A simple dynamic climate cooperation model

Supplementary Appendix

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November 11, 2020

B Appendix: Additional extensions

B.1 Extension: Finite negotiations

Here we study a modified version of our full dynamic game in which the negotiations can take place only for a finite number of periods. All other features of the model remain the same. In particular, the time horizon where the payoffs are realized is still infinite, and $\Pi_n(k)$ and $\Pi_s(k)$ represent the present values of payoffs over this infinite time horizon. However, we abstract from the possibility of signing short-term agreements, and we only consider the random membership case. The only difference to our model from Section 2 is that if no treaty is signed by the end of period T (where $T > 1$), then no treaty is signed whatsoever, and each country receives a stream of payoffs π_0 per period from period $T + 1$ onwards. This modification of the model allows us to investigate to what extent our previous results depend on the assumption of an infinite time horizon. An infinite time horizon is usually required to sustain tacit collusion in dynamic pricing games, where collusion breaks down completely if the time horizon is finite. By contrast, we show in the following that in our model, a high degree of cooperation typically emerges if the number of periods in which countries can negotiate is finite but sufficiently large.

In order to facilitate the comparison to the game with infinite negotiations, we consider symmetric subgame perfect equilibria with no delays.[62] Let us now consider such an equilibrium and denote k_t^* (where $t = 1, 2, \ldots, T$) the number of countries that sign an agreement in period t (conditional on reaching that period).

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[62]Equilibria with delays are characterized in Section B.2.

As a first observation, note that if period T is reached without signing any agreement before, then the countries are essentially in the same situation as in the static model, and due to Assumption 4, in equilibrium k^{st} countries sign an agreement. This means that $k_T^* = k^{st}$. Intuitively, one could expect a similar effect as for a finitely repeated prisoner's dilemma, where repeating the static equilibrium is the only subgame perfect outcome. However, this turns out not to be the case here, when the counties have the opportunity to delay the negotiations.

For illustration, consider Example 2 from Section 3 with the parameter values as illustrated in Figure 2 (i.e., $N = 10$ and $\delta = 0.6$). As we have argued there, the equilibrium coalition size in the static model is 2 or 3 countries, while in the dynamic model we have $\underline{k} = 5.4$ and $\overline{k} = 8.2$ with equilibrium coalition sizes 6, 7, and 8. Thus, in the last period T (if it has been reached), $k_T^* = 3$ countries sign an agreement (assuming that countries coordinate on the equilibrium with higher participation). Now, in period $T-1$ the countries anticipate the equilibrium in period T and expect $k_T^* = 3$ countries to sign an agreement. Observe that in period $T-1$, the number of countries that sign an agreement, denoted k_{T-1}^*, is at least $\hat{\tau}(k_T^*) = \hat{\tau}(3) = 4$ (see Section 2). Much like in Proposition 3, the equilibrium coalition size in period $T-1$ must be just large enough so that k_{T-1}^* countries are willing to sign an agreement in period $T-1$, but $k_{T-1}^* - 1$ countries are not. Thus, $k_{T-1}^* = \hat{\tau}(k_T^*) = 4$. Proceeding backwards, we obtain by the same argument that $k_{T-2}^* = \hat{\tau}(4) = 5$ countries sign an agreement in period $T-2$ and $k_{T-3}^* = \hat{\tau}(5) = 6$ countries sign an agreement in period $T-3$. Now since $\hat{\tau}(6) = 6$, the number of countries that would sign an agreement in earlier stages would be again 6. The following proposition provides general statements that are analogous to Proposition 4 (see Appendix B.4 for a proof).

Proposition 9. *In the game with finite negotiations (with $T > 1$), the following statements hold:*

(i) There is an equilibrium where a coalition of size k^{st} signs an agreement in the first period, if and only if $k^{st} \geq \underline{k}$.

(ii) If $k^{st} < \underline{k}$ and T is sufficiently large, then there is an equilibrium where a coalition of size $\lceil \underline{k} \rceil$ signs an agreement in the first period.

