Structural Change, Kaldor-facts and Government Services

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Abstract
The coexistence of structural change and Kaldor-facts is a key feature of the development process. We provide a theoretical framework that can explain this stylized fact by the existence of government services.

Keywords
neoclassical growth models, structural change, Kaldor-facts, government services

JEL H4, O14, O41
1. Introduction

As documented by Kongsamut et al. (2001), the coexistence of structural change and Kaldor-facts is a key feature of the development process in industrialized countries during the last century. Furthermore, as discussed by Kongsamut et al. (2001), this stylized fact cannot be explained by standard growth theory.

In our paper we suggest a theoretical explanation for the coexistence of structural change and Kaldor-facts. To do so, we have to find a model that satisfies the following requirements: (1) Kaldor-facts are satisfied while structural change takes place; (2) all key structural change determinants are included into analysis. Requirement (1) is obvious. Requirement (2) may need some explanation: The aggregate behavior of the economy (i.e. Kaldor-facts) depends on the development of sectors (i.e. structural change). Furthermore, structural change is the result of the interaction between all (key) structural change determinants. Hence, the analysis of the relationship between structural change and Kaldor-facts should include all key structural change determinants. As discussed by Schettkat and Yocarini (2008), empirical evidence implies that the key factors that determine structural change patterns are: demand-shifts across sectors (caused by non-homo-thetic preferences) and differences in technologies across sectors. The latter include differences in TFP-growth and differences in capital-intensities across sectors.

Previous literature studies growth paths that do not satisfy both requirements ((1) and (2)). This literature can be divided into two groups:

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1 Structural change stands here for labor reallocation across sectors such as agriculture, manufacturing and services. See Kongsamut et al. (2001) for stylized facts regarding structural change.
2 In general, Kaldor-facts state/require that aggregates (e.g. aggregate output, aggregate capital) grow at a constant rate during the development process; for discussion see, e.g., Kongsamut et al. (2001).
3 The relevance of cross-sector-differences in TFP-growth for structural change has been studied by, e.g., Ngai and Pissarides (2007) (in a theoretical framework) and Baumol et al. (1985) (in an empirical study). The relevance of cross-sector-differences in capital intensities for structural change has been studied by, e.g., Acemoglu and Guerrieri (2006). For new empirical evidence on strong differences in capital intensities across sectors see, e.g., Valentiniyi and Herrendorf (2008).

• The models by Kongsamut et al. (1997), pp. 27-31, and Echevarria (1997) satisfy requirement (2), but not requirement (1).

Our model is based on the multi-sector growth-models studied by Kongsamut et al. (1997), pp. 27-31, and Echevarria (1997). Furthermore, we assume the existence of a government that collects taxes, uses them productively (in part) and provides some services/goods for free and/or grants to the representative household (e.g. social welfare grants, old-age pensions, free education, free health services, infrastructure maintenance, food vouchers).

Our results imply that the extent of this grants/services-provision has an impact on aggregate growth (Kaldor-facts) when structural change takes place. In fact, there exists a grants/services-program that ensures in our model that Kaldor-facts are exactly satisfied despite of structural change. We show that this grants-program is such that it “efficiently” ensures the basic needs of a representative household. In this case our model satisfies requirements (1) and (2). Overall, our model postulates a relationship between government’s services/grants-provision and GDP-growth via structural change.

In the next section we discuss our assumptions. In section 3 we discuss the results. In section 4 we have some concluding remarks.

2. Model-assumptions

The representative household maximizes the following utility function by consuming heterogeneous goods \((i = 1,...,m)\)

\[
U = \int_{0}^{\infty} u(C^{1}, C^{2},...,C^{m})e^{-\rho t} dt, \quad \rho > 0
\]

where
where \( t \) is the time index. \( C^i_t \) denotes the “market consumption” of good \( i \) (i.e. consumption of goods that are purchased on the market). \( \bar{S}^i_t \) and \( \bar{G}^i_t \) are exogenous. If \( \bar{S}^i_t \) is negative, it can be interpreted as the basic need regarding good \( i \) (e.g. food, basic education). If \( \bar{S}^i_t \) is positive, it can be interpreted as an endowment regarding good/service \( i \), e.g. a household that can repair cars has some positive endowment regarding the service “car repairing”. The \( \bar{G}^i_t \)'s stand for the free services and grants that are provided (and guaranteed) by the government. If some \( \bar{G}^i_t \)'s are assumed to be negative, they can be interpreted as goods/services that have to be provided to the government (a sort of “tax”), e.g. military service. Some \( \bar{S}^i_t \) and/or \( \bar{G}^i_t \) could be assumed to be constant and/or equal to zero.

Since income-elasticity and price-elasticity of demand differ across goods \( i \) and are different from unity (as long as not all \( \bar{S}^i_t + \bar{G}^i_t = 0 \)), the preferences allow for structural change caused by non-homothetic preferences and relative-price-changes.

