discuss the results/extensions of our model, i.e. endogenous terms of trade, negative growth effects of offshoring, the distribution of effects across phases, implications for unemployment, “manufacturing renaissance” and “partial offshoring”. Finally, we summarize our main results and suggest some topics for further research (section 7).

2. Model assumptions

The model is a sort of disaggregated Ramsey-model with international trade. Due to model-setting there exists an aggregate balanced growth path that features balanced growth with respect to aggregates, but unbalanced growth (i.e. structural

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8 It is argued in the literature that, while offshoring of manufacturing activities included mainly lpg activities (see e.g. the discussion in Mankiw and Swagel (2006)), future services offshoring may include hpg activities; see e.g. Blinder (2005, 2007a, 2007b) and Irwin (2005).
change) with respect to disaggregated variables (for details see also Ngai and Pissarides, 2007).

We assume that there are three types of goods and services (i). However, the model could be extended for an arbitrary number of goods. To follow the recent discussion about services offshoring we could assume, e.g., that the goods and services are classified as follows: manufacturing goods (M), personal services (P) and impersonal services I, i.e. $i = M, P, I$. Blinder (2007a) suggests dividing the service jobs into personal services, i.e. services that cannot be delivered electronically from far away without degradation in quality (e.g. child care and surgery), and impersonal services that can be delivered electronically from far away without degradation in quality (e.g. typesetting and programming). Impersonal services are offshorable (or tradable), but personal services are not.

The representative household consumes all three types of goods and services. We assume the lifetime utility function suggested by Ngai and Pissarides (2007). They have proven that the lifetime utility ($U$) has to be a logarithmic function of the consumption composite in order to allow for aggregate balanced growth. The consumption composite itself is a CES-function of consumption ($C_i$) of goods and services ($i = M, P, I$):

$$
U = \int_0^\infty \ln \left( \sum_i \omega_i C_i^{(\varepsilon - 1)/\varepsilon} \right)^{\varepsilon/(\varepsilon - 1)} e^{-\rho t} dt, \quad i = M, P, I
$$

$$
0 < \varepsilon < 1; \quad \rho, \omega_i > 0 \quad \forall i
$$

$$
\sum_i \omega_i = 1
$$

where $t$ is the time index.

Since we assume here $\varepsilon < 1$, the goods are poor substitutes and relative demand is price inelastic, which is necessary for analyzing the growth slowdown associated with Baumol’s cost disease. (For further explanations with respect to the features of this utility function see Ngai and Pissarides, 2007.) These goods and services

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9 see e.g. Amiti and Wei (2005, 2006), Garner (2004) and Blinder (2005, 2007b) on the role of progress in communication and information technology for services offshoring and the future importance of services offshoring.
are produced by the corresponding domestic sectors. Each sector produces its output via a Cobb-Douglas production function by using labor, capital and intermediate inputs. The amount of labor available is constant and normalized to unity. However, exogenous population growth could be integrated into this model easily. Total factor productivity (TFP) grows in each sector at a sector specific rate \((g_i)\):

\[
Y_i = A_i n_i^{1-\alpha-\beta} (k_i K)^\alpha (z_i Z)^\beta, \quad i = M, I, P
\]

\(\alpha, \beta > 0\) and \(1 - \alpha - \beta > 0\)

\[
\frac{\dot{A}_i}{A_i} = g_i, \quad i = M, I, P
\]

where \(Y_i\) is the output of sector \(i\); \(K\) is the aggregate capital; \(Z\) is an index of the total “amount” of intermediate inputs; \(k_i, n_i\) and \(z_i\) represent respectively the fraction of capital, labor and intermediate inputs devoted to the production of sector \(i\); \(A_i\) is a sector-specific productivity parameter.

We assume that sectors M and P do not produce intermediate inputs. In the autarky, intermediate inputs are produced by sector I; hence,

\[
Z = h_i
\]

where \(h_i\) is the “amount” of domestic intermediate inputs produced by sector I. When we allow for trade, we assume that the intermediate inputs index \((Z)\) is a Cobb-Douglas-function of domestic and foreign intermediate inputs:

\[
Z = h_i^\varphi (h_F)^{1-\varphi}, \quad 0 < \varphi < 1
\]

where \(h_F\) is the “amount” of foreign intermediate inputs. Note that this function implies that foreign intermediates are not a perfect substitute for domestic intermediates, since when using foreign intermediates domestic intermediates are still necessary in production (i.e. there is only partial-offshoring). This may come from the fact that foreign and domestic intermediates are produced by different productions functions (which allows for price differences between abroad and home). For example, if software is produced less expensively abroad, it may be lower quality software (since foreign programmers have lower quality education);
hence, domestic software-programming may be necessary to repair software failures which may show up later. Anyway some domestic services are always necessary to integrate foreign services into domestic production (see also Blinder, 2007b). The assumption of equation (6b) is also consistent with the empirical findings that domestic and foreign intermediate inputs complement each other in the production of final goods (see e.g. Desai et al., 2005, or Hanson, et al., 2003). Note that the parameter $\varphi$ could overall be interpreted as a quality/tradability parameter of foreign intermediates: The lower $\varphi$, the better substitutes are foreign and domestic intermediates. $\varphi$ can represent e.g. the quality of foreign education, but also the quality of the transfer process (tradability) of services. For example, if $\varphi$ is very close to zero, the foreign services are nearly perfectly tradable and their quality is comparable to the quality of domestic services, and therefore intermediates will be produced nearly only abroad. Furthermore, $\varphi$ could also be interpreted as a measure of uncertainty regarding foreign intermediates as modeled by Choi (2007).

Since the economy imports intermediate inputs, it has to “pay” for them by exporting goods and services. Let sector P represent all goods and services that cannot be exported. That is, only the output of sector M can be exported. Alternatively, we could also assume that sector-P-output can be exported and sector-M-output cannot. However, the key-model results would be the same. (We assume here in accordance with the standard trade theory that the output of sector I is not exported, i.e. the foreign country has some comparative advantage in I-production. That is, we assume that goods and services that are exported are not imported at the same time. However, the model could be modified easily to include at the same time exports and imports of the same good.) We abstract from any other trade not associated with offshoring in order to isolate the effects of offshoring. Let $e_M$ denote the exports of sector M. Furthermore, let $p_i$ denote the price of good i ($i = M, I, P$). Thus, aggregate exports ($E$) are given by

$$E := p_M e_M$$

We assume now that aggregate exports ($E$) are related to imports ($h_F$) in the following manner:

$$E = Th_F$$
where $T$ is the ratio of exports to imports associated with offshoring. It determines how much the economy has to export in order to get one unit of intermediates-imports (offshoring). Therefore, $T$ is corresponding to the (reciprocal of) “offshoring-terms of trade”. We assume that the offshoring-terms of trade is changing at a constant rate ($g_T$):

$$\frac{\dot{T}}{T} = g_T$$

Later, in section 6, we will discuss the endogenous range of these terms of trade. Capital ($K$) is produced only in sector M. (Therefore, this sector could also be interpreted as the manufacturing sector.) Capital depreciates at rate $\delta$. Thus, overall, sector-M-output is consumed ($C_M$), exported and used as capital-input:

$$Y_M = C_M + \dot{K} + \delta K + e_M$$

As explained above, the output of sector (I) is consumed ($C_I$) and used as intermediate input:

$$Y_I = C_I + h_I$$

The output of sector P is consumed ($C_P$) only, as explained above:

$$Y_P = C_P$$

We assume that capital, labor and intermediate inputs are mobile across sectors. All capital, labor and intermediate inputs have to be used in domestic production, thus

$$\sum_i k_i = 1 \quad i = M, I, P$$

$$\sum_i n_i = 1 \quad i = M, I, P$$

$$\sum_i z_i = 1 \quad i = M, I, P$$

Furthermore, we define aggregate consumption expenditures ($C$) and aggregate output ($Y$) as follows:

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10 For empirical evidence that the manufacturing sector produces nearly all investment goods see e.g. Kongsamut et al. (1997).
\[
C := \sum_i p_i C_i \quad i = M, I, P
\]

(17) \[ Y := \sum_i p_i Y_i \quad i = M, I, P \]

We choose the output of sector M as numéraire, thus:

(18) \( p_M = 1 \)

Overall, we should keep in mind that the domestic country imports intermediate inputs (that are substitutes for sector-I-output) and exports a part of the sector-M-output. There is no labor mobility across countries and the households of the two countries can invest their savings respectively only in the capital of their domestic countries.