Hence, the outcome under a finite number of negotiation stages (T) is characterized by a *ratcheting-up* in the coalition size from later towards earlier periods (see Figure 7 for a graphical illustration). This ratcheting-up stops when the maximum coalition size is reached, that coincides with the *smallest* stable coalition size ($\lceil \underline{k} \rceil$, which is the smallest fixed point of function $\hat{\tau}$) under an infinite time horizon for the negotiations (in the case $\lceil \underline{k} \rceil > k^{st}$). Hence, the multiplicity of equilibria that we observed in the infinite horizon

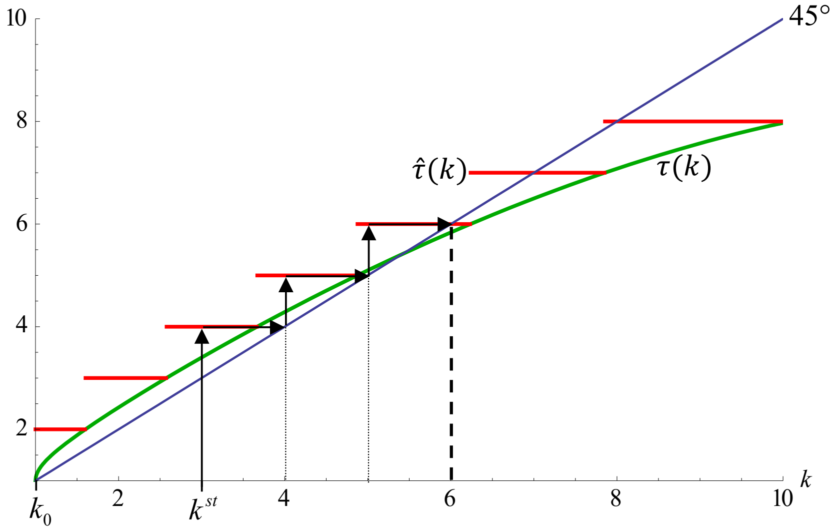

Figure 7: Illustration of *ratcheting-up* of the coalition size under finite T, for $\delta = 0.6$ and $N = 10$, random membership case (Example 2, Section 3)

case (see Figure 2) vanishes.[63] Otherwise, the results are unchanged.[64]

B.2 Extension: Non-Markov equilibria and delay

In this section we explore what other kinds of equilibria (in pure strategies) can emerge in our dynamic coalition formation model when the Markov restriction, that was imposed in most sections of this paper (except Section B.1 where a finite time horizon T was assumed), is relaxed. We do not seek to provide a full characterization of all equilibria that exist. Instead, we focus on a subset of equilibria that deliver interesting new insights. Most importantly, we preserve the payoff structure from the previous sections. Thus, we rule out collusive strategies, where countries use their *emissions* to punish deviations from some collusive agreement. Such equilibria have been studied elsewhere (e.g., Barrett 1994; Harstad, Lancia, and Russo 2019) and are not the focus of this paper. Our focus is on binding long-term agreements, and the dynamics of reaching such an agreement given the possibility to delay climate negotiations in one or several periods.

In particular, we maintain our earlier assumption that non-signatories choose their abatement efforts non-cooperatively and myopically in each period, while signatories of a long-term agreement choose their efforts so as to maximize their joint welfare. Further-

[63]Here, we refer to the multiplicity of equilibrium coalition sizes that arises when the interval $[\underline{k}, \overline{k})$ contains several integers (see Proposition 4). Because Proposition 9 does not provide a full characterization of equilibrium outcomes for the model with finite negotiations, and the Markov restriction cannot be imposed here, some multiplicity may remain, especially with regards to non-Markov equilibria (see Section B.2 for further details).