Each of the goods is produced by a sector. Each sector produces its output \((Y^i_t)\) by a Cobb-Douglas production function

\[
(3) \quad Y^i_t = B^i_t n^i_t \left( \frac{K^i_t}{B^i_t n^i_t} \right)^{\alpha^i}, \quad 0 < \alpha^i < 1, \quad \forall i = 1, \ldots, m
\]

where we have normalized the aggregate amount of labor to unity. \( K^i_t \) represents the aggregate amount of capital; \( k^i_t \) and \( n^i_t \) represent respectively the fraction of capital and labor devoted to sector \( i \); \( B^i_t \) is a sector-specific technology-parameter that grows at the exogenous, sector-specific and constant rate \( g^i_t \).

All capital and labor have to be used in production
The government levies the (non-distortionary) tax-rate $\tau$ on the output of each sector. Furthermore, like Kongsamut et al. (1997, 2001) and Ngai and Pissarides (2007), we assume that only sector $m$ produces capital (and consumption goods)

$$
(1 - \tau)Y^m_i = \dot{K}_i + \delta K_i + C^m_i
$$

$$
(1 - \tau)Y^i_i = C^i_i, \quad \forall i \neq m
$$

where $\delta$ is the depreciation rate. We define aggregate output ($Y_t$) and aggregate consumption-expenditures ($E_t$) as follows

$$
Y_t \equiv \sum_i p^i_i Y^i_t; \quad E_t \equiv \sum_i p^i_i C^i_t
$$

where $p^i_i$ denotes the relative price of good $i$. Sector $i = m$ is numéraire

$$
p^m_i = 1
$$

The government has the following “budget restriction”

$$
\{\frac{\dot{A}}{A}\} \leq g(G_t^1, \ldots, G_t^m) \leq f(\tau, \{A_j\}_{j=0}, \{Y_t^1, \ldots, Y_t^m\}_{t=0}, \{\overline{G}_t^1, \ldots, \overline{G}_t^m\}_{t=0}) \quad \forall t, \quad \frac{\dot{A}}{A} = g_A
$$

where $g(.)$ and $f(.)$ are functions of the corresponding variables. In each period the government “produces” grants and services by using (some of the) current period taxes and (if necessary) taxes that have been saved in previous periods. The exogenous variable $A_t$, that grows due to some kind of technological progress, represents the productivity-level of this “production”-process.

3. Model-results
When there is free mobility of factors across sectors, the intratemporal and intertemporal optimality conditions for this model\(^4\) are given by

\[
p_i = \frac{\partial Y^m_i}{\partial n^m_i} = \frac{\partial Y_i}{\partial \theta_i} \quad \text{and} \quad -\frac{\hat{u}_m}{\hat{u}_i} = \frac{\partial Y^m_i}{\partial \theta_i}, \quad \forall i
\]

where \(\hat{u}_m = \partial u(.) / \partial C^m_i\). These conditions imply the following equations, describing the development of aggregates and sectors

**Aggregates**

\[
(10) \quad Y_t = (B_t^m)^{1-\alpha_m}(K_t)^{\alpha_m}\left(\alpha_m \frac{n^m_i}{k^m_i} + 1 - \alpha_m\right)\left(\frac{k^m_i}{n^m_i}\right)^{\alpha_m}
\]

\[
(11) \quad \dot{K}_t = (1-\tau)Y_t - \delta K_t - E_t
\]

\[
(12) \quad \frac{\dot{E}_t + \dot{V}_t}{E_t + V_t} = \frac{\partial Y^m_i}{\partial \theta_i} - \delta - \rho = \alpha_m \frac{\dot{Y}_i}{K_i} - \frac{n^m_i}{k^m_i} - \delta - \rho
\]

\[
(13) \quad n^m_i = \frac{k^m_i}{k^m_i} = 1 - \frac{W_t}{(1-\tau)\alpha_m} + \frac{\left(1 + \sum_i \alpha_i \beta_i\right)\dot{V}_i - \left(\alpha_m - \sum_i \alpha_i \beta_i\right)E_t}{(1-\tau)\alpha_m (1-\alpha_m)\dot{Y}_t}
\]

**Sectors (represented by employment shares)**

\[
(14) \quad n^m_i = \frac{(1-\alpha_i) \beta_i}{(1-\alpha_m) (1-\tau)} \frac{(E_i + V_i)}{\dot{Y}_i} - \frac{\dot{S}_i + \dot{G}_i}{(1-\tau)\left(\frac{k^m_i}{n^m_i} \frac{1-\alpha_m}{\alpha_m} \frac{\alpha_i}{1-\alpha_i}\right)^{\alpha_m}}(B^m_i)^{1-\alpha_i}(K_t)^{\alpha_i} \quad \forall i \neq m
\]

\[
(15) \quad n^m_i = \dot{K}_t + \delta K_t + \beta_m (E_i + V_i) - \frac{\dot{S}_m + \dot{G}_m}{(1-\tau)\dot{Y}_i} - \frac{\dot{S}_m + \dot{G}_m}{(1-\tau)\dot{Y}_i}
\]

**where**

\[
(16) \quad \dot{Y}_i \equiv (B^m_i)^{1-\alpha_m}(K_t)^{\alpha_m}(k^m_i / n^m_i)^{\alpha_m}
\]

\[
(17) \quad \dot{V}_i \equiv \sum_i p_i \dot{S}_i + \sum_i p_i \dot{G}_i
\]