3. Optimum and equilibrium

Now, the model is fully specified. Equations (1)-(18) (where we use equation (6b) and not (6a)) can be optimized by using a Hamiltonian.\(^\text{11}\) Then, after some algebra the following intertemporal and intratemporal optimality conditions can be obtained for our model (we subdivide the equations describing the optimal solution into aggregated and disaggregated level equations):

**Aggregates**

(19) \[ Y = A_M K^{\frac{1}{\beta}} \eta^{\frac{\alpha}{1-\beta}} \]

(20) \[ \dot{K} = (1 - \beta)Y - C - \delta K \]

(21) \[ \frac{\dot{C}}{C} = \alpha \frac{Y}{K} - \delta - \rho \]

\(^{11}\) The optimality conditions which are obtained by the Hamiltonian, provided that there is free mobility of labor across sectors, are \(u(.)\) denotes the instantaneous utility function from equation (1), i.e. \(u(.) = \text{ln}[\ldots])\):

\[ p_i = \frac{\partial u(.)}{\partial C_i} = \frac{\partial Y_i}{\partial u(.)} \]

\[ T = \frac{\partial Y_i}{\partial (z_{i,Y})} \frac{\partial Z}{\partial h_i} = \frac{\partial Y_i}{\partial \mu_{i}} - \delta - \rho ; \quad \mu_{i} = \frac{\partial u(.)}{\partial C_{i}} \]
\[(22)\] \[Z = K^{1-\beta} \eta^{1-\beta}\]

\[(23)\] \[E = (1-\phi)\beta Y\]

where \(\eta\) is a function of exogenous model parameters growing at constant rate:

\[(24)\] \[\eta = \beta A_1 \phi^\varphi (1-\phi)^{1-\varphi} \left( \frac{p_L}{T} \right)^{1-\varphi}\]

**Sectors**

\[(25)\] \[p_i = \frac{A_M}{A_i} \ \forall i\]

\[(26)\] \[\frac{p_i \omega_i}{\varphi} = \frac{Th_F}{1-\varphi} = \beta Y\]

\[(27)\] \[n_i = k_i = z_i \ \forall i\]

\[(28)\] \[\frac{p_i C_i}{C} = \frac{x_i}{X} \ \forall i\]

\[(29)\] \[n_p = \frac{p_p Y_p}{Y} = \frac{x_p C}{X Y}\]

\[(30)\] \[n_M = \frac{p_M Y_M}{Y} = \frac{x_M C}{X Y} + (s + g^*) \frac{K}{Y} + (1-\varphi)\beta\]

\[(31)\] \[n_i = \frac{p_i Y_i}{Y} = \frac{x_i C}{X Y} + \varphi \beta\]

where \(x_i\) and \(X\) are time varying auxiliary variables and functions of exogenous model parameters:

\[(32)\] \[x_i := \frac{p_i C_i}{C_M} = \left( \frac{\omega_i}{\omega_M} \right)^\epsilon \left( \frac{A_M}{A_i} \right)^{1-\epsilon} \ \forall i\]

\[(33)\] \[X := C / C_M = \sum_i x_i\]
Now, following Ngai and Pissarides (2007), we define a **partially balanced growth path (PBGP)** such that aggregate consumption \((C)\), aggregate output \((Y)\) and capital \((K)\) grow at the same rate, thus, being consistent with the Kaldor facts. This definition requires balanced growth with respect to aggregates. However, it allows for unbalanced growth with respect to disaggregated variables such as output shares, etc., i.e. structural change can take place.

**Theorem 1a:** A unique and globally saddle-path-stable PBGP exists in our model.

**Proof:** The equations describing the aggregate optimum (especially equations (19)-(21)) resemble the ones from the “normal” one-sector Ramsey-model in all relevant features. Thus, the model in aggregates behaves like a normal Ramsey model. Therefore, we now that a unique and saddle-path-stable growth-path exists in our model, where \(Y\), \(K\) and \(C\) grow at a constant rate. For a discussion of the normal one-sector Ramsey-model (or sometimes also named Ramsey-Cass-Koopmans-model), see e.g. Barro and Sala-i-Martin (2004). Q.E.D.

**Theorem 1b:** Along the PBGP all aggregates \((Y, K, C \text{ and } E)\) grow at the constant rate \(g^*\), which is given by

\[
g^* = \frac{(1 - \beta)g_M + \beta g_\eta}{1 - \alpha - \beta}
\]

where

\[
g_\eta := \frac{\dot{\eta}}{\eta} = g_T + (1 - \varphi)\left(\frac{\dot{p}_t}{p_t} - g_T\right) = \text{const.}
\]

and where \(\frac{\dot{p}_t}{p_t}\) is given by equation (25).

**Proof:** Equation (19) implies that if \(K\) and \(Y\) grow at (the same) constant rate, their growth rate is given by equation (34). Provided that \(K\) and \(Y\) grow at rate \(g^*\), equations (20), (21) and (23) imply that aggregate consumption expenditures \((C)\) and aggregate exports \((E)\) grow at rate \(g^*\) as well. Q.E.D.
Theorem 1c: Structural change takes place along the PBGP, i.e. sectoral factor-input-shares \((n_i, k_i, z_i)\) and sectoral consumption-shares \((p_jC_j/C)\) change over time.

Proof: Equations (27)-(33) imply that although \(K, C\) and \(Y\) grow at a constant rate, \(n_i, k_i, z_i\) and \(p_jC_j/C\) change over time. This result comes from the fact that \(x_i\) and \(X\) are functions of time-varying exogenous parameters. Hence, they are not constant along the PBGP. Q.E.D.

Theorem 1d: The extent of offshoring changes along the PBGP, i.e. \(h_i/h_F\) changes.

Proof: This Theorem is implied by equations (25) and (26) as well as by Theorem 1b. Q.E.D.

Theorem 1e: \(g_\eta\) is the (implicit) TFP-growth rate of intermediates-production \((Z)\).

Proof: see APPENDIX A.

4. Effects of offshoring on growth of aggregates

First, we discuss the overall impact of offshoring on aggregate growth. Then in the next subsection we will discuss the channels along which offshoring influences GDP-growth and their relative importance. The analytical approach is as follows: we compare the PBGP of an economy, which offshores, to the PBGP of an economy, which is in autarky, ceteris paribus. We also discuss the factor reallocations during the transition period in section 5.

\[\text{It follows from equation (22) that } Z\text{ grows at a constant rate as well. However, its growth rate is different from } g^*.\]
4.1 The overall impact on aggregate growth

Up to now we modeled an economy that offshores intermediate inputs. The following Theorem implies how we can modify the model equations to describe an economy without offshoring:

**Theorem 2:** The model describes an economy without offshoring if we set \( \varphi = 1 \) and \( e_M = 0 \) in all model equations. In this case equation (6b) becomes (6a), i.e. only the output of sector 1 is used as intermediate input.

**Proof:** Note that in equation (24) \( \varphi = 1 \) does not imply that \( \eta = 0 \), since \( 0^0 = 1 \). The rest of the proof is trivial. Q.E.D.

**Theorem 3a:** Offshoring increases the growth rate of the economy \( g^* \), provided that the price of domestically produced intermediates \( p_i \) grows at higher rate than the price of imported intermediates \( T \), ceteris paribus. That is, the PBGP-growth rate of an economy, which offshores, is higher in comparison to the growth rate of an economy, which does not offshore, provided that \( \frac{\dot{p}_i}{p_i} - g_T > 0 \), ceteris paribus.

**Proof:** This Theorem is implied by equations (34) and (35) and by Theorem 2. The economy, which offshores, features \( \varphi < 1 \); the economy, which does not offshore, features \( \varphi = 1 \). Note that \( \frac{\dot{p}_i}{p_i} \) is a function of exogenous model parameters; see equation (25). Q.E.D.

**Theorem 3b:** If \( \frac{\dot{p}_i}{p_i} - g_T < 0 \), offshoring decreases the growth rate of the economy \( g^* \) in comparison to the state without offshoring, ceteris paribus. In this case, the relative amount of offshoring \( h_F / h_i \) decreases.