[64]We have also analyzed finite negotiations under deterministic membership. Using similar arguments as above, we can show that there is always an equilibrium where a coalition of size k^{st} signs an agreement in the first period under deterministic membership.

more, we do not allow for the possibility that countries can sign short-term agreements in periods where no long-term agreement has been reached yet. Hence, as in Section 2 countries' payoffs are fully captured by the functions Π_s and Π_n, by the size k of a coalition that signs a long-term agreement and the identity of the members of that coalition, as well as by the number of the period t in which the agreement is reached. We also maintain our assumption from Section 2 that the identity of coalition members (for a given coalition size k_t) is determined randomly in any period t (random membership case).

What is different when the Markov restriction is relaxed is that countries can condition their actions on the full history of the (participation) game up to that period. However, we preserve the assumption that at the signature stage the countries play a Pareto dominant equilibrium (if such exists).[65] In that respect, the countries may use only their non-participation, but not the use of veto power (during the signature stage) to punish deviations. It is well-known that strategies involving punishment (*grim trigger strategies*) can be used to sustain collusive agreements in infinitely repeated pricing games. We want to investigate if the *threat of delay* can be used in our setting to allow countries to reach a more cooperative outcome in the beginning of the game.

Before we give an answer to this question, let us first demonstrate that delay can actually occur along the equilibrium path in our setup. This is an interesting insight, given that delay has occurred many times in actual climate negotiations. To highlight this point, let us first consider the case where $k_0 = 0$ and $\Pi_s(0) = \pi_0/(1-\delta)$. Recall that by Assumption 2, the payoff functions Π_s and Π_n are increasing above k_0, which implies $\Pi_s(1) > \pi_0/(1-\delta)$. Therefore, there does not exist a trivial equilibrium where no long-term agreement is signed in any period, so the existence of equilibria with delay is clearly not based on this. Furthermore, there does not exist an equilibrium where fewer than k^{st} countries sign an agreement in the first period of the game, even if countries play non-Markov strategies that may involve delay in future periods (conditional upon reaching those periods).[66] Nevertheless, even under this simplifying assumption, subgame perfect Nash equilibria (SPNE) can exist that exhibit delay along the equilibrium path.[67]

To see this, suppose the payoff functions Π_s and Π_n are such that the static model exhibits a non-trivial amount of cooperation in equilibrium, that is: $k^{st} \geq 2$. Then by Proposition 3, there is an equilibrium (in Markov strategies) where a coalition of size $k^* \geq k^{st}$ signs a long-term agreement. Suppose, if period $\theta \geq 2$ is reached, countries indeed play Markov strategies and coordinate on the equilibrium coalition size $k^*_\theta = k^*$. Then if the discount factor is not too small, there clearly exists an equilibrium in the full dynamic model (without the Markov restriction) where no agreement is reached in

[65] This rules out equilibria where the countries use the signature behavior for punishment, for instance by joining the coalition, but not signing unless all other countries have joined.

[66] To see this, recall that for $k < k^{st}$ the external stability condition is violated, so that it would always be profitable for another outsider to join the coalition in the first period.

[67] We focus on SPNE whenever the Markov restriction is relaxed.

the first $\theta - 1$ periods, if all countries anticipate that an agreement will be reached by k_θ^* countries in period θ, yielding a payoff of

$$\pi_0 + \delta\pi_0 + \cdots + \delta^{\theta-2}\pi_0 + \delta^{\theta-1}V(k_\theta^*).$$

For this to be an equilibrium outcome, countries adopt strategies that lead to a sufficiently small coalition size (e.g., zero) in all periods $t < \theta$, so that even if an additional country would join the coalition in any of these periods, the coalition members still prefer not to sign a long-term agreement (anticipating that a more favorable outcome will be reached in period θ). Then obviously for an individual country that is assigned not to join the coalition in any of these periods, it is not profitable to deviate.