\(^4\) They can be obtained by maximizing equations (1)-(2) subject to equations (3)-(8), by using the Hamiltonian.
Equations (10)-(12) are the same as in the “normal” Ramsey-model, beside of the fact that they contain the terms $k^m_t / n^m_t$ and $V_t$. Furthermore, government grants have an impact on the growth rates of aggregates via $V_t$ and $W_t$. Equations (10)-(13) imply that, if $V_t = W_t = 0$, $\forall t$, a unique equilibrium growth path exists where aggregates ($E_t, K_t, \tilde{Y}_t$ and $Y_t$) grow at the constant rate $g_m$ and $k^m_t / n^m_t$ is constant. Furthermore, equations (14) and (15) imply that structural change takes place along this equilibrium growth path, i.e. the employment shares of sectors ($n_t^i$) change. Along this growth path the changes in the employment shares can be monotonous (increasing, decreasing or constant) or non-monotonous depending on the setting of parameters $B_t^i$, $S_t^i$ and $G_t^i$. Hence, the model can satisfy the stylized facts of structural change (see footnote 1).

Overall, Kaldor-facts are satisfied (since aggregates grow at a constant rate) and structural change takes place, provided that the government chooses the grants-program $\left\{ G_t^1, ..., G_t^n \right\}_{t=0}^{\infty}$ such that $V_t = W_t = 0$, $\forall t$. Now, we discuss the intuition behind the condition $V_t = W_t = 0$.

Assume that the directive of the government is to ensure that the representative household can cover basic needs (the negative $S_t^i$’s) by using endowments (the positive $S_t^i$’s) and by using government grants ($G_t^i$’s). Furthermore, assume that the government seeks to satisfy its
directive “efficiently”, i.e. by providing as few grants as possible. To do so, the government must choose $G_1,...,G_m$ such that $V_t = W_t = 0, \forall t$. The reason is the following: If (in the real world) an average household has to cover its basic needs by its endowments and grants, it has two possibilities: (1) it can sell the endowments and grants and buy basic-needs-goods for this “money” and/or (2) it can use the labor, that is intrinsic in its endowments, to produce the basic needs for itself (planting vegetables instead of repairing neighbors car). Condition $V_t = \sum_i p_i S_i + \sum_j p_j G_j = 0$ ensures that the household can exactly cover its basic needs by strategy (1). Condition $W_t = \sum_i \frac{S_i + G_i}{B_i \left( \frac{k_i}{n_i} \right)^\alpha} = 0$ ensures that it can cover its basic needs by strategy (2), since $B_i \left( \frac{k_i}{n_i} \right)^\alpha$ is the average productivity of labor ($Y_i / n_i$) regarding good $i$.

Both conditions ($V_t = 0$ and $W_t = 0$) must be satisfied, since otherwise the household could cover more than its basic needs by combining the two strategies (a kind of “arbitrage”); e.g. repairing a car “for money” instead of planting vegetables and afterwards buying vegetables.

4. Concluding remarks

An intuitive explanation for the fact that Kaldor-facts are satisfied in our model only if the government guarantees basic-needs-coverage is the following: The value of overall consumption-expenditures ($E_t$) is equal to the sum of the net-value of basic-needs-expenditures $5 (-V_t)$ and the net-value of “surplus-expenditures” $6 (E_t + V_t)$. Equation (12) is the same as in the “normal” Ramsey-model, beside of the fact that in our model only the growth rate of surplus-expenditures depends on the real interest rate \( \frac{\partial Y_t^m}{\partial (k^n K_t)} \). This seems

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5 This term means the value of basic needs minus the value of endowments and government grants.
6 This term means the value of consumption-expenditures that exceeds the net-value of basic-needs-expenditures, i.e. the part of the consumption-expenditures-value that is not necessary to cover basic needs.
to be plausible, since the household seeks to cover its basic needs irrespective of how high/low the real interest rate is. It is well known from the “normal” Ramsey-model (and in this respect our model is the same) that satisfaction of Kaldor-facts requires that the real interest rate and the growth rate of (overall) consumption-expenditures are constant. This requirement and equation (12) imply that Kaldor-fact can be satisfied only if $V_i = 0$.\(^7\) In other words: If the government does not ensure basic-needs-coverage, consumption-expenditures contain a basic-needs-component that is independent of the real interest rate. Therefore, consumption-expenditures do not grow at constant rate when the real interest rate is constant (Kaldor-facts are not satisfied).

It should be mentioned that equations (9) and (17)-(19) imply that the growth rate of the government-efficiency-parameter ($g_{a}$) must be sufficiently high to ensure that the government can satisfy the condition $V_i = W_i = 0$ for ever.

Needless to say that the degree of grants-system-efficiency (as defined in our paper) is only one factor among many factors that cause the coexistence of Kaldor-facts and structural change in reality. Finding further factors and testing for relative importance among factors is left for further research.

\(^7\) This is a mathematical truth: $(E_i + V_i)$ and $E_i$ can grow at constant rates only if $V_i = 0$.\(^7\)
REFERENCES