**Proof:** The proof is analogous to the proof of Theorem 3a. Note that \( h_F / h_i \) is given by equation (26). Q.E.D.
Note that, in general, the negative outcome (Theorem 3b) will not occur, since, if terms of trade develop unfavorably, the economy can return to the state of autarky (i.e. equation (6a) becomes valid instead of equation (6b)). Hence, the growth rate will never be lower than in autarky. However, in section 6 we will discuss some extensions of the model where this negative case may occur.

**Definition 1:** High-productivity-growth-sectors are sectors where the TFP-growth-rate is higher than the TFP-growth-rate of the capital-producing sector; i.e. a sector \( i \) is named high-productivity-growth-sector, provided that \( g_i > g_M, \ i \neq M \). Low-productivity-growth-sectors are sectors where the TFP-growth-rate is lower than the TFP-growth-rate of the capital-producing sector; i.e. a sector \( i \) is named low-productivity-growth-sector, provided that \( g_i < g_M, \ i \neq M \).

**Theorem 3c:** Offshoring of high-productivity-growth-activities (\( g_i > g_M \)) increases the growth rate of the economy (\( g^* \)) only if the terms of trade improve in the long run, i.e. only if \( g_T < 0 \). On the other hand, offshoring of low-productivity-growth-activities (\( g_i < g_M \)) can increase the growth rate \( g^* \) even when the terms of trade worsen in the long run, i.e. \( g_T > 0 \) (provided that \( \frac{\dot{p}_i}{p_i} - g_T > 0 \)).

**Proof:** Equation (25) implies that high-productivity-growth-activities have decreasing prices, i.e. \( \frac{\dot{p}_i}{p_i} < 0 \) for \( g_i > g_M \), and vice versa. The rest of the proof is implied by equations (34) and (35) and Theorem 2. Remember: The economy, which offshores, features \( \varphi < 1 \); the economy, which does not offshore, features \( \varphi = 1 \). **Q.E.D.**

Overall, Theorem 3c implies that even when the terms of trade worsen in the long run, offshoring can increase the growth rate of aggregate output, provided that low-productivity-growth-activities are offshored. The reason for this fact is that these activities feature increasing prices due to the “cost disease” (see also Baumol, 1967, and Ngai and Pissarides, 2007). Hence, even when the terms of
trade worsen (i.e. the price for foreign intermediates grows) it can be cheaper using foreign intermediates instead of domestic intermediates (provided that the price for domestic intermediates grows at higher rate than the price for foreign intermediates). Overall, for positive growth effects of offshoring it is not merely relevant whether the terms of trade improve or not, but rather how the terms of trade develop in comparison to the price of the domestic sector (I) that is competing with the foreign sector (Theorems 3).

### 4.2 Impact channels and their relative importance

An interesting question within this model is along which channels offshoring influences the growth rate of aggregate output ($Y$). It follows from equations (4), (17), (22) and (27) that $Y$ is determined as follows:

$$ Y = K^{1-\beta} \eta^{\rho \beta} \sum_i p_i A_i n_i. $$

This equation implies that there are five sources (or: channels) of growth within this model (note that all variables with zero as exponent denote the initial value of the corresponding variable):

1. **Intermediates production productivity (via term $\eta^{\rho \beta}$).** Remember that we have shown in Theorem 1e that $g_\eta$ can be interpreted as the TFP-growth-rate of intermediates-production ($Z$). An increase in $\eta$ increases $Y$, ceteris paribus.

2. **Price-effect of structural change/technological cross-sector-disparity (via $\sum_i p_i A_i n_i$).** Cross-sector differences in technology (TFP-growth) cause changes in relative prices (see equation (25)). In general, structural change (reallocations of labor across sectors) leads to changes in relative prices as well. Changes in relative prices cause changes in aggregate output, ceteris paribus.

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13 Note that, although in our model structural change does not directly lead to changes of relative prices, in general it does. This fact will be discussed later.
(3) **Quantity-effect of structural change** (via $\sum_i p_i^0 A_i^0 n_i^0$). Labor-reallocation across technologically distinct sectors (i.e. changes in $n_i$) leads to changes in aggregate output, ceteris paribus. In our model this effect is negative due to the growth-slowdown associated with Baumol’s cost disease: In APPENDIX B we show that factors are reallocated from sectors with high TFP-growth-rates to sectors with low-TFP-growth rates along the PBGP, provided that demand is price-inelastic ($\varepsilon < 1$). That is labor, is reallocated to low-productivity sectors; hence, average productivity of labor decreases (see also the explanations by Ngai and Pissarides, 2007, and Baumol, 1967).

(4) **Technological progress** (via $\sum_i p_i^0 A_i^0 n_i^0$). Changes in (exogenous) technology parameter(s) ($A_i$) lead to changes in aggregate output, ceteris paribus.

(5) **Capital accumulation** (via $K^{\frac{\alpha}{1-\beta}}$). An increase in capital increases aggregate output, ceteris paribus.

Effects (2) and (3) are well known form standard structural change theory. Effects (4) and (5) are known from the neoclassical growth theory. In fact, our model is neoclassical in the sense that changes in productivity (due to effects (1)-(4)) lead to changes in capital accumulation and thus to even more aggregate output-growth. This mechanism is the same as in the standard one-sector Ramsey-(Cass-Koopmans-)model.

Our model implies that offshoring has an impact on channels (1), (3) and (5):

- Equation (35) and Theorems 1e and 2 imply that offshoring increases the productivity of intermediates production by internalizing the price-difference between domestic and foreign intermediates production. (Remember that only the case $\frac{\bar{p}_t - g_T}{\bar{p}_t} > 0$ is relevant). Hence, aggregate output-growth is increased by channel (1).

- Higher productivity in intermediates-production leads also to an increase in capital accumulation and to more aggregate output-growth via channel
(5) (like in the normal Ramsey model); this fact is implied by Theorems 1 and 3 and by equation (34).

- We show in the next section that this increase in capital accumulation leads to a slowdown of structural change patterns in the long run; i.e. due to offshoring the changes in \( n_i \) over time along the PBGP become smaller (see below, Theorem 5b). In other words, the structural change patterns associated with Baumol’s cost disease are slowed down by offshoring in the long run, which implies that less labor is reallocated to low-productivity-growth-sectors. Hence, there is a positive effect of offshoring on aggregate output-growth via channel (3).

- This positive effect increases the rate of capital accumulation again and thus aggregate output-growth and so on.

It should be noted that in our model channels (2) and (3) (and (4)) always “offset” each other regarding \( Y \)-growth, since equations (14) and (25) imply that

\[
\sum_i p_i A_i n_i = \sum_i \frac{A_{M_i}}{A_i} A_i n_i = A_M \sum_i n_i = A_M .
\]

Hence, whether the changes in \( n_i \) are slowed down by offshoring or not, \( Y \) is always the same since equation (19a) reduces to

\[
(19b) \quad Y = K^{\alpha/\beta} N^{1-\beta} A_M .
\]

However, this fact does not imply that offshoring has no impact on GDP-growth via channel (3) in our model and in general. The reasons are the following:

I. Aggregate output \((Y)\), as defined in our model, is not equivalent to the real GDP, as measured in reality. Equations (17) and (18) imply that \( Y \) stands for the aggregate output expressed in manufacturing terms (i.e. the manufacturing sector is numéraire). In contrast, real GDP is not measured in manufacturing terms but there is a compound numéraire. In APPENDIX C we show that real GDP is given in our model by the following measure:

\[
(19c) \quad GDP = \frac{(1-\beta)Y}{A_M \sum_i \frac{n_i}{A_i}} .
\]
We can see from equation (19c) that changes in $n_i$ have an impact on real GDP. Furthermore, we can formulate the following theorem:

**Theorem 4:** Along the PBGP, the changes in $n_i$’s over time are such that the growth rate of real GDP declines over time, where real GDP is given by equation (19c). That is, structural change has a negative impact on the growth rate of real GDP: due to structural change the growth rate of real GDP declines over time. This negative impact is the stronger, the stronger the changes in $n_i$’s over time are.

**Proof:** see APPENDIX C.

In fact, the proof in APPENDIX C shows that structural change has a negative impact on real GDP-growth in our model, since factors are withdrawn from high-productivity-sectors and reallocated to low-productivity-sectors. We show in the next section (Theorem 5b) that this reallocation process is slowed down by offshoring in the long run. Thus, offshoring has a positive impact on real GDP-growth in the long run (see also the discussion in section 6).