For instance, in period $\theta - 1$, the critical coalition size is $\hat{\tau}(k_\theta^*)$, so that for any $k_t < \hat{\tau}(k_\theta^*) - 1$, no individual country has an incentive to deviate and join as this does not lead to the signature of a long-term agreement in that period. This logic, of course, extends readily towards earlier periods, so that if k_θ^* and δ are sufficiently large, no agreement is signed in any period $t < \theta$ even when θ is a large number, assuming that countries adopt such *delay strategies*.[68] Relaxing the assumption $k_0 = 0$ only strengthens this point, so for the rest of this section, we drop this simplifying assumption.

We are now ready to state the main result of this section. The following proposition reveals that relaxing the Markov assumption does not support larger coalition sizes (see Appendix B.4 for a proof).[69]

Proposition 10. *In any SPNE (in pure strategies), the equilibrium coalition size satisfies* $k^* \leq \max\{k^{st}, \lceil \overline{k} \rceil - 1\}$.

Intuitively, why does a strategy that involves the threat to revert to a period (or a larger number of periods) of delay not help to sustain a more cooperative outcome in the first period of the game? The answer is that if a large number of countries ($k_1^* > \max\{k^{st}, \lceil \overline{k} \rceil - 1\}$) joins the coalition in the first period on the equilibrium path to avoid the punishment phase, then each of them realizes that after a deviation, the remaining $k_1^* - 1$ countries would sign an agreement in period 1 as well. This renders the deviation profitable, as internal stability is violated. Extending the length of the punishment phase cannot help to avoid this problem, because this only reduces the continuation value so that the critical coalition size in the first period needed to sign a long-term agreement is then even smaller. The largest equilibrium coalition size in any period is obtained under the most optimistic (rational) expectations about the coalition size in the following period (in case the next period is reached). Therefore, any threat to punish by future delay only

[68] Note, however, that any country not assigned as coalition member in some period $t < \theta$, weakly prefers to join the coalition. Furthermore, given the possibility to block an agreement (unanimity rule), a country can never end up being *trapped* in an unfavorable agreement.

[69] Note that if $\overline{k} > k^{st}$, then the inequality in Proposition 10 simplifies to $k^* < \overline{k}$.

makes countries more eager to sign an agreement today, which reduces the equilibrium coalition size. Such threats are, thus, ineffective in raising the equilibrium coalition size in our model.

B.3 Extension: Random vs. deterministic membership approach

In Section 2, we analyzed our dynamic coalition formation game under the random membership approach, whereas in Section 5, we focused on the opposite extreme case with persistent identities (deterministic membership approach). Here, we analyze a modified version of our model that represents an *intermediate approach* between deterministic and random membership. In particular, we assume that in each period, the ordering *either* remains the same as in the previous period (with probability q, where $q \in [0,1]$) *or* is chosen randomly among all orderings (with probability $1-q$). In the former case, the identities of the countries persist from the previous period.[70] In the latter case, each ordering is chosen with the same probability so that each country is assigned as coalition member with a probability of k/N.

Under the above specification, the probability of a country that is assigned (resp. not assigned) to become signatory in the current period, to be assigned as a member of a coalition with k countries in the next period (in case of delay) is

$$\begin{aligned} p_s(k) &= (1-q) \cdot 1 + q \cdot \frac{k}{N}, \\ p_n(k) &= q \cdot \frac{k}{N}. \end{aligned}$$

It follows that the expected discounted welfare from next period onwards of a currently assigned resp. non-assigned country, in case of delay, becomes

$$\begin{aligned} V_s(k) &= p_s(k)\Pi_s(k) + (1-p_s(k))\Pi_n(k) = \left(1 - q + q\frac{k}{N}\right)\Pi_s(k) + q \cdot \left(1 - \frac{k}{N}\right)\Pi_n(k), \\ V_n(k) &= p_n(k)\Pi_s(k) + (1-p_n(k))\Pi_n(k) = q \cdot \frac{k}{N}\Pi_s(k) + \left(1 - q \cdot \frac{k}{N}\right)\Pi_n(k). \end{aligned}$$