II. In our model sectors are aggregated by using their current weights (i.e. by using current (relative) prices) (see equation (17)). However, in reality, real GDP is not calculated by using current weights, but by using fixed weights or chain-weights. For example, Steindel (1995) discusses the usage of such fixed weights and chain-weights in the GDP-growth calculations by the U.S. Department of Commerce’s Bureau of Economic Analysis. The discrepancy between aggregate output, as measured in our model, and real GDP as measured in reality is mentioned as well by Ngai and Pissarides (2007) and discussed in detail by Ngai and Pissarides (2004), pp. 21 ff.. (Due to this discrepancy some models use a fixed-weight definition of aggregate output, e.g. Baumol, 1967, and Echevarria, 1997.) We show in APPENDIX D that structural change determines the growth rate of real GDP, provided that real GDP is calculated by the fixed-weights-method or the chain-weights-method. Furthermore, we show in the next section that structural change is slowed down by offshoring. Hence, offshoring has an impact on real GDP-growth via channel (3), provided that real GDP is calculated by using the fixed-weights-method or the chain-weights-method.
III. In general, the effects (2), (3) and (4) do not offset each other even regarding $Y$-growth. This fact is well known from the structural change literature: More complex assumptions, e.g. the assumption that output-elasticities of inputs differ across sectors, would yield that in our model effects (2), (3) and (4) do not offset each other regarding $Y$-growth. We omit here the explicit proof, since it is well known in the literature (see e.g. the models by Acemoglu and Guerrieri (2008) and Kongsamut et al. (1997)). Nevertheless, in APPENDIX E we provide an example: we show that structural change has an impact on the growth rate of $Y$, if sector differ by output-elasticites of inputs. We show in the next section that structural change is slowed down by offshoring. Hence, we know that in general offshoring has a “final” effect on $Y$-growth via channel (3). (In our paper we do not use the more general assumption (equation (4a) from APPENDIX E), but use equation (4), since in this way we can present our results in the most comprehensible way: The assumption of equation (4a) would complicate our analysis (i.e. a closed-form solution for the model could not be derived, and we would have to rely on simulations), but would not change our key result, namely the fact that offshoring has an impact on channels (1), (3) and (5).

Note that in models, where capital accumulation is excluded from analysis and where structural change patterns associated with Baumol’s cost disease are not taken into account, offshoring influences GDP-growth only via channel (1). Hence, these models neglect the effects of offshoring via channels (3) and (5). Therefore, it may be interesting to analyze what is the relative importance of effect (1) in comparison to the other effects. Equation (19a) implies (remember that $\sum p_i A_i n_i = A_M$)

\[(36) \quad \frac{\dot{Y}}{Y} = a + b + g_M \]

where $a = \frac{\alpha}{1-\beta} g^*$ and $b = \frac{\beta}{1-\beta} g_\eta$.

Remember that offshoring acts in our model like an increase in $g_\eta$. Hence, the direct impact of offshoring on aggregate output-growth via channel (1) is covered
by $b$ and the other effects are covered by $a$ (and $g_m$). The impact of offshoring on aggregate output-growth via channel (1) is given by:
\[
\frac{\partial(\dot{Y} / Y)}{\partial g_\eta} = \frac{\beta}{1 - \beta}.
\]
Furthermore, the impact of offshoring via the other channels is given by
\[
\frac{\partial(\dot{Y} / Y)}{\partial g_\eta} = \frac{\alpha}{1 - \beta} \cdot \frac{\frac{\beta}{1 - \beta}}{1 - \alpha - \beta} = \frac{\alpha}{1 - \beta} \cdot \frac{\beta}{1 - \beta - \alpha - \beta} \quad \text{(remember equation (34))}.
\]
Hence, we can be sure that the effect via channel (1) is weaker than the other effects provided that
\[
\frac{\beta}{1 - \beta} < \frac{\alpha}{1 - \beta} \cdot \frac{\beta}{1 - \beta - \alpha - \beta},
\]
which is equivalent to $\alpha > 1 - \alpha - \beta$.

Remember that our production functions imply that $\alpha$ is the economy-wide output-elasticity of capital and $1 - \alpha - \beta$ is the economy-wide output-elasticity of labor. Hence, we can be sure that the growth effects of offshoring via channel (1) are smaller than the other effects, if output-elasticity of capital is higher than output-elasticity of labor. Note, however, that these calculations do not take channel (3) into account. That is, the effects via channel (1) are even less significant.

5. **The effects of offshoring on structural change**

In this model structural change is caused by differences in TFP-growth across sectors. The differences in TFP-growth are reflected by prices (see equation (25)). The representative household responds to the changes in prices by changing the demand-ratios across goods. Hence, producers must adapt production to changing demand, which leads to factor reallocations across sectors, i.e. structural change. (For detailed discussion see Ngai and Pissarides, 2007.) We analyze now how offshoring affects these structural change patterns.

Equations (29)-(31) are relevant for analyzing the effects of offshoring on structural change. They represent the sectoral employment shares when the economy offshores. Since labor is normalized to unity in our model, these equations also represent the sectoral employment. We compare now the structural change patterns in an economy that offshores with the structural change patterns in an economy that does not offshore (see Theorem 2), ceteris paribus. We
analyze in this section the structural change patterns when $\frac{\hat{p}_I}{p_I} - g_T > 0$. (As noted in the previous section, the case $\frac{\hat{p}_I}{p_I} - g_T < 0$ in general cannot occur, since the country could return the state of autarky if terms of trade develop unfavorably; nevertheless, all the results for the case $\frac{\hat{p}_I}{p_I} - g_T < 0$ could be derived in the same manner as in this section.)

We have to distinguish between “transitory effects” and “PBGP effects” of offshoring with respect to structural change: Remember that we assume that in the beginning the economy is in autarky and moves along the PBGP. Then opening of borders occurs, which induces a transition to a new PBGP. The term “transitory effects” denotes the factor reallocations which occur during this transition period and which come to a halt when the economy is on the new PBGP. That is, as we will see, some industry-employment-shares are constant in the old and in the new PBGP; they only change during the transition period. The reallocations which are associated with this change during the transition period are named transitory effects. On the other hand, as we will see, when comparing the new and the old PBGP the strength of the factor reallocation between some industries is not the same in the old and the new PBGP. That is, offshoring induces a change in the strength of structural change, when comparing the old and the new PBGP. This effect of offshoring is named “PBGP effects”.

“Transitory effects” of offshoring: As just explained, this term stands for the factor reallocations that are caused by offshoring and that take place only during the transition period between two PBGPs. We have to distinguish between four different “transitional effects”, which are explained in the following and summarized in Theorem 5a:

Effect 1: Offshoring increases the exports-to-output ratio (E/Y), since the economy has to “pay” for intermediate imports. This effect increases the employment share of the exporting sector $M$ (see equation (30); note that $E/Y$ is given by $(1 - \varphi)\beta$ due to equation (23)). $E/Y$ is constant along the PBGP due to Theorem 1. Thus, the increase in exports is a transitional effect with respect to structural change. That is, the changes in the
employment share of export-industries (which are a subsector of sector M) are accomplished during the transition period.

**Effect 2:** Offshoring decreases the domestic-intermediates-to-output ratio \( \left( \frac{p_I h_I}{Y} \right) \), since some intermediates are imported from abroad. (Note that due to equation (26) \( \frac{p_I h_I}{Y} \) is given by \( \phi \beta \) when offshoring takes place; in the case without offshoring, \( \frac{p_I h_I}{Y} \) is given by \( \beta \) (see Theorem 2 and equation (26)). Thus, the domestic-intermediates-production-to-output ratio decreases by \( (1-\phi)\beta \) due to offshoring.) *This effect decreases the employment share of sector I* (via \( \phi \beta \); see equation (31)), since intermediate industries are part of sector I. Note again that this effect is transitional as well, since \( \frac{p_I h_I}{Y} \) is constant along the PBGP (it is equal to \( \phi \beta \)). That is, the decrease in the domestic-intermediates-production-to-output ratio is accomplished during the transition period.

**Effect 3:** It can be shown (see APPENDIX F) that the aggregate investment-to-output ratio \( ((\delta + g^*) K / Y) \) increases due to offshoring. *This effect increases the employment share of the sector M* (see equation (30)), since capital-producing industries are a part of sector M. Since \( (\delta + g^*) K / Y \) is constant along the PBGP (see Theorem 1), this effect is transitional, i.e. the accompanying reallocations are accomplished during the transition period. The increase in the aggregate investment-to-output ratio occurs, because of the higher aggregate productivity-growth (see also the previous section).

**Effect 4:** The aggregate output of our economy is consumed, exported, used as capital-input and used as intermediate input (see equations (7), (10)-(12) and (16)-(17)). Thus, the following relation must be true:

\[
1 = \frac{C}{Y} + \frac{(\delta + g^*) K}{Y} + \frac{E}{Y} + \frac{p_I h_I}{Y}. 
\]

(This equation is implied by equations (20) and (26).) Our explanations of Effects 1 and 2 imply that \( E/Y \) increases due to offshoring by the same amount as \( \frac{p_I h_I}{Y} \) decreases due to
offshoring. Thus, \( \frac{C}{Y} + (\delta + g^*)K \) must be constant when comparing the PBGP without offshoring to the PBGP with offshoring. This fact implies that the aggregate consumption-to-output ratio \( (C/Y) \) must decrease due to offshoring during the transition period, since Effect 3 implies that \((\delta + g^*)K/Y\) increases due to offshoring. Note that, as just explained, the decrease in \( C/Y \) is caused only by the increase in the investment-to-output ratio \(((\delta + g^*)K/Y)\) (and not by exports or by domestic intermediate goods production). What are the implications of the decrease in \( C/Y \) for structural change? Theorem 1 implies that \( C/Y \) is constant along the PBGP. Thus, equations (29)-(31) imply that the decrease in \( C/Y \) is in part a transitional effect (the change in \( C/Y \) is accomplished during the transition period) which reduces the employment shares in all sectors during the transition period, since all sectors feature consumption goods industries. However, \( x_i/X \)'s are not constant along the PBGP (see equations (32)-(33)).

Hence, equations (29)-(31) imply that the decrease in \( C/Y \) has also an PBGP-effect which will be discussed now.

"PBGP effects" of offshoring (for an explicit proof see Theorem 5b): The term "PBGP effects" stands for the differences in strength of structural change when comparing the old and the new PBGP. (Strength of structural change means the amount of labor that is reallocated per unit of time. Hence, strength of structural change can be measured by strength of changes in the employment shares.) Hence, the PBGP-effects may the regarded as permanent or long-run effects of offshoring on structural change. Our discussion of Effects 1-4 above and equations (29)-(31) imply that \( \frac{x_i}{X} C \) 's are the only terms, which determine the strength of structural change along the PBGP. \( \frac{x_i}{X} C \) denotes the ratio of sectoral consumption to aggregate output, since equation (28) implies that \( \frac{x_i}{X} C = \frac{p_i C_i}{Y} \).

A decrease in \( C/Y \) (see Effect 4) decreases the strength of structural change (see

\[14\] Ie omit here the discussion of the shape of the \( x_i/X \)-curves, since they are the same as in the
equations (29)-(31) and Theorem 5b below), which means that less labor is reallocated across sectors over time. That is, offshoring causes a slowdown of structural change (or in other words: structural change smoothing) in the long run. (A discussion of the shape of the \( x_i / X \)-curves can be found in APPENDIX B and in the paper by Ngai and Pissarides, 2007.)

Now, let us summarize these results as follows:

**Theorem 5a:** Offshoring leads to

- an increase in the exports-to-output ratio \( (E/Y) \) (i.e. offshoring has a positive impact on the employment share of export industries during the transition period)
- a decrease in the domestic-intermediates-production-to-output ratio \( \left( \frac{p_i h_i}{Y} \right) \) (i.e. offshoring has a negative impact on the employment share of domestic intermediate industries during the transition period)
- an increase in the investment-to-output ratio \( ((\delta + g^*)K/Y) \) (i.e. offshoring has a positive impact on the employment share on investment-goods-industries during the transition period)
- a decrease in the consumption-to-output ratio \( (C/Y) \) (i.e. offshoring has a negative impact on the employment share of consumption-goods-industries during the transition period).

**Theorem 5b:** Structural change along the PBGP is slowed down by offshoring. That is, along the PBGP the changes in \( n_i \) over time are weaker in an economy that offshores in comparison to an economy that does not offshore, ceteris paribus.

**Proof:** Since this Theorem is important, here is an explicit proof: Remember that \( C, Y \) and \( K \) are constant along the PBGP. Hence, equations (29)-(31) imply that the following relations are true along the PBGP:

\[
(29a) \quad \frac{dn_e}{dt} = \frac{C}{Y} \frac{d(x_p/X)}{dt}
\]

where \( t \) stands for time. Theorem 5a implies that \( C/Y \) declines due to offshoring. Hence, equations (29a)-(31a) imply that the changes in \( n_i \) over time are weaker due to offshoring. Hence, since structural change stands for changes in \( n_i \) over time, structural change is slowed down by offshoring. \( Q.E.D. \)

**Theorem 5c:** The structural-change-slow-down from Theorem 5b is caused by the offshoring-induced decline in the consumption-to-output ratio.

**Proof:** This Theorem is implied by the proof of Theorem 5b. \( Q.E.D. \)

**Corollary:** Because of the positive productivity effects of offshoring, consumption becomes a (quantitatively) less important part of aggregate output (shift from consumption-goods-production to investment-goods-production). Hence, the changes in consumption demand patterns \( x_i / X \) (which are the only determinant of structural change along the PBGP) become less relevant for the reallocation of factors across sectors. Therefore, offshoring causes a structural change slowdown.

All explanations regarding the development of sectoral employment shares \( n_i \) are also true for the sectoral capital shares \( k_i \), intermediate input shares \( z_i \) (see equation (27)) and sectoral output shares \( p_i Y_i / Y \) (see equations (29)-(31)).

6. **Discussion and implications**

6.1 **A chain of dynamic effects of offshoring**

In the introduction of this paper we have postulated a chain of dynamic effects along which offshoring affects real-GDP-growth as follows:
Offshoring $\rightarrow$ Increase in TFP of intermediates production (traditional trade theory) $\rightarrow$ increase in capital-accumulation (neoclassical growth theory) $\rightarrow$ reallocation of factors from consumption-goods-production to capital-production (Uzawa’s structural change) $\rightarrow$ decline in importance of “degenerative” consumption-industries-structural change patterns (Baumol’s structural change) for real GDP-growth.

Now we show the validity of this chain by using the theorems, which have been derived in the previous sections.

Equation (35) and Theorems 1e and 2 imply that offshoring increases the TFP of intermediates production. Theorem 5a (third and fourth point) implies that this leads to an increase in capital accumulation and reallocation of factors from consumption industries to capital industries (Uzawa’s structural change). Theorems 5b,c and the Corollary imply that therefore, there is a slow-down of Baumol’s structural change. Finally, Theorem 4 implies that this has a positive impact on real GDP-growth.

### 6.2 Exogenous terms of trade

In our analysis we have assumed that terms of trade are exogenous, since the questions analyzed in our paper do not require endogenous terms of trade. As mentioned in section 3 we know that, if offshoring takes place, terms of trade must be such that

$$\frac{p_I}{g_T} > 0,$$

since otherwise the country would be better off in autarky (i.e. offshoring would not take place). Furthermore, for

$$\frac{p_I}{g_T} > 0$$

all our results are unambiguous. Hence, there is no need for endogenizing the terms of trade. Nevertheless, it may be interesting to know the (endogenous) terms of trade as function of deep parameters of the model. To derive the possible range of the terms of trade we analyze the following example: we assume for the moment that the foreign country (India) is the same as the domestic country (USA) beside of the TFP in sector I. Let the TFP in sector I in India be $A_I^f$. Hence, due to equation (25) the price for the good I is given by $p_I = \frac{A_I}{A_I}$ in the USA and by
$p_i^F = \frac{A_i}{A_i^F}$ in India. This implies that one of the countries will offshore to the other country (and export M-goods) and that both countries will be better off when trading, provided that $A_i \neq A_i^F$ and provided that $T$ (i.e. the reciprocal of the terms of trade) is somewhere between $p_i$ and $p_i^F$ (see also Theorem 3). We know that $T$ will be somewhere between $p_i$ and $p_i^F$, since otherwise both countries would be better off in autarky and in autarky both countries would lose the gains from trade. (Remember that $p_i$ is not only an indicator of technological differences between sectors, but also of the utility based relative demand for the goods, see footnote 11).