Note that for $q = 1$ we obtain the random membership case (see Section 2.1), where the above probabilities are identical and equal to $p_s(k) = p_n(k) = k/N$, and the expected values are identical and equal to $V_s(k) = V_n(k) = V(k)$ as given by (1). On the other hand, for $q = 0$, we obtain the deterministic membership case (see Section 5), where $p_s(k) = 1$, $p_n(k) = 0$, and $V_s(k) = \Pi_s(k)$, $V_n(k) = \Pi_n(k)$. Moreover, for all $q \in [0,1]$ and

[70] An alternative, but more complex, setup would be to assume that either a fixed ordering is applied or it is chosen randomly.

$k \in (k_0, N)$ it follows from the inequality $\Pi_s(k) \leq \Pi_n(k)$ that

$$\Pi_s(k) \leq V_s(k) \leq V_n(k) \leq \Pi_n(k). \tag{37}$$

Replacing $V(k)$ by $V_s(k)$, we can define the function τ in the same way as given by (23). The equilibrium conditions (for a coalition of size k^*) are then also analogous to the ones presented in Sections 2.2 and 5. For simplicity, we focus on the case where the number of assigned countries in every period is the same, irrespective of whether these identities are chosen randomly or not in the current period.[71] First, the condition for signing the agreement in the signature stage on the equilibrium path is identical to (5), with $V(k^*)$ replaced by $V_s(k^*)$, i.e.,

$$\Pi_s(k^*) \geq \pi_0 + \delta V_s(k^*). \tag{38}$$

Second, we consider external stability. Much like in the random and the deterministic membership case, the external stability condition remains the same as (ES) from the static model, i.e., $\Pi_n(k^*) \geq \Pi_s(k^*+1)$. To see this, observe that that (38) and the inequality $\Pi_n(k^*) \geq \Pi_s(k^*)$ yield $\Pi_n(k^*) \geq \pi_0 + \delta V_n(k^*)$.[72] Now consider a deviation of an outsider who joins the coalition. If (ES) holds, the deviation is not profitable, irrespective of whether the new coalition signs an agreement (yielding payoff $\Pi_s(k^*+1)$ to the deviator) or not sign an agreement (yielding payoff $\pi_0 + \delta V_n(k^*)$). On the other hand, if (ES) does not hold, then $\Pi_s(k^*+1) > \Pi_n(k^*) \geq \pi_0 + \delta V_n(k^*)$. In such a case, the new coalition of k^*+1 countries indeed signs an agreement and the deviation is profitable.

Finally, for internal stability, the same argument as under the random membership approach (in Section 2) applies. Namely, we obtain condition (IS) when the remaining countries sign an agreement after a deviation of an assigned country that does not join, i.e., when $k^* > \hat{\tau}(k^*)$, while it becomes $\Pi_s(k^*) \geq \pi_0 + \delta \Pi_s(k^*)$, which is now equivalent to (24), when the remaining countries do not sign an agreement, i.e., when $k^* = \hat{\tau}(k^*)$.

Summing up, Proposition 1 still applies and we obtain the same equilibrium conditions as under the random membership approach, namely (7) or (8), with adjusted functions V and τ (as indicated above). It then follows that Propositions 2, 3, and 4 continue to hold with the redefined function τ. Note also that the value k^{st} is derived from the static game, and is thus independent of the probability q. Furthermore, Proposition 5(i) also continues to hold and by the same proof we also obtain that $\underline{k}$ is increasing in q.

In the following we investigate, whether the results on $\overline{k}$ under this *intermediate approach* are also intermediate (for $q \in (0,1)$) between the results under the random

[71] We believe that this is without loss of generality, because what matters for the stability of today's coalition is the continuation payoff in case of a delay, and this is an expected value that does not depend on the type of the membership approach applied in the current period.