6.3 Negative impacts of offshoring

Although in the present model setting there seems to be no reason for negative effects of offshoring for GDP-growth (i.e. terms of trade must always be such that $\frac{\dot{p}_i}{p_i} - g_r > 0$), it may be possible to construct cases where offshoring negatively affects GDP-growth, i.e. $\frac{\dot{p}_i}{p_i} - g_r < 0$. For example, assume that after the departure from autarky the technology develops in such a manner that some foreign intermediates, that cannot be produced at home, become essential for state-of-art production. In this case the state of art production process becomes dependent on foreign resources (i.e. change from equation (6b) to (6a) becomes impossible). The dependency on foreign resources would allow for unfavorable terms of trade development (i.e. $\frac{\dot{p}_i}{p_i} - g_r < 0$) and hence for a GDP-growth slowdown (see Theorem 3), since the foreign country has a better bargaining position and can dictate the terms of trade. An intuitive example for this argumentation may be mineral oil: Some industrialized countries do not have (relevant) reserves of mineral oil at home. However, since they started importing mineral oil in the early 20th century their technology developed such that mineral oil is a key resource for the most products. Of course, the question here is whether the development process in these countries would have been much slower, if they
never had started importing mineral oil (e.g. by researching right from the beginning, i.e. in the early 20th century, for alternative non-oil-based technologies). Hence, there is a trade-off between the losses from using alternative technology (slower development) and the losses from offshoring-dependency (weak bargaining position due to dependency). This trade-off would have to be evaluated.

6.4 Phases of impact and manufacturing renaissance

Our distinction between “transitional effects” and “PBGP effects” of offshoring on structural change in the previous section implies that only the “PBGP effects” constitute a permanent effect on structural change in the long run. The “transitional effects” can be regarded as transitory effects of offshoring with respect to structural change.

A further interpretation of the distinction between “transitional effects” and “PBGP effects” might be that the effects of offshoring will impact the economy in two phases: In this case the “transitional effects” might be regarded as phase-1-effects and “PBGP effects” might be regarded as phase-2-effects. That is, when offshoring starts (e.g. due to technological progress or due to opening of international borders) the economy must go through phase 1 first. The reallocations during this phase are described by Effects 1-4 in the previous section: employment in domestic intermediate-inputs-industries decreases, employment in exports-industries increases, employment in capital-producing industries increases and employment in consumption-goods-industries decreases. Note that all these effects imply that labor is reallocated from all sectors to the capital-producing sector in phase 1. That is, in phase 1 there is a kind of “manufacturing sector renaissance” (provided that we interpret the capital-producing sector as the manufacturing sector; see also p.11 and footnote 10). After this phase is accomplished, phase 2 starts where smoother structural change prevails, as explained in the previous section.

Thus, overall, this interpretation implies that the economy faces a turbulent phase 1 (where strong labor reallocations take place) due to offshoring. This result supports Blinder (2005, 2007a, 2007b) who emphasizes the possible negative (transitory) effects of offshoring. He argues that the reallocations during the
transitory phase can cause high unemployment, since they require that the labor force changes its skill sets. This is especially true if the labor force has to be reallocated across sectors rather than within sectors, since different skill sets are required across sectors (for example, in the services sector soft skills are much more important than in the manufacturing sector). Our discussion of phase 1 implies that most of the labor force will have to be reallocated across sectors during this phase, which implies that indeed high unemployment may arise in reality. Furthermore, our discussion of phase 1 implies as well that unemployment might be even higher than expected by now: Unemployment may not only arise in the intermediates-industries, but also in the consumption-goods-industries (Effect 4).

6.5 Partial offshoring

A further interesting result from the previous section is that a sort of “partial offshoring” occurs. That is, only a part of the intermediates-production is offshored: the labor employed in the domestic intermediates production ($\phi\beta$) does not decrease in the long run, but is constant (see discussion of Effect 2), i.e. the intermediates-production is not completely taken over by the foreign labor force. This is consistent with the experience from manufacturing sector offshoring: developed economies are still producing manufacturing goods.\textsuperscript{15} To our knowledge, the only paper that models partial offshoring is the one by Choi (2007), where partial offshoring occurs due to uncertainty. Our results imply that partial offshoring occurs even when there is no uncertainty, provided that foreign intermediates are not perfect substitutes for domestic intermediates. In our model this result comes from the fact that the relative extent of offshoring ($h_f / h_p$) depends not only on price relations between domestic and foreign producers, but also on quality of foreign products (indicated by $\varphi$); see also equation (26) and discussion in section 2.

\textsuperscript{15} See, e.g. Blinder (2007b).
7. Concluding remarks

Overall, our model implies that the inclusion of capital and structural change associated with Baumol’s cost disease is crucial for an adequate assessment of the productivity effect of offshoring. Standard trade theory in general does not include these factors. Hence, the decision on the overall effect of offshoring (i.e. whether the negative terms-of-trade effect is stronger or weaker than the positive productivity effect) may be biased in this literature. In all our results capital plays the key role: capital accumulation does not only create additional growth by itself, but also makes the existence of growth effects via Baumol’s cost disease possible (if there was no capital in our model, offshoring would not have any effects on GDP-growth via Baumol’s cost disease).

These effects are based on the mechanism that an offshoring-induced increase in domestic productivity accelerates domestic capital-production. Therefore, the effects, which are shown in our model, are weaker, if some capital-goods are imported from abroad. (In this case domestic capital production is less relevant for domestic aggregate dynamics.) Nevertheless, we know that every country produces some capital goods at home; thus, the effects that are modeled in our paper exist in reality. Furthermore, in the discussion about North-South-offshoring\textsuperscript{16} the assumption that the North produces the largest part of its capital-goods on its own seems to be plausible, since the South does not posses the technology to produce the high-tech capital-goods of the North.

Our results imply that offshoring has the potential to influence the long run growth of industrialized economies (positively). Of course this influence persists only as long as different countries use different technologies to produce similar (not perfectly substitutable) goods.

Our result regarding the (long-run) structural change slowdown associated with offshoring implies that offshoring has an impact on the key feature of the modern development process, namely the transition from a manufacturing economy to a services economy. Hence, many topics associated with this transition (ranging from labor market policy to education-system-design) are influenced by

\textsuperscript{16} North-South-offshoring means here offshoring between industrialized countries and less developed countries, which is in the focus of the actual offshoring debate.
offshoring. Note that structural change arises in our model due to cross-sector differences in TFP-growth. We focused on this structural change determinant, since it has important implications for aggregate growth (via Baumol’s cost disease). However, this is not the only structural change determinant studied in the literature; e.g. Kongsamut et al. (2001) show that the consumption demand patterns associated with non-homothetic preferences can cause structural change as well. Our model results imply that these structural change patterns are slowed down by offshoring as well, since in our model the structural change slowdown comes from a decrease in the importance of consumption demand patterns for factor allocation.

We made several assumptions which are simplifying the model, but which are not necessary for our key results: Our subdivision of the economy into impersonal services (which are offshored), personal services and manufacturing goods is not necessary. Actually any sort of offshoring could be analyzed within our model. Furthermore, manufacturing goods need not to be exported but other goods can be exported. As discussed in the previous section, the terms-of-trade can be endogenized in our model; however, endogenous terms of trade do not change our key results.

If we used more complicated assumptions in our model (e.g. cross-sector differences in output-elasticities of inputs), the effects, which are studied in our model, would still exist, while the analysis would become much more complicated (simulations would be necessary). However, we cannot exclude that more complicated assumptions would yield additional growth-effects of offshoring. Further research could focus on the study of the existence of such effects.

Regarding further research the following points may be of interest as well:

First, as discussed in the previous section our model implies that offshoring could theoretically have negative growth effects, provided that some kind of dependency on foreign intermediates arises. Further research could try do develop this argumentation further. However, as discussed in the previous section, this case is difficult to analyze, since it requires comparing different paths of technological development.

Second, as discussed in our paper, offshoring can increase GDP-growth via structural change, since capital production becomes more important in comparison
to consumption goods production, where consumption goods production (but not capital production) causes the growth slowdown associated with Baumol’s cost disease. The question is whether the structural change patterns associated with Baumol’s cost disease can also arise in the capital producing sector. This would require, that different sorts of capital are produced by different technology and that capital demand is price inelastic. If this is possible, the offshoring-induced increase in capital production would also cause by itself some growth effects via Baumol’s cost disease. Hence, the final effects would depend on the relative strength of Baumol’s cost disease in the consumption goods production in comparison to the capital goods production.

Third, unemployment could be explicitly analyzed in our model by assuming some barriers to inter-sectoral reallocation of labor. This framework could be used to analyze the effects of such barriers on the duration of the transition period and on the growth rate of aggregates.

Fourth, there are of course several further potential impact channels of offshoring on growth which we did not analyze in our model (e.g. offshoring could influence the endogenous technological progress). All these topics are left for further research.
APPENDIX A

We know from equation (11) that \( h_i \) is produced by sector I. Thus, the total factor productivity (TFP) of \( h_i \)-production is given by the TFP of sector I. It follows from equation (4) that the TFP of sector I is given by \( A_i \). Thus, we can formulate the following Theorem:

**Theorem A1:** The TFP of the domestic intermediates-production (\( h_i \)) is given by \( A_i \).

Equations (7), (8) and (18) imply

\[
A_1 = \frac{e_M}{T}
\]

We know from equation (10) that \( e_M \) is produced by sector M. Thus, the TFP of \( e_M \)-production is given by the TFP of sector M. Thus, we can formulate the following Theorem due to equation (4):

**Theorem A2:** The TFP of exports-production (\( e_M \)) is given by \( A_M \).

It follows from equation (A.1) and Theorem A2:

**Theorem A3:** The implicit TFP of intermediates-imports (\( h_e \)) is given by \( A_M \)

Implicit TFP means here the TFP which is implied by the terms of trade and by the TFP of the export sector.

Equation (6) and Theorems A1 and A3 imply that the implicit TFP of intermediates-production (\( Z \)) is given by \( A_i \phi \left( \frac{A_M}{T} \right)^{1-\phi} \). The growth-rate of this term (i.e. the implicit TFP-growth-rate of \( Z \)-production) is given by \( \phi g_i + (1-\phi)(g_M - g_T) \). This term is equal to \( g_n \), because of equations (25) and (35). Q.E.D.


APPENDIX B

In the proof of Theorem 5b we have shown that the dynamics of the employment shares along the PBGP are given by the following equations:

(B.1) \[ \frac{dn_p}{dt} = \frac{C}{Y} \frac{d(x_p / X)}{dt} \]

(B.2) \[ \frac{dn_M}{dt} = \frac{C}{Y} \frac{d(x_M / X)}{dt} \]

(B.3) \[ \frac{dn_l}{dt} = \frac{C}{Y} \frac{d(x_l / X)}{dt} \]

Remember that C/Y is constant along the PBGP, due to Theorem 1b. Hence, equations (B.1)-(B.3) imply that the dynamics of the employment shares are determined by the dynamics of \( x_i / X \) 's.

Equations (32) and (33) imply

(B.4) \[ \frac{x_p}{X} = \frac{1}{\left( \frac{\omega_l}{\omega_p} \right) \left( \frac{A_p}{A_l} \right)^{1-\varepsilon} + \left( \frac{\omega_M}{\omega_p} \right) \left( \frac{A_P}{A_M} \right)^{1-\varepsilon} + 1} \]

(B.5) \[ \frac{x_M}{X} = \frac{1}{\left( \frac{\omega_p}{\omega_M} \right) \left( \frac{A_M}{A_P} \right)^{1-\varepsilon} + \left( \frac{\omega_l}{\omega_M} \right) \left( \frac{A_L}{A_M} \right)^{1-\varepsilon} + 1} \]

(B.6) \[ \frac{x_l}{X} = \frac{1}{\left( \frac{\omega_p}{\omega_I} \right) \left( \frac{A_I}{A_P} \right)^{1-\varepsilon} + \left( \frac{\omega_M}{\omega_I} \right) \left( \frac{A_M}{A_I} \right)^{1-\varepsilon} + 1} \]

We analyze here two cases: (1) high-productivity-goods are offshored and (2) low-productivity-goods are offshored (see also Definition 1).

**Case (1): In this case the sectoral productivity ranking is as follows:**

(B.7) \( A_p < A_M < A_I, \quad g_p < g_M < g_I \)
Hence, the sector P is a low-productivity-sector and sector I is a high-productivity-sector. (B.4), (B.6) and (B.7) imply that \( \frac{d(x_p / X)}{dt} > 0 \) and \( \frac{d(x_I / X)}{dt} < 0 \), provided that \( \varepsilon < 1 \). Hence, equations (B.1) and (B.3) imply that the employment share of sector P increases and the employment share of sector I decreases, provided that \( \varepsilon < 1 \). That is, factors are reallocated from high-productivity-sector(s) to low-productivity-sector(s), provided that demand is price-inelastic.

Case (2): In this case the sectoral productivity ranking is as follows:

(B.8) \( A_p < A_I < A_M, \ g_p < g_I < g_M \)

In this case sector M has the highest TFP-growth rate and sector P has the lowest TFP-growth rate. (B.4), (B.5) and (B.8) imply that \( \frac{d(x_p / X)}{dt} > 0 \) and \( \frac{d(x_M / X)}{dt} < 0 \), provided that \( \varepsilon < 1 \). Hence equations (B.1) and (B.2) imply that \( \frac{dn_p}{dt} > 0 \) and \( \frac{dn_M}{dt} < 0 \), provided that \( \varepsilon < 1 \). That is, again factors are reallocated from the sector with the highest-TFP-growth rate to the sector with the lowest-TFP-growth rate, provided that demand is price-inelastic.
APPENDIX C

In reality real GDP is calculated by using an average price as GDP-deflator; i.e. in general, a basket of all goods that have been produced is used as numéraire. (See also Ngai and Pissarides (2007), p. 435, and Ngai and Pissarides (2004), p. 21.) In our model we choose the manufacturing output as numéraire, since in this way we can analyze equilibrium growth paths in the most convenient manner. Nevertheless, we can calculate the real GDP by using an average price deflator as well. We use the following compound deflator which may be regarded as the theoretical mirror image of the deflators that are used to calculate real GDP in reality:

\[
\bar{p} = \sum_{i} \frac{p_{i}Y_{i}^{N}}{Y_{N}} - p_{i}
\]

where \( Y_{i}^{N} \) and \( Y_{N} \) denote respectively the net-output of sector \( i \) and aggregate net-output. \( p_{i} \) are the prices of our model (where sector-M-output is numéraire). “Net-output” means here gross-output minus real value of intermediates inputs (which is equal to “real-value added”). Hence, \( Y_{i}^{N} \) is given by the following relation:

\[
p_{i}Y_{i}^{N} = p_{i}Y_{i} - z_{i}H
\]

where \( H \) is the aggregate value of all intermediates that have been used for production. Hence,

\[
H = p_{i}h_{i} + T h_{F}
\]

(Remember that, \( T \) is the price of foreign intermediates as e.g. defined in equation (8)).

Hence,

\[
Y_{N} = \sum_{i} p_{i}Y_{i}^{N}
\]

We use “net output”, since in reality GDP does not include intermediates production in order to avoid “double counting of intermediates production”. (See, e.g., Landefeld et al. (2008) on intermediate inputs and GDP.)
Overall, our GDP-deflator ((C.1)) is simply a weighted-average of prices, where we used net-output-shares as weights (and where prices are in manufacturing terms). Thus, if we divide our aggregate net-output (which is expressed in manufacturing terms) by this deflator we have a GDP-measure that is similar to that used in reality.

(C.5) \[ GDP \equiv \frac{Y_N}{\bar{p}} \]

Note that instead of definition (C.1) we could use the following definition:

(C.6) \[ \bar{p}^t \equiv \sum_i \frac{p_i^t Y_i}{Y} p_i \]

where not net-output-shares are used as weights, but output-shares. However, it does not matter, i.e. in our model \( \bar{p} \) and \( \bar{p}^t \) are the same. (We omit here the proof, since it is trivial.)