[72] This follows from a straightforward computation: $[\Pi_n(k) - \delta V_s(k)] - [\Pi_s(k) - \delta V_s(k)] = [\Pi_n(k) - \Pi_s(k)]\,[1 - \delta(1-q)] \geq 0$, since $\delta, q \in [0,1]$.

and the deterministic membership approach (as we would expect), or if some additional effects may arise. We content ourselves with an analysis based on our simplest example, Example 1 from Section 3. Using the functional forms for this example, (11), and noting that $\pi_0 = 0$, it is straight-forward to show that:

$$\tau(k) = \sqrt{\delta}\, k \sqrt{1 + q\left(1 - \frac{k}{N}\right)}.$$

Note that for $q = 0$, we again obtain $\tau(k) = \sqrt{\delta}\, k$, the same result that we found for the deterministic membership case in Section 5 for this example, and for $q = 1$, we get $\tau(k) = k\sqrt{\delta(2 - k/N)}$, the same result that we found in Section 3 for the random membership case.

A simple numerical investigation shows that there are no novel effects. The results are indeed intermediate between the random and the deterministic membership approach. For example, for $\delta = 0.7$ and $N = 10$, we obtain $\overline{k} = 6.12$ if $q = 0$, so that the largest equilibrium coalition size under the deterministic membership approach is $k^* = 6$. For $q = 1$, the largest equilibrium coalition size is $k^* = 8$.[73] For $q = 1/4$, the largest equilibrium coalition size is $k^* = 7$, i.e., intermediate between the random and deterministic membership approach.

B.4 Proofs for Appendix B

Proof of Proposition 9. (i) Consider the case $k^{st} \geq \tau(k^{st})$. As argued above the proposition, upon arriving in the final period with negotiations, $t = T$, the equilibrium coalition size is $k^*_T = k^{st}$. Much like in Proposition 2, in period $T - 1$ (if this period is reached), k^{st} countries are willing to sign an agreement. Thus, $k^*_{T-1} = k^{st}$ and the same arguments can readily be applied also to all other periods $t < T - 1$. This completes the proof of (i).

(ii) Before proceeding with the proof we state the following lemma. Its proof follows below the proof of Proposition 9.

Lemma 5. *Assume that $k^{st} < \underline{k}$. Consider the following sequence defined recursively:*[74]

$$l_0 = k^{st} = \lceil \tilde{k} \rceil, \qquad l_\beta = \hat{\tau}(l_{\beta-1}) \quad \text{for } \beta = 1, 2, \ldots. \tag{39}$$

Then there is some $\theta \geq 0$ such that $l_0 < l_1 < \cdots < l_{\theta-1} < l_\theta = l_{\theta+1} = \cdots = \lceil \underline{k} \rceil$.

[73]Note that under the deterministic membership approach, the upper bound on the stable coalition size, $\overline{k}$, does not depend on the number of countries, unless it is equal to N (corner solution). By contrast, under the random membership approach, $\overline{k}$ depends on N. Hence, there is no unambiguous ranking of the maximum stable coalition size in these two cases.

[74]We use the subscript β for counting *backwards* in time (see below).

Now we show that $k_t^* = l_{T-t}$ for $t = 1, 2, \dots, T$. The proof proceeds in the same way as the argument preceding the proposition. As argued there, $k_T^* = \lceil \tilde{k} \rceil = l_0$. For any $t \leq T-1$, if the countries in period t anticipate that $k_{t+1}^* = l_{T-t-1}$ countries sign an agreement in the next period, then $k_t^* \geq \hat{\tau}(k_{t+1}) = \hat{\tau}(l_{T-t-1}) = l_{T-t}$. Thus, the countries prefer to sign the agreement in period t (when this period is reached). In addition, since $k_t^* \geq k^{st}$ for all t, external stability is satisfied in all periods.