By using equations (C.1)-(C.5) and (15),(19), (22) and (25)-(27) it can be shown that

(C.7) \[ GDP = \frac{(1-\beta)Y}{\bar{p}} = \frac{(1-\beta)Y}{A_M \sum_i \frac{n_i}{A_i}} \]

Q.E.D.

Note that \( Y \) is given by equations (19) and (24). Furthermore the price index is given by

(C.8) \[ \bar{p} = A_M \sum_i \frac{n_i}{A_i} \]

We can see that the real GDP is determined by factor-allocation across sectors (where sectors differ by the productivity parameters \( A_i \)). Hence, structural change (i.e. changes \( n_i \)) affect the growth rate of GDP.

Now, we show that the impact of structural change on real GDP is negative. (C.7), (C.8), (19) and (24) imply that the impact of structural change on GDP depends only on the development of \( \bar{p} \). If we show that reallocation of factors across sectors (i.e. changes in \( n_i \)) increases \( \bar{p} \), we know that structural change has a negative impact on real GDP-growth. To do so we derive the following total differential of \( \bar{p} \):

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Due to equation (14) the following relation must be true

(B.10) \[ \sum_i d n_i = 0 \]

where we have calculated the total differential of equation (14).

Like in APPENDIX B we distinguish between two cases: (1) high-productivity-goods are offshored and (2) low-productivity-goods are offshored (see also Definition 1).

Case (1): In this case the relations between sectoral total-factor-productivities are as follows:

(C.11) \[ A_p < A_M < A_I, \quad g_p < g_M < g_I \]

By using equation (C.10), equation (C.9) can be reformulated as follows:

(C.12) \[ d\bar{p} = \left( \frac{A_M}{A_p} - 1 \right) dn_p + \left( \frac{A_M}{A_I} - 1 \right) dn_i \]

In APPENDIX B we have shown that in this case the employment share of sector P increases and the employment share of sector I decreases; hence

(C.13) \[ dn_p > 0, \quad dn_i < 0 \]

(C.11)-(C.13) imply that \[ d\bar{p} > 0 \].

Case (2): In this case the relations between sectoral total-factor-productivities are as follows:

(C.14) \[ A_p < A_I < A_M, \quad g_p < g_I < g_M \]

By using equation (C.10), equation (C.9) can be reformulated as follows:

(C.15) \[ d\bar{p} = \left( \frac{A_M}{A_p} - A_M \right) dn_p + \left( 1 - \frac{A_M}{A_I} \right) dn_M \]
In APPENDIX B we have shown that in this case the employment share of sector P increases and the employment share of sector M decreases; hence

\[(C.15) \quad d:\!n_P > 0, \quad d:\!n_M < 0\]

(C.14)-(C.15) imply that \( d\!\bar{p} > 0 \)

Overall, we have shown that in both cases labor is reallocated from the low-productivity-sector to the high-productivity sector. This reallocation process leads to increases in \( \bar{p} \). Increases in \( \bar{p} \) lead to decreases in real GDP (according to equation (C.7)). Hence, structural change has a negative impact on real GDP-growth. Furthermore, the stronger structural change is, the stronger is this negative impact (where the strength of structural change is indicated by \( dn_{\text{'s}} \)).

Q.E.D.
APPENDIX D

Following Steindel (1995), GDP measured by the fixed-weights method is given in our model by

\[
\sum_{i} p_{i}^{0} Y_i = K^{1-\beta} \eta^{1-\beta} \sum_{i} p_{i}^{0} A_i n_i \quad \text{where we used the prices in manufacturing terms in } t = 0 \text{ as fixed sector-weights; however, any other fixed weights could be used here). This equation implies that effects (2) and (3) (and (4)) do not offset each other regarding GDP-growth (since } \sum_{i} p_{i}^{0} A_i n_i = A_m^{0} \sum_{i} \frac{A_i n_i}{A_i^{0}}). \text{ That is, changes in } n_i \text{ have an impact on the growth rate of } GDP_f. \text{ Hence, our discussion in section 4.2 implies that in our model offshoring influences } GDP_f \text{-growth via channel (3).}

We study the impacts of offshoring on chain-weighted GDP-growth in our model by using the following example: Assume that we want to calculate the growth rate of chain-weighted GDP between two points in time (e.g. } t = 1 \text{ and } t = 2 \text{ ). Define

\[
Y^1 = \sum_{i} p_{i} Y_i = K^{1-\beta} \eta^{1-\beta} \sum_{i} p_{i} A_i n_i \quad \text{and} \quad Y^2 = \sum_{i} p_{i}^{2} Y_i = K^{1-\beta} \eta^{1-\beta} \sum_{i} p_{i}^{2} A_i n_i,
\]

where \( p_{i}^{1} \) and \( p_{i}^{2} \) denote respectively the prices of goods \( i \) in \( t = 1 \) and \( t = 2 \). Following Steindel (1995), the growth rate of chain-weighted GDP between \( t = 1 \) and \( t = 2 \) is given by:

\[
\begin{align*}
\frac{\Delta GDP_c}{GDP_c} &= \frac{1}{2} \left( \frac{\dot{Y}^1}{Y^1} + \frac{\dot{Y}^2}{Y^2} \right) = \frac{\alpha}{1-\beta} g^{*} + \frac{\beta}{1-\beta} g_{\eta} + \frac{1}{2} \left( \frac{\sum_{i} p_{i}^{1} (A_i \dot{n_i})}{\sum_{i} p_{i} A_i n_i} + \frac{\sum_{i} p_{i}^{2} (A_i \dot{n_i})}{\sum_{i} p_{i}^{2} A_i n_i} \right)
\end{align*}
\]

Hence, we can see that factor reallocation (via \( n_i \)) has an impact on } GDP_c \text{-growth, i.e. the effects (2) and (3) (and (4)) do not offset each other regarding } GDP_c \text{-growth. Thus, we know from the discussion in section 4.2 that offshoring influences } GDP_c \text{-growth via channel (3).}
Assume that production functions are given by the following equation:

\[
Y_i = A_i (n_i)^{1-\alpha_i-\beta} (k_i K)^{\alpha_i} (z_i Z)^{\beta_i}, \quad \forall i
\]

This equation implies that input-elasticities of output differ across sectors.

Footnote 11 implies that the price of a good is given by:

\[
p_i = \frac{Y_M}{n_i Y_i} \frac{1-\alpha_M - \beta_M}{1-\alpha_i - \beta_i}.
\]

Hence, equations (14) and (17) imply that

\[
Y = (1-\alpha_M - \beta_M)Y_M \frac{\sum_{i \in M} n_i}{1-\sum_{i \in M} n_i}
\]

Note that this equation can be restructured further (i.e. the growth path of \(Y_M\) could be derived as function of exogenous parameters in the optimum); in this respect see the paper by Kongsamut et al. (1997) and the paper by Acemoglu and Guerrieri (2008) as well. However, for our purposes the function in (19b) is sufficient: we can see now that the allocation of labor across sectors (via \(n_i\)) has an impact on \(Y\). That is, structural change has an impact on the growth-rate of \(Y\), i.e. the effects (2), (3) and (4) do not offset each other regarding \(Y\)-growth. Hence, the discussion in section 4.2 implies that offshoring has a “final” impact on \(Y\)-growth when output-elasticities differ across sectors.
APPENDIX F

Theorem 1 and equation (21) imply that the following relation is true along the PBGP:

\[(F.1) \quad g^* = \alpha \frac{Y}{K} - \delta - \rho \]

This equation can be rearranged as follows:

\[(F.2) \quad (\delta + g^*) \frac{K}{Y} = \frac{(\delta + g^*)\alpha}{g^* + \delta + \rho} \]

The first derivative of \(\frac{(\delta + g^*)\alpha}{g^* + \delta + \rho}\) with respect to \(g^*\) is given by

\[(F.3) \quad \frac{\partial \left( \frac{(\delta + g^*)\alpha}{g^* + \delta + \rho} \right)}{\partial g^*} = \frac{\alpha \rho}{g^* + \delta + \rho} > 0 \]

Equations (F.2) and (F.3) and Theorem 3 imply that offshoring increases \((\delta + g^*) \frac{K}{Y}\) provided that \(\frac{\hat{p}_l}{\rho_l} - g_r > 0\). Q.E.D.
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