Now consider internal stability. Similarly as in the arguments preceding Proposition 2 and 3, we distinguish two cases: Either $k_t^* = \hat{\tau}(k_{t+1}^*)$ or $k_t^* > \hat{\tau}(k_{t+1}^*)$. We show that the former case applies. Otherwise, if $k_t^* > \hat{\tau}(k_{t+1}^*)$, the coalition size k_t^* needs to satisfy both the external and internal stability conditions (ES) and (IS), and would thus be an equilibrium coalition size of the static game (i.e., $k_t^* = l_0$). This is a contradiction, since $k_t^* > \hat{\tau}(k_{t+1}^*) = \hat{\tau}(l_{T-t-1}) = l_{T-t} \geq l_0 = \lceil \tilde{k} \rceil = k^{st}$. Thus, indeed the former case applies, which yields $k_t^* = \hat{\tau}(k_{t+1}^*) = \hat{\tau}(l_{T-t-1}) = l_{T-t}$.

In order to complete the proof of (ii), it is sufficient to set $T > \theta + 1$, where θ is introduced in Lemma 5. Then $k_1^* = l_{T-1} = l_\theta = \lceil \underline{k} \rceil$. □

Proof of Lemma 5. Before proceeding with the actual proof, recall that due to Assumption 5, $k_0 < k < \underline{k}$ implies $k < \tau(k) < \underline{k}$.

First, we show that $l_{\beta-1} < l_\beta \leq \lceil \underline{k} \rceil$ when $l_{\beta-1} < \lceil \underline{k} \rceil$. Since $l_{\beta-1}$ is an integer, the inequality $l_{\beta-1} < \lceil \underline{k} \rceil$ implies $l_{\beta-1} < \underline{k}$. Then it follows that $l_{\beta-1} < \tau(l_{\beta-1}) < \underline{k}$. Since $l_\beta = \hat{\tau}(l_{\beta-1}) = \lceil \tau(l_{\beta-1}) \rceil$, we obtain $l_{\beta-1} < l_\beta \leq \lceil \underline{k} \rceil$.

Next, we show that $l_\beta = l_{\beta-1}$ when $l_{\beta-1} = \lceil \underline{k} \rceil$. Since $l_{\beta-1}$ is a positive integer and $l_{\beta-1} \in [\underline{k}, \overline{k})$, it follows from the discussion preceding Proposition 4 that $l_{\beta-1}$ is a fixed point of $\hat{\tau}$. Thus, $l_\beta = \hat{\tau}(l_{\beta-1}) = l_{\beta-1}$.

Summing up, since $l_0 = k^{st} < \lceil \underline{k} \rceil$, the sequence $l_0, l_1, l_2, \dots$ is bounded from above by $\lceil \underline{k} \rceil$ and is increasing before attaining this bound. Let us set θ such that $l_{\theta-1} < \lceil \underline{k} \rceil = l_\theta$. Then $l_\beta = l_\theta = \lceil \underline{k} \rceil$ for $\beta \geq \theta$, which completes the proof. □

Proof of Proposition 10. Proof by contradiction. Let k^{max} be the largest equilibrium coalition size in the full set of SPNE (in pure strategies), and suppose (to the contrary of the statement in the proposition) that $k^{max} > \max\{k^{st}, \lceil \overline{k} \rceil - 1\}$, which is equivalent to $k^{max} \geq \max\{k^{st} + 1, \overline{k}\}$.

Now consider an equilibrium where a coalition of $k_t = k^{max}$ countries signs an agreement at some stage t. We show that there there is a profitable deviation not to join the coalition for some member. If the remaining $k_t - 1$ coalition members do not sign an agreement, then $V(k^{max})$ is the maximal payoff the deviating country can expect in the next round. Thus, the payoff of each country after such a deviation is at most $\pi_0 + \delta V(k^{max})$. However, since $k_t = k^{max} \geq \overline{k}$, we have $\Pi_s(k_t - 1) \geq \pi_0 + \delta V(k^{max})$ and thus, the remaining countries would sign an agreement in period t. However, anticipating that the remaining countries sign an agreement, not joining the coalition is indeed a profitable deviation, since $k_t = k^{max} > k^{st}$ violates internal stability. $\square$